EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 09

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If the fundamental period of $f(x)$ is 6, the period of $-f(x)$ is

A. 3 **B.** 12 **C.** 6 **D.** $\frac{1}{6}$

2. The constant function $f(x) = 9$ is

A. periodic, with no fundamental period

B. periodic, with fundamental period 1

C. periodic, with fundamental period 9

D. not periodic

3. Among $\sin x$, $\sec x$, $|\operatorname{cosec} x|$ and $\cos x$, the function with period π is

A. $\sin x$ **B.** $\cos x$ **C.** $|\operatorname{cosec} x|$ **D.** $\sec x$

4. The period of $\cot x$ is

A. 4π **B.** $\frac{\pi}{2}$ **C.** 2π **D.** π

5. The period of $\cos \frac{4x}{5}$ is

A. $\frac{8\pi}{5}$ **B.** 2π **C.** $\frac{5\pi}{2}$ **D.** $\frac{5\pi}{4}$

6. The period of $\operatorname{cosec}\left(6x + \frac{\pi}{3}\right)$ is

A. 2π **B.** $\frac{\pi}{3}$ **C.** $\frac{\pi}{6}$ **D.** 12π

7. The period of $\sec \frac{x}{3}$ is

A. 2π **B.** 6π **C.** $\frac{2\pi}{3}$ **D.** 3π

8. The period of $\cot 3x - \operatorname{cosec} 4x$ is

A. π **B.** $\frac{\pi}{6}$ **C.** $\frac{5\pi}{6}$ **D.** $\frac{\pi}{3}$

9. Among $\cot x + \sec 2\pi x$, $\tan 3x + \tan 4x$, $\cot \frac{x}{2} + \sec 4x$ and $\sin x + \sec 3x$, the function that is NOT periodic is

A. $\sin x + \sec 3x$ **B.** $\cot \frac{x}{2} + \sec 4x$

C. $\tan 3x + \tan 4x$ **D.** $\cot x + \sec 2\pi x$

10. The period of $\tan\frac{2x}{3} - \sin 2x$ is

A. $\frac{\pi}{2}$ **B.** $\frac{5\pi}{2}$ **C.** $\frac{3\pi}{2}$ **D.** 3π

11. The period of $\cot^3 x$ is

A. π **B.** $\frac{\pi}{3}$ **C.** 3π **D.** 2π

12. The period of $\operatorname{cosec}^6\frac{x}{3}$ is

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{3}$ **C.** 6π **D.** 3π

13. The period of $4\operatorname{cosec}^4 x + 4\sec^4 x$ is

A. $\frac{\pi}{4}$ **B.** 2π **C.** π **D.** $\frac{\pi}{2}$

14. The period of $3\sin^6 x + 6\cos^6 x$ is

A. 2π **B.** π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

15. The period of $3|\tan x| + 3|\cot x|$ is

A. 2π **B.** π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

16. The period of $5x - [5x]$ is

A. 1 **B.** $\frac{1}{5}$ **C.** 5 **D.** $\frac{2}{5}$

17. Among $\sin x^2$, $\tan\frac{x}{2}$, $|\cos 3x|$ and $\sin^2 x$, the function that is NOT periodic is

A. $|\cos 3x|$ **B.** $\tan\frac{x}{2}$

C. $\sin x^2$ **D.** $\sin^2 x$

18. The range of $2 + 3|\sin x|$ is

A. $[0, 3]$ **B.** $[2, 5]$ **C.** $[-2, 1]$ **D.** $[-1, 2]$

19. The range of $-2 - 2\sin^4 x$ is

A. $[0, 2]$ **B.** $[-4, -2]$ **C.** $[-2, -1]$ **D.** $[-2, 0]$

20. The maximum value of $6\cos x + 8\sin x$ is

A. 10 **B.** -10 **C.** 100 **D.** 14

21. The minimum value of $7\cos x + 24\sin x - 3$ is

A. -25 **B.** -34 **C.** -28 **D.** 22

22. The range of $\dfrac{1}{-7\cos x + 24\sin x + 23}$ is

A. $[-\frac{1}{25}, \frac{1}{25}]$ **B.** $[-2, 48]$

C. $(-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$ **D.** $[-\frac{1}{2}, \frac{1}{48}]$

23. The range of $\dfrac{1}{-15\cos x - 8\sin x + 21}$ is

A. $[\frac{1}{38}, \infty)$ **B.** $[\frac{1}{38}, \frac{1}{4}]$

C. $[-\frac{1}{17}, \frac{1}{17}]$ **D.** $[4, 38]$

24. The minimum value of $49\sin^2\theta + 16\,\mathrm{cosec}^2\,\theta$ is

A. 56 **B.** 65 **C.** 22 **D.** 28

25. The minimum value of $25\cos^2\theta + 4\sec^2\theta$ is

A. 20 **B.** 29 **C.** 14 **D.** 10

Key sheet

1. C	**2.** A	**3.** C	**4.** D	**5.** C
6. B	**7.** B	**8.** A	**9.** D	**10.** D
11. A	**12.** D	**13.** D	**14.** B	**15.** D
16. B	**17.** C	**18.** B	**19.** B	**20.** A
21. C	**22.** C	**23.** B	**24.** A	**25.** A

Solutions

1. If the fundamental period of $f(x)$ is 6, the period of $-f(x)$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is 6.

