EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 05

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The constant function $f(x) = 13$ is

A. periodic, with no fundamental period

B. not periodic

C. periodic, with fundamental period 13

D. periodic, with fundamental period 1

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2. If $f(x+2) = f(x)$ for all real x and $f(1) = 12$, then $f(11) =$

A. 60 **B.** 14 **C.** 12 **D.** 22

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3. Among $\sin x$, $\operatorname{cosec} x$, $\cos x$ and $|\sec x|$, the function with period π is

A. $\sin x$ **B.** $\cos x$ **C.** $\operatorname{cosec} x$ **D.** $|\sec x|$

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4. The period of $|\operatorname{cosec} x|$ is

A. π **B.** $\frac{\pi}{2}$ **C.** 2π **D.** 4π

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5. The period of $\tan\left(\frac{3x}{4} + \frac{\pi}{4}\right)$ is

A. $\frac{8\pi}{3}$ **B.** $\frac{4\pi}{3}$ **C.** π **D.** $\frac{3\pi}{4}$

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6. The period of $\sec\left(\frac{2x}{3} + \frac{\pi}{3}\right)$ is

A. $\frac{3\pi}{2}$ **B.** 3π **C.** 2π **D.** $\frac{4\pi}{3}$

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7. The period of $\operatorname{cosec}\left(4x + \frac{\pi}{6}\right)$ is

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{4}$ **C.** 2π **D.** 8π

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8. The period of $\operatorname{cosec} 4x - \operatorname{cosec} \frac{3x}{4}$ is

A. $\frac{4\pi}{3}$ **B.** $\frac{19\pi}{6}$ **C.** $\frac{\pi}{6}$ **D.** 8π

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9. Among $\cot 2x + \tan \frac{x}{2}$, $\cos 3x + \sin 2\pi x$, $\cot \frac{x}{3} + \tan 2x$ and $\sec x + \tan 4x$, the function that is NOT periodic is

A. $\cot 2x + \tan \frac{x}{2}$ **B.** $\sec x + \tan 4x$

C. $\cos 3x + \sin 2\pi x$ **D.** $\cot \frac{x}{3} + \tan 2x$

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10. The period of $\sin\frac{x}{3}+\cos\frac{x}{2}$ is

A. 24π **B.** 10π **C.** 12π **D.** 2π

11. The period of $\cot^3 3x$ is

A. $\frac{\pi}{9}$ **B.** $\frac{2\pi}{3}$ **C.** 3π **D.** $\frac{\pi}{3}$

12. The period of $\cos^4 x$ is

A. π **B.** $\frac{\pi}{4}$ **C.** 4π **D.** 2π

13. The period of $\operatorname{cosec}^2 x+\sec^2 x$ is

A. π **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{2}$ **D.** 2π

14. The period of $3\sin^3 x+5\cos^3 x$ is

A. $\frac{\pi}{2}$ **B.** π **C.** 2π **D.** $\frac{\pi}{4}$

15. The period of $2|\operatorname{cosec} x|+2|\sec x|$ is

A. π **B.** 2π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

16. The period of $6x-[6x]$ is

A. 6 **B.** $\frac{1}{6}$ **C.** $\frac{1}{3}$ **D.** 1

17. Among $\sin x+\cos 2x$, $x-[x]$, $\sin\sqrt{x}$ and $2x-[2x]$, the function that is NOT periodic is

A. $x-[x]$ **B.** $2x-[2x]$

C. $\sin x+\cos 2x$ **D.** $\sin\sqrt{x}$

18. The range of $1+\cos^5 x$ is

A. $[0,2]$ **B.** $[-1,1]$ **C.** $[1,2]$ **D.** $[-2,0]$

19. The range of $2-\cot^2 x$ is

A. $[2,\infty)$ **B.** $(-\infty,2]$ **C.** $(-\infty,-2]$ **D.** $(-\infty,0]$

20. The minimum value of $2\cos x+3\sin x-3$ is

A. $-3+\sqrt{13}$ **B.** -8 **C.** $-\sqrt{13}$ **D.** $-\sqrt{13}-3$

21. The range of $2\cos x+3\sin x+3$ is

A. $[-2,8]$ **B.** $[3-\sqrt{13},3+\sqrt{13}]$

C. $[-\sqrt{13},\sqrt{13}]$ **D.** $[2,4]$

22. The range of $\dfrac{1}{-8\cos x-15\sin x-20}$ is

A. $[-\frac{1}{17},\frac{1}{17}]$ **B.** $(-\infty,-\frac{1}{3}]$

C. $[-\frac{1}{3},-\frac{1}{37}]$ **D.** $[-37,-3]$

23. The range of $\dfrac{1}{-7\cos x + 24\sin x + 29}$ is

A. $[\frac{1}{54}, \frac{1}{4}]$ **B.** $[-\frac{1}{25}, \frac{1}{25}]$

C. $[4, 54]$ **D.** $[\frac{1}{54}, \infty)$

24. The minimum value of $4\cos^2\theta + 9\sec^2\theta$ is

A. 12 **B.** 10 **C.** 13 **D.** 6

25. The minimum value of $4\tan\theta + 1\cot\theta$ for acute θ is

A. 6 **B.** 4 **C.** 5 **D.** 2

Key sheet

1. A	**2.** C	**3.** D	**4.** A	**5.** B
6. B	**7.** A	**8.** D	**9.** C	**10.** C
11. D	**12.** A	**13.** C	**14.** C	**15.** D
16. B	**17.** D	**18.** A	**19.** B	**20.** D
21. B	**22.** C	**23.** A	**24.** C	**25.** B

Solutions

1. The constant function $f(x) = 13$ is

$f(x+p) = f(x)$ for every $p > 0$, so f is periodic.
There is no least positive p, so it has no fundamental period.

