EXPRESSWALA ACADEMY

Maths 1A: Periodicity and Extreme Values

MCQ Test, Set 01

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $f(x+8) = f(x)$ for all real x and $f(1) = -7$, then $f(-31) =$

A. -7 **B.** -39 **C.** 28 **D.** 1

2. If the fundamental period of $f(x)$ is $\frac{7\pi}{2}$, the period of $-f(x)$ is

A. 7π **B.** $\frac{7\pi}{2}$ **C.** $\frac{7\pi}{4}$ **D.** $\frac{2}{7\pi}$

3. Among $\cos x$, $|\sec x|$, $\sin x$ and $\operatorname{cosec} x$, the function with period π is

A. $\operatorname{cosec} x$ **B.** $\sin x$ **C.** $\cos x$ **D.** $|\sec x|$

4. The fundamental period of $\tan x$ is

A. 2π **B.** 4π **C.** π **D.** $\frac{\pi}{2}$

5. The period of $\sec(4x+1)$ is

A. $\frac{\pi}{2}$ **B.** 2π **C.** $\frac{\pi}{4}$ **D.** 8π

6. The period of $\tan\left(\frac{4x}{5} - \frac{\pi}{4}\right)$ is

A. $\frac{4\pi}{5}$ **B.** π **C.** $\frac{5\pi}{2}$ **D.** $\frac{5\pi}{4}$

7. The period of $\sin\left(5x - \frac{\pi}{4}\right)$ is

A. $\frac{\pi}{5}$ **B.** $\frac{2\pi}{5}$ **C.** 10π **D.** 2π

8. The period of $\cot\frac{3x}{2} + \cos 4x$ is

A. $\frac{\pi}{3}$ **B.** $\frac{7\pi}{6}$ **C.** 2π **D.** $\frac{\pi}{6}$

9. Among $\operatorname{cosec} 3x + \sin \pi x$, $\sec 2x + \cot 4x$, $\tan 2x + \operatorname{cosec}\frac{x}{3}$ and $\sec\frac{x}{3} + \operatorname{cosec} 3x$, the function that is NOT periodic is

