EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 10

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $f(x+7) = f(x)$ for all real x and $f(-4) = 6$, then $f(-25) =$

A. -18 **B.** -15 **C.** 13 **D.** 6

2. If the fundamental period of $f(x)$ is $\frac{2\pi}{9}$, the period of $-f(x)$ is

A. $\frac{\pi}{9}$ **B.** $\frac{2\pi}{9}$ **C.** $\frac{4\pi}{9}$ **D.** $\frac{9}{2\pi}$

3. Among $\operatorname{cosec} x$, $\cot x$, $\sin x$ and $\cos x$, the function with period π is

A. $\cot x$ **B.** $\operatorname{cosec} x$ **C.** $\sin x$ **D.** $\cos x$

4. The fundamental period of $|\operatorname{cosec} x|$ is

A. 2π **B.** π **C.** 4π **D.** $\frac{\pi}{2}$

5. The period of $\cot\left(3x + \frac{\pi}{4}\right)$ is

A. $\frac{\pi}{3}$ **B.** 3π **C.** π **D.** $\frac{2\pi}{3}$

6. The period of $\operatorname{cosec} 6x$ is

A. 2π **B.** 12π **C.** $\frac{\pi}{3}$ **D.** $\frac{\pi}{6}$

7. The period of $\sin(7x + 5)$ is

A. $\frac{2\pi}{7}$ **B.** $\frac{\pi}{7}$ **C.** 14π **D.** 2π

8. The period of $\cos\frac{x}{2} - \operatorname{cosec} 3x$ is

A. $\frac{14\pi}{3}$ **B.** $\frac{2\pi}{3}$ **C.** $\frac{8\pi}{3}$ **D.** 4π

9. Among $\sin 3x + \sin\frac{x}{3}$, $\tan 3x + \cos \pi x$, $\cos\frac{x}{2} + \cot 2x$ and $\cot\frac{x}{3} + \operatorname{cosec} x$, the function that is NOT periodic is

A. $\cos\frac{x}{2} + \cot 2x$ **B.** $\sin 3x + \sin\frac{x}{3}$

C. $\tan 3x + \cos \pi x$ **D.** $\cot\frac{x}{3} + \operatorname{cosec} x$

10. The period of $\sec 4x + \cos\frac{3x}{4}$ is

A. $\frac{19\pi}{6}$ **B.** $\frac{4\pi}{3}$ **C.** 8π **D.** $\frac{\pi}{6}$

11. The period of $\sec^4 2x$ is

A. $\frac{\pi}{8}$ **B.** $\frac{\pi}{2}$ **C.** π **D.** 2π

12. The period of $\tan^3 5x$ is

A. $\frac{2\pi}{5}$ **B.** $\frac{\pi}{5}$ **C.** $\frac{\pi}{15}$ **D.** 5π

13. The period of $5\tan^6 x + 5\cot^6 x$ is

A. $\frac{\pi}{4}$ **B.** π **C.** 2π **D.** $\frac{\pi}{2}$

14. The period of $4\sin^4 x + 5\cos^4 x$ is

A. π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** 2π

15. The period of $5|\tan x| + 5|\cot x|$ is

A. π **B.** 2π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

16. The period of $\frac{x}{4} - [\frac{x}{4}]$ is

A. 4 **B.** 1 **C.** $\frac{1}{4}$ **D.** 8

17. Among $\cos\sqrt{x}$, $x - [x]$, $\cos^4 2x$ and $\sin^2 x$, the function that is NOT periodic is

A. $\cos^4 2x$ **B.** $\sin^2 x$

C. $\cos\sqrt{x}$ **D.** $x - [x]$

18. The range of $3 + 4\sin^3 x$ is

A. $[-4, 4]$ **B.** $[2, 4]$ **C.** $[-1, 7]$ **D.** $[-7, 1]$

19. The range of $2 + 4\sin^2 x$ is

A. $[-2, 2]$ **B.** $[2, 6]$ **C.** $[0, 4]$ **D.** $[2, 3]$

20. The range of $2\cos x + 3\sin x + 6$ is

A. $[-\sqrt{13}, \sqrt{13}]$ **B.** $[6 - \sqrt{13}, \sqrt{13} + 6]$

C. $[1, 11]$ **D.** $[5, 7]$

21. The maximum value of $3\cos x + 4\sin x$ is

A. 5 **B.** 25 **C.** -5 **D.** 7

22. The range of $\dfrac{1}{-3\cos x + 4\sin x + 1}$ is

A. $[-4, 6]$ **B.** $(-\infty, -\frac{1}{4}] \cup [\frac{1}{6}, \infty)$

C. $[-\frac{1}{4}, \frac{1}{6}]$ **D.** $[-\frac{1}{5}, \frac{1}{5}]$

23. The range of $\dfrac{1}{-8\cos x + 15\sin x - 18}$ is

A. $[-\frac{1}{17}, \frac{1}{17}]$ **B.** $(-\infty, -1]$

C. $[-35, -1]$ **D.** $[-1, -\frac{1}{35}]$

24. The minimum value of $36\tan\theta + 25\cot\theta$ for acute θ is

A. 30 **B.** 61 **C.** 22 **D.** 60

25. The minimum value of $36\sin^2\theta + 9\,\mathrm{cosec}^2\theta$ is

A. 18 **B.** 27 **C.** 36 **D.** 45

Key sheet

1. D	**2.** B	**3.** A	**4.** B	**5.** A
6. C	**7.** A	**8.** D	**9.** C	**10.** C
11. B	**12.** B	**13.** D	**14.** A	**15.** D
16. A	**17.** C	**18.** C	**19.** B	**20.** B
21. A	**22.** B	**23.** D	**24.** D	**25.** C

Solutions

1. If $f(x+7) = f(x)$ for all real x and $f(-4) = 6$, then $f(-25) =$

$-25 = -4 - 3 \times 7$

$f(-25) = f(-4) = 6$

Ans: (D) 6

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2. If the fundamental period of $f(x)$ is $\frac{2\pi}{9}$, the period of $-f(x)$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is $\frac{2\pi}{9}$.

