EXPRESSWALA ACADEMY

# Maths 1A: Periodicity and Extreme Values

## MCQ Test, Set 02

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

**1.** The constant function $f(x) = 5$ is

**A.** periodic, with fundamental period $1$

**B.** periodic, with fundamental period $5$

**C.** periodic, with no fundamental period

**D.** not periodic

**2.** If $f(x+2) = f(x)$ for all real $x$ and $f(3) = 11$, then $f(9) =$

**A.** $11$ **B.** $33$ **C.** $13$ **D.** $17$

**3.** Among $|\sin x|$, $|\cot x|$, $\sec x$ and $\tan x$, the function with period $2\pi$ is

**A.** $|\cot x|$ **B.** $|\sin x|$ **C.** $\tan x$ **D.** $\sec x$

**4.** The period of $|\cos x|$ is

**A.** $2\pi$ **B.** $4\pi$ **C.** $\frac{\pi}{2}$ **D.** $\pi$

**5.** The period of $\sin\left(4x + \frac{\pi}{6}\right)$ is

**A.** $\frac{\pi}{2}$ **B.** $2\pi$ **C.** $8\pi$ **D.** $\frac{\pi}{4}$

**6.** The period of $\sin(8x - 2)$ is

**A.** $\frac{\pi}{8}$ **B.** $2\pi$ **C.** $\frac{\pi}{4}$ **D.** $16\pi$

**7.** The period of $\tan(7x - 2)$ is

**A.** $7\pi$ **B.** $\frac{2\pi}{7}$ **C.** $\pi$ **D.** $\frac{\pi}{7}$

**8.** The period of $\operatorname{cosec}\frac{2x}{3} - \cos\frac{x}{3}$ is

**A.** $18\pi$ **B.** $3\pi$ **C.** $9\pi$ **D.** $6\pi$

**9.** Among $\tan 4x + \operatorname{cosec} x$, $\operatorname{cosec}\frac{x}{3} + \sec 4x$, $\cos 3x + \tan 2x$ and $\tan 2x + \operatorname{cosec} 3\pi x$, the function that is NOT periodic is

**A.** $\cos 3x + \tan 2x$ **B.** $\operatorname{cosec}\frac{x}{3} + \sec 4x$

**C.** $\tan 2x + \operatorname{cosec} 3\pi x$ **D.** $\tan 4x + \operatorname{cosec} x$

**10.** The period of $\cos\frac{3x}{4} - \cot\frac{3x}{2}$ is

**A.** $\frac{10\pi}{3}$ **B.** $\frac{8\pi}{3}$ **C.** $\frac{2\pi}{3}$ **D.** $\frac{16\pi}{9}$

**11.** The period of $\tan^3 4x$ is

**A.** $4\pi$ **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{12}$

**12.** The period of $\sec^6 x$ is

**A.** $\pi$ **B.** $\frac{\pi}{6}$ **C.** $2\pi$ **D.** $6\pi$

**13.** The period of $2\,\mathrm{cosec}^6 x + 2\sec^6 x$ is

**A.** $\frac{\pi}{2}$ **B.** $2\pi$ **C.** $\frac{\pi}{4}$ **D.** $\pi$

**14.** The period of $4\tan^4 x + \cot^4 x$ is

**A.** $\frac{\pi}{4}$ **B.** $\frac{\pi}{2}$ **C.** $\pi$ **D.** $2\pi$

**15.** The period of $2|\,\mathrm{cosec}\,x| + 3|\sec x|$ is

**A.** $\frac{\pi}{4}$ **B.** $2\pi$ **C.** $\frac{\pi}{2}$ **D.** $\pi$

**16.** The period of $7x - [7x]$ is

**A.** $7$ **B.** $1$ **C.** $\frac{2}{7}$ **D.** $\frac{1}{7}$

**17.** Among $\tan\frac{x}{2}$, $\sec^3 x$, $|\tan x| + |\cot x|$ and $\sin\sqrt{x}$, the function that is NOT periodic is

