EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 04

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $f(x+2) = f(x)$ for all real x and $f(5) = 4$, then $f(1) =$

A. 4 **B.** 0 **C.** 6 **D.** -8

2. If the fundamental period of $f(x)$ is 6, the period of $\frac{1}{f(x)}$ is

A. 12 **B.** 6 **C.** 3 **D.** $\frac{1}{6}$

3. Among $\operatorname{cosec} x$, $\cos x$, $|\cot x|$ and $\sec x$, the function with period π is

A. $|\cot x|$ **B.** $\cos x$ **C.** $\sec x$ **D.** $\operatorname{cosec} x$

4. The period of $|\sec x|$ is

A. 4π **B.** π **C.** $\frac{\pi}{2}$ **D.** 2π

5. The period of $\cot\left(\frac{x}{3} + \frac{\pi}{4}\right)$ is

A. 3π **B.** π **C.** 6π **D.** $\frac{\pi}{3}$

6. The period of $\operatorname{cosec}(4x + 7)$ is

A. 2π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** 8π

7. The period of $\cot\left(\frac{4x}{5} + 7\right)$ is

A. π **B.** $\frac{5\pi}{2}$ **C.** $\frac{5\pi}{4}$ **D.** $\frac{4\pi}{5}$

8. The period of $\cos 3x + \tan\frac{2x}{3}$ is

A. 6π **B.** π **C.** $\frac{13\pi}{6}$ **D.** $\frac{\pi}{6}$

9. Among $\sec 2x + \cot\frac{x}{2}$, $\tan\frac{x}{3} + \operatorname{cosec} 3x$, $\operatorname{cosec} x + \tan \pi x$ and $\sec 4x + \sec\frac{x}{2}$, the function that is NOT periodic is

A. $\sec 2x + \cot\frac{x}{2}$ **B.** $\sec 4x + \sec\frac{x}{2}$

C. $\operatorname{cosec} x + \tan \pi x$ **D.** $\tan\frac{x}{3} + \operatorname{cosec} 3x$

10. The period of $\tan 4x - \sec 5x$ is

A. $\frac{\pi}{10}$ **B.** $\frac{\pi}{20}$ **C.** $\frac{13\pi}{20}$ **D.** 2π

11. The period of $\cot^5\frac{x}{3}$ is

A. 3π **B.** $\frac{3\pi}{5}$ **C.** $\frac{\pi}{3}$ **D.** 6π

12. The period of $\sec^2 2x$ is

A. 2π **B.** π **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{4}$

13. The period of $3\sin^6 x + 3\cos^6 x$ is

A. $\frac{\pi}{2}$ **B.** $\frac{\pi}{4}$ **C.** 2π **D.** π

14. The period of $\sin^5 x + 4\cos^5 x$ is

A. π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** 2π

15. The period of $5|\sin x| + 6|\cos x|$ is

A. $\frac{\pi}{2}$ **B.** 2π **C.** $\frac{\pi}{4}$ **D.** π

16. The period of $\frac{x}{2} - \left[\frac{x}{2}\right]$ is

A. 1 **B.** 2 **C.** $\frac{1}{2}$ **D.** 4

17. Among $\sin x + \cos 2x$, $\sin x^2$, $|\cos 3x|$ and $2x - [2x]$, the function that is NOT periodic is

A. $\sin x + \cos 2x$ **B.** $|\cos 3x|$

C. $\sin x^2$ **D.** $2x - [2x]$

18. The range of $4 - 2\cos^5 x$ is

A. $[3, 5]$ **B.** $[-6, -2]$ **C.** $[-2, 2]$ **D.** $[2, 6]$

19. The range of $4 - 2|\sin x|$ is

A. $[-2, 0]$ **B.** $[4, 6]$ **C.** $[2, 4]$ **D.** $[-6, -4]$

20. The range of $3\cos x + 5\sin x + 1$ is

A. $[-7, 9]$ **B.** $[0, 2]$

C. $[1 - \sqrt{34}, 1 + \sqrt{34}]$ **D.** $[-\sqrt{34}, \sqrt{34}]$

21. The maximum value of $12\cos x + 5\sin x$ is

A. 17 **B.** -13 **C.** 169 **D.** 13

22. The range of $\dfrac{1}{-5\cos x - 12\sin x + 1}$ is

A. $[-\frac{1}{13}, \frac{1}{13}]$ **B.** $(-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$

C. $[-\frac{1}{12}, \frac{1}{14}]$ **D.** $[-12, 14]$

23. The range of $\dfrac{1}{12\cos x - 5\sin x + 1}$ is

A. $(-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$ **B.** $[-12, 14]$

C. $[-\frac{1}{13}, \frac{1}{13}]$ **D.** $[-\frac{1}{12}, \frac{1}{14}]$

24. The minimum value of $25\tan\theta + 64\cot\theta$ for acute θ is

A. 40 **B.** 26 **C.** 89 **D.** 80

25. The minimum value of $25\sin^2\theta + 49\,\mathrm{cosec}^2\theta$ is

A. 24 **B.** 74 **C.** 35 **D.** 70

Key sheet

1. A	**2.** B	**3.** A	**4.** B	**5.** A
6. B	**7.** C	**8.** A	**9.** C	**10.** D
11. A	**12.** C	**13.** A	**14.** D	**15.** D
16. B	**17.** C	**18.** D	**19.** C	**20.** C
21. D	**22.** B	**23.** A	**24.** D	**25.** B

Solutions

1. If $f(x+2) = f(x)$ for all real x and $f(5) = 4$, then $f(1) =$

$1 = 5 - 2 \times 2$

$f(1) = f(5) = 4$

Ans: (A) 4

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2. If the fundamental period of $f(x)$ is 6, the period of $\frac{1}{f(x)}$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is 6.

