EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 07

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $f(x+3) = f(x)$ for all real x and $f(-2) = 8$, then $f(-11) =$

A. -1 **B.** 11 **C.** 8 **D.** -24

2. If the fundamental period of $f(x)$ is $\frac{2\pi}{9}$, the period of $\frac{1}{f(x)}$ is

A. $\frac{\pi}{9}$ **B.** $\frac{4\pi}{9}$ **C.** $\frac{9}{2\pi}$ **D.** $\frac{2\pi}{9}$

3. Among $\cos x$, $\sec x$, $|\operatorname{cosec} x|$ and $\sin x$, the function with period π is

A. $\cos x$ **B.** $|\operatorname{cosec} x|$ **C.** $\sec x$ **D.** $\sin x$

4. The fundamental period of $|\sin x|$ is

A. π **B.** 4π **C.** 2π **D.** $\frac{\pi}{2}$

5. The period of $\cot\left(\frac{x}{3} + 3\right)$ is

A. 3π **B.** 6π **C.** π **D.** $\frac{\pi}{3}$

6. The period of $\sec\left(5x - \frac{\pi}{4}\right)$ is

A. 2π **B.** $\frac{\pi}{5}$ **C.** $\frac{2\pi}{5}$ **D.** 10π

7. The period of $\cos\left(\frac{x}{3} + \frac{\pi}{4}\right)$ is

A. 6π **B.** 3π **C.** 2π **D.** $\frac{2\pi}{3}$

8. The period of $\sin\frac{3x}{2} - \sec 5x$ is

A. 4π **B.** $\frac{26\pi}{15}$ **C.** $\frac{8\pi}{15}$ **D.** $\frac{2\pi}{15}$

9. Among $\cos x + \sin\frac{x}{2}$, $\tan 3x + \operatorname{cosec}\frac{x}{3}$, $\tan 3x + \cot 2\pi x$ and $\sin\frac{x}{3} + \tan 2x$, the function that is NOT periodic is

A. $\sin\frac{x}{3} + \tan 2x$ **B.** $\tan 3x + \cot 2\pi x$

C. $\tan 3x + \operatorname{cosec}\frac{x}{3}$ **D.** $\cos x + \sin\frac{x}{2}$

10. The period of $\operatorname{cosec} 4x + \tan\frac{x}{2}$ is

A. $\frac{\pi}{2}$ **B.** π **C.** $\frac{5\pi}{2}$ **D.** 2π

11. The period of $\sin^7 3x$ is

A. $\frac{\pi}{3}$ **B.** 6π **C.** $\frac{2\pi}{21}$ **D.** $\frac{2\pi}{3}$

12. The period of $\cot^4 2x$ is

A. π **B.** $\frac{\pi}{8}$ **C.** $\frac{\pi}{2}$ **D.** 2π

13. The period of $4\sin^6 x + 4\cos^6 x$ is

A. $\frac{\pi}{2}$ **B.** 2π **C.** π **D.** $\frac{\pi}{4}$

14. The period of $2\tan^6 x + 4\cot^6 x$ is

A. 2π **B.** π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

15. The period of $5|\operatorname{cosec} x| + 6|\sec x|$ is

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{4}$ **C.** π **D.** 2π

16. The period of $\frac{x}{3} - [\frac{x}{3}]$ is

A. 6 **B.** $\frac{1}{3}$ **C.** 1 **D.** 3

17. Among $x - [x]$, $\sin x + \cos 2x$, $|\cos 3x|$ and $\sin x^2$, the function that is NOT periodic is

A. $x - [x]$ **B.** $\sin x + \cos 2x$

C. $|\cos 3x|$ **D.** $\sin x^2$

18. The range of $3 - 2\sin x$ is

A. $[2, 4]$ **B.** $[-5, -1]$ **C.** $[1, 5]$ **D.** $[-2, 2]$

19. The range of $4 + 4|\cos x|$ is

A. $[-4, 0]$ **B.** $[4, 8]$ **C.** $[4, 5]$ **D.** $[0, 4]$

20. The range of $6\cos x + 8\sin x - 2$ is

A. $[-3, -1]$ **B.** $[-10, 10]$ **C.** $[-12, 8]$ **D.** $[-16, 12]$

21. The maximum value of $4\cos x + 3\sin x + 2$ is

A. 5 **B.** -3 **C.** 9 **D.** 7

22. The range of $\dfrac{1}{-15\cos x + 8\sin x + 15}$ is

A. $[-2, 32]$ **B.** $(-\infty, -\frac{1}{2}] \cup [\frac{1}{32}, \infty)$

C. $[-\frac{1}{2}, \frac{1}{32}]$ **D.** $[-\frac{1}{17}, \frac{1}{17}]$

23. The range of $\dfrac{1}{8\cos x + 6\sin x + 1}$ is

A. $[-\frac{1}{10}, \frac{1}{10}]$ **B.** $[-\frac{1}{9}, \frac{1}{11}]$

C. $(-\infty, -\frac{1}{9}] \cup [\frac{1}{11}, \infty)$ **D.** $[-9, 11]$

24. The minimum value of $25\tan\theta + 1\cot\theta$ for acute θ is

A. 26 **B.** 10 **C.** 12 **D.** 5

25. The minimum value of $4\sin^2\theta + 36\operatorname{cosec}^2\theta$ is

A. 40 **B.** 24 **C.** 16 **D.** 12

Key sheet

1. C	**2.** D	**3.** B	**4.** A	**5.** A
6. C	**7.** A	**8.** A	**9.** B	**10.** D
11. D	**12.** C	**13.** A	**14.** B	**15.** C
16. D	**17.** D	**18.** C	**19.** B	**20.** C
21. D	**22.** B	**23.** C	**24.** B	**25.** A

Solutions

1. If $f(x+3) = f(x)$ for all real x and $f(-2) = 8$, then $f(-11) =$

$-11 = -2 - 3 \times 3$

$f(-11) = f(-2) = 8$

Ans: (C) 8

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2. If the fundamental period of $f(x)$ is $\frac{2\pi}{9}$, the period of $\frac{1}{f(x)}$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is $\frac{2\pi}{9}$.