A. $\tan 2x + \operatorname{cosec}\frac{x}{3}$ **B.** $\operatorname{cosec} 3x + \sin \pi x$

C. $\sec 2x + \cot 4x$ **D.** $\sec\frac{x}{3} + \operatorname{cosec} 3x$

10. The period of $\tan 6x - \tan 2x$ is

A. $\frac{\pi}{2}$ **B.** $\frac{2\pi}{3}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{\pi}{12}$

11. The period of $\cos^3\frac{x}{2}$ is

A. 4π **B.** π **C.** $\frac{4\pi}{3}$ **D.** 2π

12. The period of $\operatorname{cosec}^3 5x$ is

A. 10π **B.** $\frac{2\pi}{5}$ **C.** $\frac{2\pi}{15}$ **D.** $\frac{\pi}{5}$

13. The period of $4\tan^2 x + 4\cot^2 x$ is

A. 2π **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{2}$ **D.** π

14. The period of $\operatorname{cosec}^4 x + 5\sec^4 x$ is

A. π **B.** $\frac{\pi}{2}$ **C.** 2π **D.** $\frac{\pi}{4}$

15. The period of $3|\tan x| + 4|\cot x|$ is

A. 2π **B.** $\frac{\pi}{4}$ **C.** π **D.** $\frac{\pi}{2}$

16. The period of $2x - [2x]$ is

A. 2 **B.** $\frac{\pi}{2}$ **C.** $\frac{1}{2}$ **D.** 1

17. Among $\cos^4 2x$, $\sec^3 x$, $|\cos 3x|$ and $\tan\sqrt{x}$, the function that is NOT periodic is

A. $\sec^3 x$ **B.** $\tan\sqrt{x}$

C. $\cos^4 2x$ **D.** $|\cos 3x|$

18. The range of $-2 - \cot^2 x$ is

A. $(-\infty, 0]$ **B.** $(-\infty, 2]$ **C.** $[-2, \infty)$ **D.** $(-\infty, -2]$

19. The range of $-1 + 2\operatorname{cosec}^2 x$ is

A. $[3, \infty)$ **B.** $[2, \infty)$ **C.** $(-\infty, -3]$ **D.** $[1, \infty)$

20. The range of $6\cos x + 8\sin x + 1$ is

A. $[0, 2]$ **B.** $[-9, 11]$ **C.** $[-13, 15]$ **D.** $[-10, 10]$

21. The maximum value of $\cos x + \sin x - 3$ is

A. -1 **B.** $-3 - \sqrt{2}$ **C.** $\sqrt{2}$ **D.** $-3 + \sqrt{2}$

22. The range of $\dfrac{1}{8\cos x + 6\sin x + 14}$ is

A. $[4, 24]$ **B.** $[\frac{1}{24}, \infty)$

C. $[\frac{1}{24}, \frac{1}{4}]$ **D.** $[-\frac{1}{10}, \frac{1}{10}]$

23. The range of $\dfrac{1}{-24\cos x + 7\sin x + 27}$ is

A. $[\frac{1}{52}, \infty)$ **B.** $[-\frac{1}{25}, \frac{1}{25}]$

C. $[2, 52]$ **D.** $[\frac{1}{52}, \frac{1}{2}]$

24. The minimum value of $25\tan\theta + 4\cot\theta$ for acute θ is

A. 10 **B.** 20 **C.** 14 **D.** 29

25. The minimum value of $1\sin^2\theta + 49\operatorname{cosec}^2\theta$ is

A. 50 **B.** 14 **C.** 7 **D.** 16

Key sheet

1. A	**2.** B	**3.** D	**4.** C	**5.** A
6. D	**7.** B	**8.** C	**9.** B	**10.** A
11. A	**12.** B	**13.** C	**14.** A	**15.** C
16. C	**17.** B	**18.** D	**19.** D	**20.** B
21. D	**22.** C	**23.** D	**24.** B	**25.** A

Solutions

1. If $f(x+8) = f(x)$ for all real x and $f(1) = -7$, then $f(-31) =$

$-31 = 1 - 4 \times 8$

$f(-31) = f(1) = -7$

Ans: (A) -7

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2. If the fundamental period of $f(x)$ is $\frac{7\pi}{2}$, the period of $-f(x)$ is

$-f$ and $\frac{1}{f}$ have the same period as f.

So the period is $\frac{7\pi}{2}$.

Ans: (B) $\frac{7\pi}{2}$

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3. Among $\cos x$, $|\sec x|$, $\sin x$ and $\operatorname{cosec} x$, the function with period π is

$\tan x$, $\cot x$ and $|\text{any of the six}|$ have period π; the other four have period 2π.

So $|\sec x|$ has period π.

Ans: (D) $|\sec x|$

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4. The fundamental period of $\tan x$ is

$\tan$ and $\cot$ repeat after π.

Period $= \pi$.

Ans: (C) π

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5. The period of $\sec(4x+1)$ is

Period of $\sec x$ is 2π; divide by $|a| = 4$.

Period $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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6. The period of $\tan\left(\frac{4x}{5} - \frac{\pi}{4}\right)$ is

Period of $\tan x$ is π; divide by $|a| = \frac{4}{5}$.

Period $= \frac{5\pi}{4}$.

Ans: (D) $\frac{5\pi}{4}$

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7. The period of $\sin\left(5x - \frac{\pi}{4}\right)$ is

Period of $\sin x$ is 2π; divide by $|a| = 5$.

Period $= \frac{2\pi}{5}$.

Ans: (B) $\frac{2\pi}{5}$

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8. The period of $\cot\frac{3x}{2} + \cos 4x$ is

Periods: $\frac{2\pi}{3}$ and $\frac{\pi}{2}$.

L.C.M. $= 2\pi$.

Ans: (C) 2π

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9. Among $\operatorname{cosec} 3x + \sin \pi x$, $\sec 2x + \cot 4x$, $\tan 2x + \operatorname{cosec} \frac{x}{3}$ and $\sec \frac{x}{3} + \operatorname{cosec} 3x$, the function that is NOT periodic is

In $\operatorname{cosec} 3x + \sin \pi x$ one period is a multiple of π (irrational) and the other is rational.
A rational and an irrational number have no L.C.M., so it is not periodic.

