EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 06

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If the fundamental period of $f(x)$ is 10, the period of $\frac{1}{f(x)}$ is

A. 10 **B.** 5 **C.** 20 **D.** $\frac{1}{10}$

2. The constant function $f(x) = -6$ is

A. not periodic

B. periodic, with fundamental period 1

C. periodic, with no fundamental period

D. periodic, with fundamental period 6

3. Among $\sin x$, $\sec x$, $\cos x$ and $|\cot x|$, the function with period π is

A. $\sec x$ **B.** $\cos x$ **C.** $|\cot x|$ **D.** $\sin x$

4. The fundamental period of $\sec x$ is

A. 2π **B.** π **C.** $\frac{\pi}{2}$ **D.** 4π

5. The period of $\sec(7x + 1)$ is

A. $\frac{\pi}{7}$ **B.** 2π **C.** $\frac{2\pi}{7}$ **D.** 14π

6. The period of $\cot\left(\frac{x}{2} - 2\right)$ is

A. π **B.** 4π **C.** $\frac{\pi}{2}$ **D.** 2π

7. The period of $\cos\left(4x + \frac{\pi}{6}\right)$ is

A. 2π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** 8π

8. The period of $\cot\frac{x}{2} + \sin 6x$ is

A. $\frac{7\pi}{3}$ **B.** $\frac{\pi}{3}$ **C.** 2π **D.** $\frac{2\pi}{3}$

9. Among $\tan x + \tan 3x$, $\sin\frac{x}{2} + \cot 3x$, $\cot 2x + \operatorname{cosec} \pi x$ and $\tan x + \cos 4x$, the function that is NOT periodic is

A. $\tan x + \cos 4x$ **B.** $\sin\frac{x}{2} + \cot 3x$

C. $\tan x + \tan 3x$ **D.** $\cot 2x + \operatorname{cosec} \pi x$

10. The period of $\sin 4x + \cos 5x$ is

A. 2π **B.** $\frac{9\pi}{10}$ **C.** $\frac{\pi}{10}$ **D.** $\frac{\pi}{5}$

11. The period of $\tan^4 4x$ is

A. 4π **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{16}$ **D.** $\frac{\pi}{2}$

12. The period of $\operatorname{cosec}^2 4x$ is

A. 4π **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{8}$ **D.** $\frac{\pi}{2}$

13. The period of $5\tan^4 x + 5\cot^4 x$ is

A. 2π **B.** $\frac{\pi}{4}$ **C.** π **D.** $\frac{\pi}{2}$

14. The period of $4\tan^3 x + \cot^3 x$ is

A. $\frac{\pi}{4}$ **B.** π **C.** $\frac{\pi}{2}$ **D.** 2π

15. The period of $3|\operatorname{cosec} x| + 3|\sec x|$ is

A. $\frac{\pi}{4}$ **B.** 2π **C.** π **D.** $\frac{\pi}{2}$

16. The period of $4x - [4x]$ is

A. 1 **B.** $\frac{1}{4}$ **C.** 4 **D.** $\frac{1}{2}$

17. Among $\cos\sqrt{x}$, $x - [x]$, $\sin^2 x$ and $\cos^4 2x$, the function that is NOT periodic is

A. $\sin^2 x$ **B.** $x - [x]$

C. $\cos\sqrt{x}$ **D.** $\cos^4 2x$

18. The range of $-1 - 2\sin^5 x$ is

A. $[-2, 0]$ **B.** $[-1, 3]$ **C.** $[-2, 2]$ **D.** $[-3, 1]$

19. The range of $4 - \cot^2 x$ is

A. $[4, \infty)$ **B.** $(-\infty, 4]$ **C.** $(-\infty, 0]$ **D.** $(-\infty, -4]$

20. The maximum value of $12\cos x + 5\sin x + 6$ is

A. 19 **B.** 13 **C.** -7 **D.** 23

21. The minimum value of $8\cos x + 6\sin x + 4$ is

A. -6 **B.** 14 **C.** -10 **D.** -96

22. The range of $\dfrac{1}{-6\cos x - 8\sin x + 12}$ is

A. $[-\frac{1}{10}, \frac{1}{10}]$ **B.** $[2, 22]$

C. $[\frac{1}{22}, \infty)$ **D.** $[\frac{1}{22}, \frac{1}{2}]$

23. The range of $\dfrac{1}{-3\cos x + 4\sin x - 6}$ is

A. $[-11, -1]$ **B.** $[-1, -\frac{1}{11}]$

C. $[-\frac{1}{5}, \frac{1}{5}]$ **D.** $(-\infty, -1]$

24. The minimum value of $64\sin^2\theta + 4\operatorname{cosec}^2\theta$ is

A. 32 **B.** 68 **C.** 16 **D.** 20

25. The minimum value of $9\cos^2\theta + 4\sec^2\theta$ is

A. 13 **B.** 10 **C.** 12 **D.** 6

Key sheet

1. A	**2.** C	**3.** C	**4.** A	**5.** C
6. D	**7.** B	**8.** C	**9.** D	**10.** A
11. B	**12.** B	**13.** D	**14.** B	**15.** D
16. B	**17.** C	**18.** D	**19.** B	**20.** A
21. A	**22.** D	**23.** B	**24.** A	**25.** C

Solutions

1. If the fundamental period of $f(x)$ is 10, the period of $\frac{1}{f(x)}$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is 10.

