EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 03

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If the fundamental period of $f(x)$ is 10, the period of $-f(x)$ is

A. 20 **B.** 10 **C.** $\frac{1}{10}$ **D.** 5

2. The constant function $f(x) = -9$ is

A. periodic, with fundamental period 9

B. not periodic

C. periodic, with no fundamental period

D. periodic, with fundamental period 1

3. Among $\tan x$, $|\sin x|$, $|\cos x|$ and $\operatorname{cosec} x$, the function with period 2π is

A. $|\sin x|$ **B.** $\tan x$ **C.** $\operatorname{cosec} x$ **D.** $|\cos x|$

4. The fundamental period of $\cos x$ is

A. 2π **B.** 4π **C.** π **D.** $\frac{\pi}{2}$

5. The period of $\tan\left(3x + \frac{\pi}{6}\right)$ is

A. $\frac{2\pi}{3}$ **B.** 3π **C.** π **D.** $\frac{\pi}{3}$

6. The period of $\sec(6x + 3)$ is

A. $\frac{\pi}{3}$ **B.** 2π **C.** $\frac{\pi}{6}$ **D.** 12π

7. The period of $\operatorname{cosec}\left(\frac{x}{3} + 7\right)$ is

A. 6π **B.** 2π **C.** $\frac{2\pi}{3}$ **D.** 3π

8. The period of $\cot 6x - \sin 4x$ is

A. $\frac{\pi}{12}$ **B.** $\frac{\pi}{6}$ **C.** $\frac{2\pi}{3}$ **D.** $\frac{\pi}{2}$

9. Among $\cos\frac{x}{2} + \cot 4x$, $\sec 2x + \cos 3\pi x$, $\sec 3x + \cot\frac{x}{2}$ and $\cot\frac{x}{2} + \tan x$, the function that is NOT periodic is

A. $\sec 2x + \cos 3\pi x$ **B.** $\cot\frac{x}{2} + \tan x$

C. $\sec 3x + \cot\frac{x}{2}$ **D.** $\cos\frac{x}{2} + \cot 4x$

10. The period of $\cos 4x + \operatorname{cosec} \frac{2x}{3}$ is

A. $\frac{7\pi}{2}$ **B.** $\frac{\pi}{2}$ **C.** 3π **D.** $\frac{3\pi}{2}$

11. The period of $\sin^4 3x$ is

A. $\frac{2\pi}{3}$ **B.** $\frac{\pi}{3}$ **C.** 3π **D.** $\frac{\pi}{12}$

12. The period of $\cos^7 x$ is

A. π **B.** $\frac{2\pi}{7}$ **C.** 2π **D.** 14π

13. The period of $\tan^4 x + \cot^4 x$ is

A. $\frac{\pi}{4}$ **B.** $\frac{\pi}{2}$ **C.** π **D.** 2π

14. The period of $\operatorname{cosec}^5 x + 3\sec^5 x$ is

A. $\frac{\pi}{2}$ **B.** π **C.** 2π **D.** $\frac{\pi}{4}$

15. The period of $|\operatorname{cosec} x| + |\sec x|$ is

A. $\frac{\pi}{4}$ **B.** $\frac{\pi}{2}$ **C.** 2π **D.** π

16. The period of $\frac{2x}{5} - \left[\frac{2x}{5}\right]$ is

A. $\frac{2}{5}$ **B.** 5 **C.** 1 **D.** $\frac{5}{2}$

17. Among $\cos^4 2x$, $\sin\sqrt{x}$, $\sin x + \cos 2x$ and $\sin^2 x$, the function that is NOT periodic is

A. $\sin^2 x$ **B.** $\sin\sqrt{x}$

C. $\cos^4 2x$ **D.** $\sin x + \cos 2x$

18. The range of $-1 + 4\cos^6 x$ is

A. $[1, 5]$ **B.** $[0, 4]$ **C.** $[-1, 3]$ **D.** $[-5, -1]$

19. The range of $1 + 3\tan^2 x$ is

A. $[0, \infty)$ **B.** $[-1, \infty)$ **C.** $(-\infty, 1]$ **D.** $[1, \infty)$

20. The maximum value of $8\cos x + 15\sin x + 2$ is

A. 25 **B.** 17 **C.** -15 **D.** 19

21. The minimum value of $15\cos x + 8\sin x + 1$ is

A. -16 **B.** 18 **C.** -22 **D.** -17

22. The range of $\dfrac{1}{-24\cos x - 7\sin x + 27}$ is

A. $[\frac{1}{52}, \infty)$ **B.** $[\frac{1}{52}, \frac{1}{2}]$

C. $[-\frac{1}{25}, \frac{1}{25}]$ **D.** $[2, 52]$

23. The range of $\dfrac{1}{24\cos x + 7\sin x + 23}$ is

A. $(-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$ **B.** $[-\frac{1}{25}, \frac{1}{25}]$

C. $[-\frac{1}{2}, \frac{1}{48}]$ **D.** $[-2, 48]$

24. The minimum value of $16\sin^2\theta + 36\,\mathrm{cosec}^2\,\theta$ is

A. 48 **B.** 24 **C.** 52 **D.** 20

25. The minimum value of $36\cos^2\theta + 49\sec^2\theta$ is

A. 84 **B.** 42 **C.** 26 **D.** 85

Key sheet

1. B	**2.** C	**3.** C	**4.** A	**5.** D
6. A	**7.** A	**8.** D	**9.** A	**10.** C
11. B	**12.** C	**13.** B	**14.** C	**15.** B
16. D	**17.** B	**18.** C	**19.** D	**20.** D
21. A	**22.** B	**23.** A	**24.** C	**25.** D

Solutions

1. If the fundamental period of $f(x)$ is 10, the period of $-f(x)$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is 10.