Ans: (B) 6

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3. Among $\operatorname{cosec} x$, $\cos x$, $|\cot x|$ and $\sec x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $|\cot x|$ has period π.

Ans: (A) $|\cot x|$

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4. The period of $|\sec x|$ is

Taking the absolute value folds the graph, so it repeats after π.

Period $= \pi$.

Ans: (B) π

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5. The period of $\cot\left(\frac{x}{3} + \frac{\pi}{4}\right)$ is

Period of $\cot x$ is π; divide by $|a| = \frac{1}{3}$.

Period $= 3\pi$.

Ans: (A) 3π

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6. The period of $\operatorname{cosec}(4x + 7)$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = 4$.

Period $= \frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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7. The period of $\cot\left(\frac{4x}{5} + 7\right)$ is

Period of $\cot x$ is π; divide by $|a| = \frac{4}{5}$.

Period $= \frac{5\pi}{4}$.

Ans: (C) $\frac{5\pi}{4}$

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8. The period of $\cos 3x + \tan\frac{2x}{3}$ is

Periods: $\frac{2\pi}{3}$ and $\frac{3\pi}{2}$.

L.C.M. $= 6\pi$.

Ans: (A) 6π

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9. Among $\sec 2x + \cot \frac{x}{2}$, $\tan \frac{x}{3} + \operatorname{cosec} 3x$, $\operatorname{cosec} x + \tan \pi x$ and $\sec 4x + \sec \frac{x}{2}$, the function that is NOT periodic is

In $\operatorname{cosec} x + \tan \pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (C) $\operatorname{cosec} x + \tan \pi x$

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10. The period of $\tan 4x - \sec 5x$ is

Periods: $\frac{\pi}{4}$ and $\frac{2\pi}{5}$.
L.C.M. $= 2\pi$.

Ans: (D) 2π

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11. The period of $\cot^5 \frac{x}{3}$ is

$\cot^5 x$ has period π.
So the period is 3π.

Ans: (A) 3π

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12. The period of $\sec^2 2x$ is

$\sec^2 x$ has period π.
So the period is $\frac{\pi}{2}$.

Ans: (C) $\frac{\pi}{2}$

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13. The period of $3 \sin^6 x + 3 \cos^6 x$ is

Table: $n = 6$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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14. The period of $\sin^5 x + 4 \cos^5 x$ is

Table: $n = 5$, coefficients unequal.
Period $= 2\pi$.

Ans: (D) 2π

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15. The period of $5|\sin x| + 6|\cos x|$ is

Absolute values with unequal coefficients.
Period $= \pi$.

Ans: (D) π

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16. The period of $\frac{x}{2} - \left[\frac{x}{2}\right]$ is

$x - [x]$ has period 1.
So $\frac{x}{2} - \left[\frac{x}{2}\right]$ has period $\frac{1}{\frac{1}{2}} = 2$.

Ans: (B) 2

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17. Among $\sin x + \cos 2x$, $\sin x^2$, $|\cos 3x|$ and $2x - [2x]$, the function that is NOT periodic is

The zeros of $\sin x^2$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (C) $\sin x^2$

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18. The range of $4 - 2\cos^5 x$ is

$\cos^5 x$ has range $[-1, 1]$.

So $4 - 2\cos^5 x$ has range $[2, 6]$.

Ans: (D) $[2, 6]$

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19. The range of $4 - 2|\sin x|$ is

$|\sin x|$ has range $[0, 1]$.

So $4 - 2|\sin x|$ has range $[2, 4]$.

Ans: (C) $[2, 4]$

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20. The range of $3\cos x + 5\sin x + 1$ is

$\sqrt{9 + 25} = \sqrt{34}$

Range $= [1 - \sqrt{34}, 1 + \sqrt{34}]$.

Ans: (C) $[1 - \sqrt{34}, 1 + \sqrt{34}]$

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21. The maximum value of $12\cos x + 5\sin x$ is

$\sqrt{a^2 + b^2} = 13$

Maximum $= 0 + 13 = 13$.

Ans: (D) 13

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22. The range of $\dfrac{1}{-5\cos x - 12\sin x + 1}$ is

The denominator has range $[-12, 14]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$.

Ans: (B) $(-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$

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23. The range of $\dfrac{1}{12\cos x - 5\sin x + 1}$ is

The denominator has range $[-12, 14]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$.

Ans: (A) $(-\infty, -\frac{1}{12}] \cup [\frac{1}{14}, \infty)$

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24. The minimum value of $25\tan\theta + 64\cot\theta$ for acute θ is

$\geq 2\sqrt{25 \cdot 64} = 80$

Equality is possible, so the minimum is 80.

Ans: (D) 80

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25. The minimum value of $25\sin^2\theta + 49\,\mathrm{cosec}^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 70, and equality needs $\sin^2\theta = \frac{7}{5}$.

Here $\frac{7}{5} > 1$, so the minimum is at $\sin^2\theta = 1$: 74.

Ans: (B) 74