Ans: (D) $\frac{2\pi}{9}$

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3. Among $\cos x$, $\sec x$, $|\operatorname{cosec} x|$ and $\sin x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $|\operatorname{cosec} x|$ has period π.

Ans: (B) $|\operatorname{cosec} x|$

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4. The fundamental period of $|\sin x|$ is

Taking the absolute value folds the graph, so it repeats after π.

Period $= \pi$.

Ans: (A) π

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5. The period of $\cot\left(\frac{x}{3} + 3\right)$ is

Period of $\cot x$ is π; divide by $|a| = \frac{1}{3}$.

Period $= 3\pi$.

Ans: (A) 3π

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6. The period of $\sec\left(5x - \frac{\pi}{4}\right)$ is

Period of $\sec x$ is 2π; divide by $|a| = 5$.

Period $= \frac{2\pi}{5}$.

Ans: (C) $\frac{2\pi}{5}$

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7. The period of $\cos\left(\frac{x}{3} + \frac{\pi}{4}\right)$ is

Period of $\cos x$ is 2π; divide by $|a| = \frac{1}{3}$.

Period $= 6\pi$.

Ans: (A) 6π

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8. The period of $\sin\frac{3x}{2} - \sec 5x$ is

Periods: $\frac{4\pi}{3}$ and $\frac{2\pi}{5}$.

L.C.M. $= 4\pi$.

Ans: (A) 4π

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9. Among $\cos x + \sin \frac{x}{2}$, $\tan 3x + \operatorname{cosec} \frac{x}{3}$, $\tan 3x + \cot 2\pi x$ and $\sin \frac{x}{3} + \tan 2x$, the function that is NOT periodic is

In $\tan 3x + \cot 2\pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (B) $\tan 3x + \cot 2\pi x$

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10. The period of $\operatorname{cosec} 4x + \tan \frac{x}{2}$ is

Periods: $\frac{\pi}{2}$ and 2π.
L.C.M. $= 2\pi$.

Ans: (D) 2π

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11. The period of $\sin^7 3x$ is

$\sin^7 x$ has period 2π.
So the period is $\frac{2\pi}{3}$.

Ans: (D) $\frac{2\pi}{3}$

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12. The period of $\cot^4 2x$ is

$\cot^4 x$ has period π.
So the period is $\frac{\pi}{2}$.

Ans: (C) $\frac{\pi}{2}$

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13. The period of $4\sin^6 x + 4\cos^6 x$ is

Table: $n = 6$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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14. The period of $2\tan^6 x + 4\cot^6 x$ is

Table: $n = 6$, coefficients unequal.
Period $= \pi$.

Ans: (B) π

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15. The period of $5|\operatorname{cosec} x| + 6|\sec x|$ is

Absolute values with unequal coefficients.
Period $= \pi$.

Ans: (C) π

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16. The period of $\frac{x}{3} - \left[\frac{x}{3}\right]$ is

$x - [x]$ has period 1.
So $\frac{x}{3} - \left[\frac{x}{3}\right]$ has period $\frac{1}{\frac{1}{3}} = 3$.

Ans: (D) 3

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17. Among $x - [x]$, $\sin x + \cos 2x$, $|\cos 3x|$ and $\sin x^2$, the function that is NOT periodic is

The zeros of $\sin x^2$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (D) $\sin x^2$

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18. The range of $3 - 2\sin x$ is

$\sin x$ has range $[-1, 1]$.

So $3 - 2\sin x$ has range $[1, 5]$.

Ans: (C) $[1, 5]$

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19. The range of $4 + 4|\cos x|$ is

$|\cos x|$ has range $[0, 1]$.

So $4 + 4|\cos x|$ has range $[4, 8]$.

Ans: (B) $[4, 8]$

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20. The range of $6\cos x + 8\sin x - 2$ is

$\sqrt{36 + 64} = 10$

Range $= [-12, 8]$.

Ans: (C) $[-12, 8]$

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21. The maximum value of $4\cos x + 3\sin x + 2$ is

$\sqrt{a^2 + b^2} = 5$

Maximum $= 2 + 5 = 7$.

Ans: (D) 7

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22. The range of $\dfrac{1}{-15\cos x + 8\sin x + 15}$ is

The denominator has range $[-2, 32]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{2}] \cup [\frac{1}{32}, \infty)$.

Ans: (B) $(-\infty, -\frac{1}{2}] \cup [\frac{1}{32}, \infty)$

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23. The range of $\dfrac{1}{8\cos x + 6\sin x + 1}$ is

The denominator has range $[-9, 11]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{9}] \cup [\frac{1}{11}, \infty)$.

Ans: (C) $(-\infty, -\frac{1}{9}] \cup [\frac{1}{11}, \infty)$

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24. The minimum value of $25\tan\theta + 1\cot\theta$ for acute θ is

$\geq 2\sqrt{25 \cdot 1} = 10$

Equality is possible, so the minimum is 10.

Ans: (B) 10

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25. The minimum value of $4\sin^2\theta + 36\,\mathrm{cosec}^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 24, and equality needs $\sin^2\theta = 3$.

Here $3 > 1$, so the minimum is at $\sin^2\theta = 1$: 40.

Ans: (A) 40