Ans: (B) 10

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2. The constant function $f(x) = -9$ is

$f(x+p) = f(x)$ for every $p > 0$, so f is periodic.

There is no least positive p, so it has no fundamental period.

Ans: (C) periodic, with no fundamental period

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3. Among $\tan x$, $|\sin x|$, $|\cos x|$ and $\operatorname{cosec} x$, the function with period 2π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $\operatorname{cosec} x$ has period 2π.

Ans: (C) $\operatorname{cosec} x$

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4. The fundamental period of $\cos x$ is

It repeats after 2π.

Period $= 2\pi$.

Ans: (A) 2π

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5. The period of $\tan\left(3x + \frac{\pi}{6}\right)$ is

Period of $\tan x$ is π; divide by $|a| = 3$.

Period $= \frac{\pi}{3}$.

Ans: (D) $\frac{\pi}{3}$

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6. The period of $\sec(6x + 3)$ is

Period of $\sec x$ is 2π; divide by $|a| = 6$.

Period $= \frac{\pi}{3}$.

Ans: (A) $\frac{\pi}{3}$

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7. The period of $\operatorname{cosec}\left(\frac{x}{3} + 7\right)$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = \frac{1}{3}$.

Period $= 6\pi$.

Ans: (A) 6π

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8. The period of $\cot 6x - \sin 4x$ is

Periods: $\frac{\pi}{6}$ and $\frac{\pi}{2}$.

L.C.M. $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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9. Among $\cos\frac{x}{2}+\cot 4x$, $\sec 2x+\cos 3\pi x$, $\sec 3x+\cot\frac{x}{2}$ and $\cot\frac{x}{2}+\tan x$, the function that is NOT periodic is

In $\sec 2x+\cos 3\pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (A) $\sec 2x+\cos 3\pi x$

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10. The period of $\cos 4x+\operatorname{cosec}\frac{2x}{3}$ is

Periods: $\frac{\pi}{2}$ and 3π.
L.C.M. $=3\pi$.

Ans: (C) 3π

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11. The period of $\sin^4 3x$ is

$\sin^4 x$ has period π.
So the period is $\frac{\pi}{3}$.

Ans: (B) $\frac{\pi}{3}$

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12. The period of $\cos^7 x$ is

$\cos^7 x$ has period 2π.
So the period is 2π.

Ans: (C) 2π

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13. The period of $\tan^4 x+\cot^4 x$ is

Table: $n=4$, coefficients equal.
Period $=\frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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14. The period of $\operatorname{cosec}^5 x+3\sec^5 x$ is

Table: $n=5$, coefficients unequal.
Period $=2\pi$.

Ans: (C) 2π

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15. The period of $|\operatorname{cosec} x|+|\sec x|$ is

Absolute values with equal coefficients.
Period $=\frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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16. The period of $\frac{2x}{5}-\left[\frac{2x}{5}\right]$ is

$x-[x]$ has period 1.
So $\frac{2x}{5}-\left[\frac{2x}{5}\right]$ has period $\frac{1}{\frac{2}{5}}=\frac{5}{2}$.

Ans: (D) $\frac{5}{2}$

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17. Among $\cos^4 2x$, $\sin\sqrt{x}$, $\sin x + \cos 2x$ and $\sin^2 x$, the function that is NOT periodic is

The zeros of $\sin\sqrt{x}$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (B) $\sin\sqrt{x}$

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18. The range of $-1 + 4\cos^6 x$ is

$\cos^6 x$ has range $[0, 1]$.

So $-1 + 4\cos^6 x$ has range $[-1, 3]$.

Ans: (C) $[-1, 3]$

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19. The range of $1 + 3\tan^2 x$ is

$\tan^2 x$ has range $[0, \infty)$.

So $1 + 3\tan^2 x$ has range $[1, \infty)$.

Ans: (D) $[1, \infty)$

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20. The maximum value of $8\cos x + 15\sin x + 2$ is

$\sqrt{a^2 + b^2} = 17$

Maximum $= 2 + 17 = 19$.

Ans: (D) 19

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21. The minimum value of $15\cos x + 8\sin x + 1$ is

$\sqrt{a^2 + b^2} = 17$

Minimum $= 1 - 17 = -16$.

Ans: (A) -16

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22. The range of $\dfrac{1}{-24\cos x - 7\sin x + 27}$ is

The denominator has range $[2, 52]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{52}, \frac{1}{2}]$.

Ans: (B) $[\frac{1}{52}, \frac{1}{2}]$

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23. The range of $\dfrac{1}{24\cos x + 7\sin x + 23}$ is

The denominator has range $[-2, 48]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$.

Range $= (-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$.

Ans: (A) $(-\infty, -\frac{1}{2}] \cup [\frac{1}{48}, \infty)$

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24. The minimum value of $16\sin^2\theta + 36\,\mathrm{cosec}^2\,\theta$ is

A.M. $\geq$ G.M. gives ≥ 48, and equality needs $\sin^2\theta = \frac{3}{2}$.

Here $\frac{3}{2} > 1$, so the minimum is at $\sin^2\theta = 1$: 52.

Ans: (C) 52

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25. The minimum value of $36\cos^2\theta + 49\sec^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 84, and equality needs $\cos^2\theta = \frac{7}{6}$.

Here $\frac{7}{6} > 1$, so the minimum is at $\cos^2\theta = 1$: 85.

Ans: (D) 85