EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 08

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The constant function $f(x) = -4$ is

A. periodic, with fundamental period 1

B. periodic, with no fundamental period

C. periodic, with fundamental period 4

D. not periodic

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2. If $f(x+6) = f(x)$ for all real x and $f(4) = -4$, then $f(28) =$

A. 20 **B.** -4 **C.** 2 **D.** -16

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3. Among $\cos x$, $|\cot x|$, $\operatorname{cosec} x$ and $\sin x$, the function with period π is

A. $|\cot x|$ **B.** $\cos x$ **C.** $\operatorname{cosec} x$ **D.** $\sin x$

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4. The fundamental period of $|\cot x|$ is

A. 2π **B.** π **C.** $\frac{\pi}{2}$ **D.** 4π

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5. The period of $\cot\left(6x + \frac{\pi}{6}\right)$ is

A. 6π **B.** π **C.** $\frac{\pi}{3}$ **D.** $\frac{\pi}{6}$

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6. The period of $\tan\left(7x + \frac{\pi}{4}\right)$ is

A. 7π **B.** $\frac{\pi}{7}$ **C.** $\frac{2\pi}{7}$ **D.** π

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7. The period of $\operatorname{cosec}\left(\frac{x}{3} - \frac{\pi}{4}\right)$ is

A. 2π **B.** $\frac{2\pi}{3}$ **C.** 6π **D.** 3π

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8. The period of $\cot 2x + \cot 4x$ is

A. $\frac{\pi}{8}$ **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{3\pi}{4}$

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9. Among $\operatorname{cosec} x + \cot 2\pi x$, $\tan 4x + \tan 2x$, $\cos 3x + \cot x$ and $\sin 2x + \sec\frac{x}{2}$, the function that is NOT periodic is

A. $\sin 2x + \sec\frac{x}{2}$ **B.** $\tan 4x + \tan 2x$

C. $\operatorname{cosec} x + \cot 2\pi x$ **D.** $\cos 3x + \cot x$

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10. The period of $\cos\frac{3x}{2}+\sin 5x$ is

A. $\frac{2\pi}{15}$ **B.** $\frac{26\pi}{15}$ **C.** $\frac{8\pi}{15}$ **D.** 4π

11. The period of $\sec^3 2x$ is

A. π **B.** 4π **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{3}$

12. The period of $\tan^2\frac{x}{3}$ is

A. $\frac{\pi}{3}$ **B.** 6π **C.** 3π **D.** $\frac{3\pi}{2}$

13. The period of $\sin^6 x+\cos^6 x$ is

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{4}$ **C.** π **D.** 2π

14. The period of $3\tan^2 x+5\cot^2 x$ is

A. 2π **B.** π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

15. The period of $4|\operatorname{cosec} x|+5|\sec x|$ is

A. π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** 2π

16. The period of $\frac{3x}{2}-\left[\frac{3x}{2}\right]$ is

A. $\frac{2}{3}$ **B.** 1 **C.** $\frac{4}{3}$ **D.** $\frac{3}{2}$

17. Among $\sec^3 x$, $x-[x]$, $\sin x^2$ and $|\tan x|+|\cot x|$, the function that is NOT periodic is

A. $\sec^3 x$ **B.** $x-[x]$

C. $|\tan x|+|\cot x|$ **D.** $\sin x^2$

18. The range of $4+3\sin^5 x$ is

A. $[1,7]$ **B.** $[3,5]$ **C.** $[-3,3]$ **D.** $[-7,-1]$

19. The range of $2+3\cos x$ is

A. $[1,3]$ **B.** $[-5,1]$ **C.** $[-1,5]$ **D.** $[-3,3]$

20. The minimum value of $24\cos x+7\sin x+2$ is

A. 27 **B.** -29 **C.** -25 **D.** -23

21. The range of $4\cos x+3\sin x-2$ is

A. $[-5,5]$ **B.** $[-9,5]$ **C.** $[-3,-1]$ **D.** $[-7,3]$

22. The range of $\dfrac{1}{-24\cos x+7\sin x+29}$ is

A. $[4,54]$ **B.** $[\frac{1}{54},\infty)$

C. $[\frac{1}{54},\frac{1}{4}]$ **D.** $[-\frac{1}{25},\frac{1}{25}]$

23. The range of $\dfrac{1}{-8\cos x+15\sin x+1}$ is

A. $[-\frac{1}{17},\frac{1}{17}]$ **B.** $[-16,18]$

C. $(-\infty,-\frac{1}{16}]\cup[\frac{1}{18},\infty)$ **D.** $[-\frac{1}{16},\frac{1}{18}]$

24. The minimum value of $36\cos^2\theta + 25\sec^2\theta$ is

A. 30 **B.** 22 **C.** 61 **D.** 60

25. The minimum value of $49\tan\theta + 1\cot\theta$ for acute θ is

A. 7 **B.** 50 **C.** 16 **D.** 14

Key sheet

1. B	**2.** B	**3.** A	**4.** B	**5.** D
6. B	**7.** C	**8.** B	**9.** C	**10.** D
11. A	**12.** C	**13.** A	**14.** B	**15.** A
16. A	**17.** D	**18.** A	**19.** C	**20.** D
21. D	**22.** C	**23.** C	**24.** D	**25.** D

Solutions

1. The constant function $f(x) = -4$ is

$f(x+p) = f(x)$ for every $p > 0$, so f is periodic.
There is no least positive p, so it has no fundamental period.