Ans: (B) $\frac{2\pi}{9}$

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3. Among $\operatorname{cosec} x$, $\cot x$, $\sin x$ and $\cos x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $\cot x$ has period π.

Ans: (A) $\cot x$

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4. The fundamental period of $|\operatorname{cosec} x|$ is

Taking the absolute value folds the graph, so it repeats after π.

Period $= \pi$.

Ans: (B) π

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5. The period of $\cot\left(3x + \frac{\pi}{4}\right)$ is

Period of $\cot x$ is π; divide by $|a| = 3$.

Period $= \frac{\pi}{3}$.

Ans: (A) $\frac{\pi}{3}$

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6. The period of $\operatorname{cosec} 6x$ is

Period of $\operatorname{cosec} x$ is 2π; divide by $|a| = 6$.

Period $= \frac{\pi}{3}$.

Ans: (C) $\frac{\pi}{3}$

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7. The period of $\sin(7x + 5)$ is

Period of $\sin x$ is 2π; divide by $|a| = 7$.

Period $= \frac{2\pi}{7}$.

Ans: (A) $\frac{2\pi}{7}$

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8. The period of $\cos\frac{x}{2} - \operatorname{cosec} 3x$ is

Periods: 4π and $\frac{2\pi}{3}$.

L.C.M. $= 4\pi$.

Ans: (D) 4π

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9. Among $\sin 3x + \sin \frac{x}{3}$, $\tan 3x + \cos \pi x$, $\cos \frac{x}{2} + \cot 2x$ and $\cot \frac{x}{3} + \operatorname{cosec} x$, the function that is NOT periodic is

In $\tan 3x + \cos \pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (C) $\tan 3x + \cos \pi x$

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10. The period of $\sec 4x + \cos \frac{3x}{4}$ is

Periods: $\frac{\pi}{2}$ and $\frac{8\pi}{3}$.
L.C.M. $= 8\pi$.

Ans: (C) 8π

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11. The period of $\sec^4 2x$ is

$\sec^4 x$ has period π.
So the period is $\frac{\pi}{2}$.

Ans: (B) $\frac{\pi}{2}$

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12. The period of $\tan^3 5x$ is

$\tan^3 x$ has period π.
So the period is $\frac{\pi}{5}$.

Ans: (B) $\frac{\pi}{5}$

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13. The period of $5 \tan^6 x + 5 \cot^6 x$ is

Table: $n = 6$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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14. The period of $4 \sin^4 x + 5 \cos^4 x$ is

Table: $n = 4$, coefficients unequal.
Period $= \pi$.

Ans: (A) π

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15. The period of $5|\tan x| + 5|\cot x|$ is

Absolute values with equal coefficients.
Period $= \frac{\pi}{2}$.

Ans: (D) $\frac{\pi}{2}$

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16. The period of $\frac{x}{4} - \left[\frac{x}{4}\right]$ is

$x - [x]$ has period 1.
So $\frac{x}{4} - \left[\frac{x}{4}\right]$ has period $\frac{1}{\frac{1}{4}} = 4$.

Ans: (A) 4

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17. Among $\cos\sqrt{x}$, $x-[x]$, $\cos^4 2x$ and $\sin^2 x$, the function that is NOT periodic is

The zeros of $\cos\sqrt{x}$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (C) $\cos\sqrt{x}$

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18. The range of $3+4\sin^3 x$ is

$\sin^3 x$ has range $[-1,1]$.

So $3+4\sin^3 x$ has range $[-1,7]$.

Ans: (C) $[-1,7]$

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19. The range of $2+4\sin^2 x$ is

$\sin^2 x$ has range $[0,1]$.

So $2+4\sin^2 x$ has range $[2,6]$.

Ans: (B) $[2,6]$

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20. The range of $2\cos x+3\sin x+6$ is

$\sqrt{4+9}=\sqrt{13}$

Range $=[6-\sqrt{13},\sqrt{13}+6]$.

Ans: (B) $[6-\sqrt{13},\sqrt{13}+6]$

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21. The maximum value of $3\cos x+4\sin x$ is

$\sqrt{a^2+b^2}=5$

Maximum $=0+5=5$.

Ans: (A) 5

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22. The range of $\dfrac{1}{-3\cos x+4\sin x+1}$ is

The denominator has range $[-4,6]$.

The range of f contains 0 inside it, so the range of $\frac{1}{f}$ is $\left(-\infty,\frac{1}{a}\right]\cup\left[\frac{1}{b},\infty\right)$.

Range $=(-\infty,-\frac{1}{4}]\cup[\frac{1}{6},\infty)$.

Ans: (B) $(-\infty,-\frac{1}{4}]\cup[\frac{1}{6},\infty)$

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23. The range of $\dfrac{1}{-8\cos x+15\sin x-18}$ is

The denominator has range $[-35,-1]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$.

Range $=[-1,-\frac{1}{35}]$.

Ans: (D) $[-1,-\frac{1}{35}]$

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24. The minimum value of $36\tan\theta+25\cot\theta$ for acute θ is

$\geq 2\sqrt{36\cdot 25}=60$

Equality is possible, so the minimum is 60.

Ans: (D) 60

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25. The minimum value of $36\sin^2\theta + 9\,\mathrm{cosec}^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 36, and equality needs $\sin^2\theta = \frac{1}{2}$.

Here $\frac{1}{2} \leq 1$, so the minimum is 36.

Ans: (C) 36