**A.** $\sec^3 x$ **B.** $\sin\sqrt{x}$

**C.** $\tan\frac{x}{2}$ **D.** $|\tan x| + |\cot x|$

**18.** The range of $2 + \tan^2 x$ is

**A.** $(-\infty, 2]$ **B.** $[-2, \infty)$ **C.** $[2, \infty)$ **D.** $[0, \infty)$

**19.** The range of $3 - 2\,\mathrm{cosec}^2 x$ is

**A.** $[5, \infty)$ **B.** $(-\infty, -2]$ **C.** $(-\infty, 1]$ **D.** $(-\infty, -5]$

**20.** The minimum value of $3\cos x + 4\sin x - 2$ is

**A.** $-9$ **B.** $-7$ **C.** $-5$ **D.** $3$

**21.** The range of $15\cos x + 8\sin x + 4$ is

**A.** $[-17, 17]$ **B.** $[-13, 21]$ **C.** $[-19, 27]$ **D.** $[3, 5]$

**22.** The range of $\dfrac{1}{-4\cos x + 3\sin x + 7}$ is

**A.** $[\frac{1}{12}, \frac{1}{2}]$ **B.** $[\frac{1}{12}, \infty)$

**C.** $[-\frac{1}{5}, \frac{1}{5}]$ **D.** $[2, 12]$

**23.** The range of $\dfrac{1}{5\cos x - 12\sin x + 14}$ is

**A.** $[\frac{1}{27}, 1]$ **B.** $[1, 27]$

**C.** $[-\frac{1}{13}, \frac{1}{13}]$ **D.** $[\frac{1}{27}, \infty)$

**24.** The minimum value of $49\cos^2\theta + 36\sec^2\theta$ is

**A.** $85$ **B.** $84$ **C.** $42$ **D.** $26$

**25.** The minimum value of $64\tan\theta + 36\cot\theta$ for acute $\theta$ is

**A.** $28$ **B.** $100$ **C.** $48$ **D.** $96$

## Key sheet

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|---|---|---|---|---|
| **1.** C | **2.** A | **3.** D | **4.** D | **5.** A |
| **6.** C | **7.** D | **8.** D | **9.** C | **10.** B |
| **11.** B | **12.** A | **13.** A | **14.** C | **15.** D |
| **16.** D | **17.** B | **18.** C | **19.** C | **20.** B |
| **21.** B | **22.** A | **23.** A | **24.** B | **25.** D |

## Solutions

**1.** The constant function $f(x) = 5$ is

$f(x+p) = f(x)$ for every $p > 0$, so $f$ is periodic.
There is no least positive $p$, so it has no fundamental period.

**Ans: (C) periodic, with no fundamental period**

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**2.** If $f(x+2) = f(x)$ for all real $x$ and $f(3) = 11$, then $f(9) =$

$9 = 3 + 3 \times 2$
$f(9) = f(3) = 11$

**Ans: (A)** $11$

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**3.** Among $|\sin x|$, $|\cot x|$, $\sec x$ and $\tan x$, the function with period $2\pi$ is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period $\pi$; the other four have period $2\pi$.
So $\sec x$ has period $2\pi$.

**Ans: (D)** $\sec x$

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**4.** The period of $|\cos x|$ is

Taking the absolute value folds the graph, so it repeats after $\pi$.
Period $= \pi$.

**Ans: (D)** $\pi$

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**5.** The period of $\sin\left(4x + \frac{\pi}{6}\right)$ is

Period of $\sin x$ is $2\pi$; divide by $|a| = 4$.
Period $= \frac{\pi}{2}$.

**Ans: (A)** $\frac{\pi}{2}$

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**6.** The period of $\sin(8x - 2)$ is

Period of $\sin x$ is $2\pi$; divide by $|a| = 8$.
Period $= \frac{\pi}{4}$.

**Ans: (C)** $\frac{\pi}{4}$

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**7.** The period of $\tan(7x - 2)$ is

Period of $\tan x$ is $\pi$; divide by $|a| = 7$.
Period $= \frac{\pi}{7}$.

**Ans: (D)** $\frac{\pi}{7}$

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**8.** The period of $\text{cosec}\,\frac{2x}{3} - \cos\frac{x}{3}$ is

Periods: $3\pi$ and $6\pi$.
L.C.M. $= 6\pi$.