Ans: (B) $\operatorname{cosec} 3x + \sin \pi x$

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10. The period of $\tan 6x - \tan 2x$ is

Periods: $\frac{\pi}{6}$ and $\frac{\pi}{2}$.
L.C.M. $= \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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11. The period of $\cos^3 \frac{x}{2}$ is

$\cos^3 x$ has period 2π.
So the period is 4π.

Ans: (A) 4π

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12. The period of $\operatorname{cosec}^3 5x$ is

$\operatorname{cosec}^3 x$ has period 2π.
So the period is $\frac{2\pi}{5}$.

Ans: (B) $\frac{2\pi}{5}$

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13. The period of $4 \tan^2 x + 4 \cot^2 x$ is

Table: $n = 2$, coefficients equal.
Period $= \frac{\pi}{2}$.

Ans: (C) $\frac{\pi}{2}$

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14. The period of $\operatorname{cosec}^4 x + 5 \sec^4 x$ is

Table: $n = 4$, coefficients unequal.
Period $= \pi$.

Ans: (A) π

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15. The period of $3|\tan x| + 4|\cot x|$ is

Absolute values with unequal coefficients.
Period $= \pi$.

Ans: (C) π

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16. The period of $2x - [2x]$ is

$x - [x]$ has period 1.
So $2x - [2x]$ has period $\frac{1}{2}$.

Ans: (C) $\frac{1}{2}$

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17. Among $\cos^4 2x$, $\sec^3 x$, $|\cos 3x|$ and $\tan\sqrt{x}$, the function that is NOT periodic is

The zeros of $\tan\sqrt{x}$ get further apart (or closer together) as x grows.

So no shift maps its graph onto itself: it is not periodic.

Ans: (B) $\tan\sqrt{x}$

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18. The range of $-2-\cot^2 x$ is

$\cot^2 x$ has range $[0,\infty)$.

So $-2-\cot^2 x$ has range $(-\infty,-2]$.

Ans: (D) $(-\infty,-2]$

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19. The range of $-1+2\operatorname{cosec}^2 x$ is

$\operatorname{cosec}^2 x$ has range $[1,\infty)$.

So $-1+2\operatorname{cosec}^2 x$ has range $[1,\infty)$.

Ans: (D) $[1,\infty)$

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20. The range of $6\cos x+8\sin x+1$ is

$\sqrt{36+64}=10$

Range $=[-9,11]$.

Ans: (B) $[-9,11]$

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21. The maximum value of $\cos x+\sin x-3$ is

$\sqrt{a^2+b^2}=\sqrt{2}$

Maximum $=-3+\sqrt{2}=-3+\sqrt{2}$.

Ans: (D) $-3+\sqrt{2}$

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22. The range of $\dfrac{1}{8\cos x+6\sin x+14}$ is

The denominator has range $[4,24]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$.

Range $=[\frac{1}{24},\frac{1}{4}]$.

Ans: (C) $[\frac{1}{24},\frac{1}{4}]$

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23. The range of $\dfrac{1}{-24\cos x+7\sin x+27}$ is

The denominator has range $[2,52]$.

Both ends have the same sign, so the range of $\frac{1}{f}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$.

Range $=[\frac{1}{52},\frac{1}{2}]$.

Ans: (D) $[\frac{1}{52},\frac{1}{2}]$

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24. The minimum value of $25\tan\theta+4\cot\theta$ for acute θ is

$\geq 2\sqrt{25\cdot 4}=20$

Equality is possible, so the minimum is 20.

Ans: (B) 20

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25. The minimum value of $1\sin^2\theta + 49\,\mathrm{cosec}^2\theta$ is

A.M. $\geq$ G.M. gives ≥ 14, and equality needs $\sin^2\theta = 7$.

Here $7 > 1$, so the minimum is at $\sin^2\theta = 1$: 50.

Ans: (A) 50