Ans: (B) periodic, with no fundamental period

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2. If $f(x+6) = f(x)$ for all real x and $f(4) = -4$, then $f(28) =$

$28 = 4 + 4 \times 6$
$f(28) = f(4) = -4$

Ans: (B) -4

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3. Among $\cos x$, $|\cot x|$, $\operatorname{cosec} x$ and $\sin x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.
So $|\cot x|$ has period π.

Ans: (A) $|\cot x|$

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4. The fundamental period of $|\cot x|$ is

Taking the absolute value folds the graph, so it repeats after π.
Period $= \pi$.

Ans: (B) π

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5. The period of $\cot\left(6x + \frac{\pi}{6}\right)$ is

Period of $\cot x$ is π; divide by $|a| = 6$.
Period $= \frac{\pi}{6}$.

Ans: (D) $\frac{\pi}{6}$

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6. The period of $\tan\left(7x + \frac{\pi}{4}\right)$ is

Period of $\tan x$ is π; divide by $|a| = 7$.
Period $= \frac{\pi}{7}$.

Ans: (B) $\frac{\pi}{7}$

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7. The period of $\operatorname{cosec}\left(\frac{x}{3} - \frac{\pi}{4}\right)$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = \frac{1}{3}$.
Period $= 6\pi$.

Ans: (C) 6π

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8. The period of $\cot 2x + \cot 4x$ is

Periods: $\frac{\pi}{2}$ and $\frac{\pi}{4}$.
L.C.M. $= \frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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9. Among $\operatorname{cosec} x + \cot 2\pi x$, $\tan 4x + \tan 2x$, $\cos 3x + \cot x$ and $\sin 2x + \sec \frac{x}{2}$, the function that is NOT periodic is

In $\operatorname{cosec} x + \cot 2\pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (C) $\operatorname{cosec} x + \cot 2\pi x$

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10. The period of $\cos \frac{3x}{2} + \sin 5x$ is

Periods: $\frac{4\pi}{3}$ and $\frac{2\pi}{5}$.
L.C.M. $= 4\pi$.

Ans: (D) 4π

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11. The period of $\sec^3 2x$ is

$\sec^3 x$ has period 2π.
So the period is π.

Ans: (A) π

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12. The period of $\tan^2 \frac{x}{3}$ is

$\tan^2 x$ has period π.
So the period is 3π.

Ans: (C) 3π

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13. The period of $\sin^6 x + \cos^6 x$ is

Table: $n = 6$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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14. The period of $3\tan^2 x + 5\cot^2 x$ is

Table: $n = 2$, coefficients unequal.
Period $= \pi$.

Ans: (B) π

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15. The period of $4|\operatorname{cosec} x| + 5|\sec x|$ is

Absolute values with unequal coefficients.
Period $= \pi$.

Ans: (A) π

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16. The period of $\frac{3x}{2} - [\frac{3x}{2}]$ is

$x - [x]$ has period 1.
So $\frac{3x}{2} - [\frac{3x}{2}]$ has period $\frac{1}{\frac{3}{2}} = \frac{2}{3}$.

Ans: (A) $\frac{2}{3}$

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17. Among $\sec^3 x$, $x - [x]$, $\sin x^2$ and $|\tan x| + |\cot x|$, the function that is NOT periodic is

The zeros of $\sin x^2$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (D) $\sin x^2$

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18. The range of $4 + 3\sin^5 x$ is

$\sin^5 x$ has range $[-1, 1]$.

So $4 + 3\sin^5 x$ has range $[1, 7]$.

Ans: (A) $[1, 7]$

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19. The range of $2 + 3\cos x$ is

$\cos x$ has range $[-1, 1]$.

So $2 + 3\cos x$ has range $[-1, 5]$.

Ans: (C) $[-1, 5]$

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20. The minimum value of $24\cos x + 7\sin x + 2$ is

$\sqrt{a^2 + b^2} = 25$

Minimum $= 2 - 25 = -23$.

Ans: (D) -23

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21. The range of $4\cos x + 3\sin x - 2$ is

$\sqrt{16 + 9} = 5$

Range $= [-7, 3]$.

Ans: (D) $[-7, 3]$

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22. The range of $\dfrac{1}{-24\cos x + 7\sin x + 29}$ is

The denominator has range $[4, 54]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{54}, \frac{1}{4}]$.

Ans: (C) $[\frac{1}{54}, \frac{1}{4}]$

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23. The range of $\dfrac{1}{-8\cos x + 15\sin x + 1}$ is

The denominator has range $[-16, 18]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{16}] \cup [\frac{1}{18}, \infty)$.

Ans: (C) $(-\infty, -\frac{1}{16}] \cup [\frac{1}{18}, \infty)$

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24. The minimum value of $36\cos^2\theta + 25\sec^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 60, and equality needs $\cos^2\theta = \frac{5}{6}$.

Here $\frac{5}{6} \leq 1$, so the minimum is 60.

Ans: (D) 60

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25. The minimum value of $49\tan\theta + 1\cot\theta$ for acute θ is

$\geq 2\sqrt{49 \cdot 1} = 14$

Equality is possible, so the minimum is 14.

Ans: (D) 14