**Ans: (D)** $6\pi$

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**9.** Among $\tan 4x + \operatorname{cosec} x$, $\operatorname{cosec} \frac{x}{3} + \sec 4x$, $\cos 3x + \tan 2x$ and $\tan 2x + \operatorname{cosec} 3\pi x$, the function that is NOT periodic is

In $\tan 2x + \operatorname{cosec} 3\pi x$ one period is a multiple of $\pi$ (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

**Ans: (C)** $\tan 2x + \operatorname{cosec} 3\pi x$

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**10.** The period of $\cos \frac{3x}{4} - \cot \frac{3x}{2}$ is

Periods: $\frac{8\pi}{3}$ and $\frac{2\pi}{3}$.
L.C.M. $= \frac{8\pi}{3}$.

**Ans: (B)** $\frac{8\pi}{3}$

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**11.** The period of $\tan^3 4x$ is

$\tan^3 x$ has period $\pi$.
So the period is $\frac{\pi}{4}$.

**Ans: (B)** $\frac{\pi}{4}$

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**12.** The period of $\sec^6 x$ is

$\sec^6 x$ has period $\pi$.
So the period is $\pi$.

**Ans: (A)** $\pi$

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**13.** The period of $2\operatorname{cosec}^6 x + 2\sec^6 x$ is

Table: $n = 6$, coefficients equal.
Period $= \frac{\pi}{2}$.

**Ans: (A)** $\frac{\pi}{2}$

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**14.** The period of $4\tan^4 x + \cot^4 x$ is

Table: $n = 4$, coefficients unequal.
Period $= \pi$.

**Ans: (C)** $\pi$

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**15.** The period of $2|\operatorname{cosec} x| + 3|\sec x|$ is

Absolute values with unequal coefficients.
Period $= \pi$.

**Ans: (D)** $\pi$

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**16.** The period of $7x - [7x]$ is

$x - [x]$ has period $1$.
So $7x - [7x]$ has period $\frac{1}{7}$.

**Ans: (D)** $\frac{1}{7}$

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**17.** Among $\tan\frac{x}{2}$, $\sec^3 x$, $|\tan x| + |\cot x|$ and $\sin\sqrt{x}$, the function that is NOT periodic is

The zeros of $\sin\sqrt{x}$ get further apart (or closer together) as $x$ grows.

So no shift maps its graph onto itself: it is not periodic.

**Ans: (B)** $\sin\sqrt{x}$

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**18.** The range of $2 + \tan^2 x$ is

$\tan^2 x$ has range $[0, \infty)$.

So $2 + \tan^2 x$ has range $[2, \infty)$.

**Ans: (C)** $[2, \infty)$

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**19.** The range of $3 - 2\,\mathrm{cosec}^2 x$ is

$\mathrm{cosec}^2 x$ has range $[1, \infty)$.

So $3 - 2\,\mathrm{cosec}^2 x$ has range $(-\infty, 1]$.

**Ans: (C)** $(-\infty, 1]$

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**20.** The minimum value of $3\cos x + 4\sin x - 2$ is

$\sqrt{a^2 + b^2} = 5$

Minimum $= -2 - 5 = -7$.

**Ans: (B)** $-7$

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**21.** The range of $15\cos x + 8\sin x + 4$ is

$\sqrt{225 + 64} = 17$

Range $= [-13, 21]$.

**Ans: (B)** $[-13, 21]$

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**22.** The range of $\dfrac{1}{-4\cos x + 3\sin x + 7}$ is

The denominator has range $[2, 12]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{12}, \frac{1}{2}]$.

**Ans: (A)** $[\frac{1}{12}, \frac{1}{2}]$

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**23.** The range of $\dfrac{1}{5\cos x - 12\sin x + 14}$ is

The denominator has range $[1, 27]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b}, \frac{1}{a}\right]$.

Range $= [\frac{1}{27}, 1]$.

**Ans: (A)** $[\frac{1}{27}, 1]$

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**24.** The minimum value of $49\cos^2\theta + 36\sec^2\theta$ is

A.M. $\geq$ G.M. gives $\geq 84$, and equality needs $\cos^2\theta = \frac{6}{7}$.

Here $\frac{6}{7} \leq 1$, so the minimum is $84$.

**Ans: (B)** $84$

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**25.** The minimum value of $64\tan\theta + 36\cot\theta$ for acute $\theta$ is

$\geq 2\sqrt{64 \cdot 36} = 96$

Equality is possible, so the minimum is $96$.

**Ans: (D)** $96$