Ans: (A) 10

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2. The constant function $f(x) = -6$ is

$f(x+p) = f(x)$ for every $p > 0$, so f is periodic.

There is no least positive p, so it has no fundamental period.

Ans: (C) periodic, with no fundamental period

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3. Among $\sin x$, $\sec x$, $\cos x$ and $|\cot x|$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $|\cot x|$ has period π.

Ans: (C) $|\cot x|$

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4. The fundamental period of $\sec x$ is

It repeats after 2π.

Period $= 2\pi$.

Ans: (A) 2π

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5. The period of $\sec(7x+1)$ is

Period of $\sec x$ is 2π; divide by $|a| = 7$.

Period $= \frac{2\pi}{7}$.

Ans: (C) $\frac{2\pi}{7}$

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6. The period of $\cot\left(\frac{x}{2} - 2\right)$ is

Period of $\cot x$ is π; divide by $|a| = \frac{1}{2}$.

Period $= 2\pi$.

Ans: (D) 2π

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7. The period of $\cos\left(4x + \frac{\pi}{6}\right)$ is

Period of $\cos x$ is 2π; divide by $|a| = 4$.

Period $= \frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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8. The period of $\cot\frac{x}{2} + \sin 6x$ is

Periods: 2π and $\frac{\pi}{3}$.

L.C.M. $= 2\pi$.

Ans: (C) 2π

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9. Among $\tan x + \tan 3x$, $\sin \frac{x}{2} + \cot 3x$, $\cot 2x + \operatorname{cosec} \pi x$ and $\tan x + \cos 4x$, the function that is NOT periodic is

In $\cot 2x + \operatorname{cosec} \pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (D) $\cot 2x + \operatorname{cosec} \pi x$

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10. The period of $\sin 4x + \cos 5x$ is

Periods: $\frac{\pi}{2}$ and $\frac{2\pi}{5}$.
L.C.M. $= 2\pi$.

Ans: (A) 2π

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11. The period of $\tan^4 4x$ is

$\tan^4 x$ has period π.
So the period is $\frac{\pi}{4}$.

Ans: (B) $\frac{\pi}{4}$

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12. The period of $\operatorname{cosec}^2 4x$ is

$\operatorname{cosec}^2 x$ has period π.
So the period is $\frac{\pi}{4}$.

Ans: (B) $\frac{\pi}{4}$

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13. The period of $5\tan^4 x + 5\cot^4 x$ is

Table: $n = 4$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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14. The period of $4\tan^3 x + \cot^3 x$ is

Table: $n = 3$, coefficients unequal.
Period $= \pi$.

Ans: (B) π

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15. The period of $3|\operatorname{cosec} x| + 3|\sec x|$ is

Absolute values with equal coefficients.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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16. The period of $4x - [4x]$ is

$x - [x]$ has period 1.
So $4x - [4x]$ has period $\frac{1}{4}$.

Ans: (B) $\frac{1}{4}$

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17. Among $\cos\sqrt{x}$, $x-[x]$, $\sin^2 x$ and $\cos^4 2x$, the function that is NOT periodic is

The zeros of $\cos\sqrt{x}$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (C) $\cos\sqrt{x}$

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18. The range of $-1-2\sin^5 x$ is

$\sin^5 x$ has range $[-1,1]$.

So $-1-2\sin^5 x$ has range $[-3,1]$.

Ans: (D) $[-3,1]$

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19. The range of $4-\cot^2 x$ is

$\cot^2 x$ has range $[0,\infty)$.

So $4-\cot^2 x$ has range $(-\infty,4]$.

Ans: (B) $(-\infty,4]$

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20. The maximum value of $12\cos x+5\sin x+6$ is

$\sqrt{a^2+b^2}=13$

Maximum $=6+13=19$.

Ans: (A) 19

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21. The minimum value of $8\cos x+6\sin x+4$ is

$\sqrt{a^2+b^2}=10$

Minimum $=4-10=-6$.

Ans: (A) -6

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22. The range of $\dfrac{1}{-6\cos x-8\sin x+12}$ is

The denominator has range $[2,22]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$.

Range $=[\frac{1}{22},\frac{1}{2}]$.

Ans: (D) $[\frac{1}{22},\frac{1}{2}]$

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23. The range of $\dfrac{1}{-3\cos x+4\sin x-6}$ is

The denominator has range $[-11,-1]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$.

Range $=[-1,-\frac{1}{11}]$.

Ans: (B) $[-1,-\frac{1}{11}]$

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24. The minimum value of $64\sin^2\theta+4\,\mathrm{cosec}^2\,\theta$ is

A.M. $\geq$ G.M. gives ≥ 32, and equality needs $\sin^2\theta=\frac{1}{4}$.

Here $\frac{1}{4}\leq 1$, so the minimum is 32.

Ans: (A) 32

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25. The minimum value of $9\cos^2\theta + 4\sec^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 12, and equality needs $\cos^2\theta = \frac{2}{3}$.

Here $\frac{2}{3} \leq 1$, so the minimum is 12.

Ans: (C) 12