Ans: (C) 6

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2. The constant function $f(x) = 9$ is

$f(x+p) = f(x)$ for every $p > 0$, so f is periodic.

There is no least positive p, so it has no fundamental period.

Ans: (A) periodic, with no fundamental period

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3. Among $\sin x$, $\sec x$, $|\operatorname{cosec} x|$ and $\cos x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $|\operatorname{cosec} x|$ has period π.

Ans: (C) $|\operatorname{cosec} x|$

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4. The period of $\cot x$ is

$\tan$ and $\cot$ repeat after π.

Period $= \pi$.

Ans: (D) π

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5. The period of $\cos \frac{4x}{5}$ is

Period of $\cos x$ is 2π; divide by $|a| = \frac{4}{5}$.

Period $= \frac{5\pi}{2}$.

Ans: (C) $\frac{5\pi}{2}$

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6. The period of $\operatorname{cosec}\left(6x + \frac{\pi}{3}\right)$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = 6$.

Period $= \frac{\pi}{3}$.

Ans: (B) $\frac{\pi}{3}$

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7. The period of $\sec \frac{x}{3}$ is

Period of $\sec x$ is 2π; divide by $|a| = \frac{1}{3}$.

Period $= 6\pi$.

Ans: (B) 6π

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8. The period of $\cot 3x - \operatorname{cosec} 4x$ is

Periods: $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

L.C.M. $= \pi$.

Ans: (A) π

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9. Among $\cot x + \sec 2\pi x$, $\tan 3x + \tan 4x$, $\cot \frac{x}{2} + \sec 4x$ and $\sin x + \sec 3x$, the function that is NOT periodic is

In $\cot x + \sec 2\pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (D) $\cot x + \sec 2\pi x$

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10. The period of $\tan \frac{2x}{3} - \sin 2x$ is

Periods: $\frac{3\pi}{2}$ and π.
L.C.M. $= 3\pi$.

Ans: (D) 3π

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11. The period of $\cot^3 x$ is

$\cot^3 x$ has period π.
So the period is π.

Ans: (A) π

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12. The period of $\operatorname{cosec}^6 \frac{x}{3}$ is

$\operatorname{cosec}^6 x$ has period π.
So the period is 3π.

Ans: (D) 3π

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13. The period of $4\operatorname{cosec}^4 x + 4\sec^4 x$ is

Table: $n = 4$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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14. The period of $3\sin^6 x + 6\cos^6 x$ is

Table: $n = 6$, coefficients unequal.
Period $= \pi$.

Ans: (B) π

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15. The period of $3|\tan x| + 3|\cot x|$ is

Absolute values with equal coefficients.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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16. The period of $5x - [5x]$ is

$x - [x]$ has period 1.
So $5x - [5x]$ has period $\frac{1}{5}$.

Ans: (B) $\frac{1}{5}$

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17. Among $\sin x^2$, $\tan \frac{x}{2}$, $|\cos 3x|$ and $\sin^2 x$, the function that is NOT periodic is

The zeros of $\sin x^2$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (C) $\sin x^2$

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18. The range of $2 + 3|\sin x|$ is

$|\sin x|$ has range $[0, 1]$.

So $2 + 3|\sin x|$ has range $[2, 5]$.

Ans: (B) $[2, 5]$

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19. The range of $-2 - 2\sin^4 x$ is

$\sin^4 x$ has range $[0, 1]$.

So $-2 - 2\sin^4 x$ has range $[-4, -2]$.

Ans: (B) $[-4, -2]$

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20. The maximum value of $6\cos x + 8\sin x$ is

$\sqrt{a^2 + b^2} = 10$

Maximum $= 0 + 10 = 10$.

Ans: (A) 10

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21. The minimum value of $7\cos x + 24\sin x - 3$ is

$\sqrt{a^2 + b^2} = 25$

Minimum $= -3 - 25 = -28$.

Ans: (C) -28

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22. The range of $\dfrac{1}{-7\cos x + 24\sin x + 23}$ is

The denominator has range $[-2, 48]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$.

Ans: (C) $(-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$

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23. The range of $\dfrac{1}{-15\cos x - 8\sin x + 21}$ is

The denominator has range $[4, 38]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{38}, \frac{1}{4}]$.

Ans: (B) $[\frac{1}{38}, \frac{1}{4}]$

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24. The minimum value of $49\sin^2\theta + 16\,\mathrm{cosec}^2\,\theta$ is

A.M. $\geq$ G.M. gives ≥ 56, and equality needs $\sin^2\theta = \frac{4}{7}$.

Here $\frac{4}{7} \leq 1$, so the minimum is 56.

Ans: (A) 56

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25. The minimum value of $25\cos^2\theta + 4\sec^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 20, and equality needs $\cos^2\theta = \frac{2}{5}$.

Here $\frac{2}{5} \leq 1$, so the minimum is 20.

Ans: (A) 20