Ans: (A) periodic, with no fundamental period

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2. If $f(x+2) = f(x)$ for all real x and $f(1) = 12$, then $f(11) =$

$11 = 1 + 5 \times 2$
$f(11) = f(1) = 12$

Ans: (C) 12

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3. Among $\sin x$, $\operatorname{cosec} x$, $\cos x$ and $|\sec x|$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.
So $|\sec x|$ has period π.

Ans: (D) $|\sec x|$

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4. The period of $|\operatorname{cosec} x|$ is

Taking the absolute value folds the graph, so it repeats after π.
Period $= \pi$.

Ans: (A) π

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5. The period of $\tan\left(\frac{3x}{4} + \frac{\pi}{4}\right)$ is

Period of $\tan x$ is π; divide by $|a| = \frac{3}{4}$.
Period $= \frac{4\pi}{3}$.

Ans: (B) $\frac{4\pi}{3}$

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6. The period of $\sec\left(\frac{2x}{3} + \frac{\pi}{3}\right)$ is

Period of $\sec x$ is 2π; divide by $|a| = \frac{2}{3}$.
Period $= 3\pi$.

Ans: (B) 3π

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7. The period of $\operatorname{cosec}\left(4x + \frac{\pi}{6}\right)$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = 4$.
Period $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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8. The period of $\operatorname{cosec} 4x - \operatorname{cosec} \frac{3x}{4}$ is

Periods: $\frac{\pi}{2}$ and $\frac{8\pi}{3}$.
L.C.M. $= 8\pi$.

Ans: (D) 8π

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9. Among $\cot 2x + \tan \frac{x}{2}$, $\cos 3x + \sin 2\pi x$, $\cot \frac{x}{3} + \tan 2x$ and $\sec x + \tan 4x$, the function that is NOT periodic is

In $\cos 3x + \sin 2\pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (C) $\cos 3x + \sin 2\pi x$

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10. The period of $\sin \frac{x}{3} + \cos \frac{x}{2}$ is

Periods: 6π and 4π.
L.C.M. $= 12\pi$.

Ans: (C) 12π

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11. The period of $\cot^3 3x$ is

$\cot^3 x$ has period π.
So the period is $\frac{\pi}{3}$.

Ans: (D) $\frac{\pi}{3}$

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12. The period of $\cos^4 x$ is

$\cos^4 x$ has period π.
So the period is π.

Ans: (A) π

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13. The period of $\operatorname{cosec}^2 x + \sec^2 x$ is

Table: $n = 2$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (C) $\frac{\pi}{2}$

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14. The period of $3\sin^3 x + 5\cos^3 x$ is

Table: $n = 3$, coefficients unequal.
Period $= 2\pi$.

Ans: (C) 2π

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15. The period of $2|\operatorname{cosec} x| + 2|\sec x|$ is

Absolute values with equal coefficients.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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16. The period of $6x - [6x]$ is

$x - [x]$ has period 1.
So $6x - [6x]$ has period $\frac{1}{6}$.

Ans: (B) $\frac{1}{6}$

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17. Among $\sin x + \cos 2x$, $x - [x]$, $\sin\sqrt{x}$ and $2x - [2x]$, the function that is NOT periodic is

The zeros of $\sin\sqrt{x}$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (D) $\sin\sqrt{x}$

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18. The range of $1 + \cos^5 x$ is

$\cos^5 x$ has range $[-1, 1]$.

So $1 + \cos^5 x$ has range $[0, 2]$.

Ans: (A) $[0, 2]$

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19. The range of $2 - \cot^2 x$ is

$\cot^2 x$ has range $[0, \infty)$.

So $2 - \cot^2 x$ has range $(-\infty, 2]$.

Ans: (B) $(-\infty, 2]$

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20. The minimum value of $2\cos x + 3\sin x - 3$ is

$\sqrt{a^2 + b^2} = \sqrt{13}$

Minimum $= -3 - \sqrt{13} = -\sqrt{13} - 3$.

Ans: (D) $-\sqrt{13} - 3$

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21. The range of $2\cos x + 3\sin x + 3$ is

$\sqrt{4 + 9} = \sqrt{13}$

Range $= [3 - \sqrt{13}, 3 + \sqrt{13}]$.

Ans: (B) $[3 - \sqrt{13}, 3 + \sqrt{13}]$

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22. The range of $\dfrac{1}{-8\cos x - 15\sin x - 20}$ is

The denominator has range $[-37, -3]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [-\frac{1}{3}, -\frac{1}{37}]$.

Ans: (C) $[-\frac{1}{3}, -\frac{1}{37}]$

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23. The range of $\dfrac{1}{-7\cos x + 24\sin x + 29}$ is

The denominator has range $[4, 54]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{54}, \frac{1}{4}]$.

Ans: (A) $[\frac{1}{54}, \frac{1}{4}]$

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24. The minimum value of $4\cos^2\theta + 9\sec^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 12, and equality needs $\cos^2\theta = \frac{3}{2}$.

Here $\frac{3}{2} > 1$, so the minimum is at $\cos^2\theta = 1$: 13.

Ans: (C) 13

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25. The minimum value of $4\tan\theta + 1\cot\theta$ for acute θ is

$\geq 2\sqrt{4 \cdot 1} = 4$

Equality is possible, so the minimum is 4.

Ans: (B) 4