EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = \frac{2}{5}$, find $\sin 2A$. [1 mark]

2. If $\cos A = -\frac{15}{17}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. If $\cos 2A = \frac{1}{5}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin A$. [2 marks]

4. If $\tan A = \frac{1}{6}$, find $\tan 3A$. [2 marks]

5. If $\cos A = -\frac{8}{9}$ and $0 < A < \pi$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3 marks]

6. If $\tan \frac{A}{2} = -\frac{2}{3}$, find $\cos A$. [1 mark]

7. Show that $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$. [5 marks]

8. Write the exact value of $\sin 198^\circ$. [1 mark]

9. Find $\sin 18^\circ$ from $\sin 2\theta = \cos 3\theta$ with $\theta = 18^\circ$, and hence evaluate $\sec 36^\circ$. [5 marks]

10. Find the exact value of $\tan 82\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = \frac{4}{5}$, find $\sin^4\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\cot 30^\circ - \tan 30^\circ$. [2 marks]

13. Prove that $\cot A - \tan A - 2\tan 2A = 4\cot 4A$. [5 marks]

14. Find the value of $(2\cos 67\frac{1}{2}^\circ - 1)(2\cos 67\frac{1}{2}^\circ + 1)(2\cos 135^\circ - 1)$. [3 marks]

15. Find the value of $\cos 20^\circ + \cos 60^\circ + \cos 100^\circ + \cos 140^\circ$. [3 marks]

16. Find the value of $4\left(\operatorname{cosec}^2 \frac{\pi}{7} + \operatorname{cosec}^2 \frac{2\pi}{7} + \operatorname{cosec}^2 \frac{3\pi}{7}\right)$. [1 mark]

17. Show that $\cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta$. Hence evaluate $\cos 5^\circ \cos 55^\circ \cos 65^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\tan 142\frac{1}{2}^\circ}{\tan 7\frac{1}{2}^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 60^\circ$, is $\cos \frac{A}{2} + \sin \frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1 + \sin A}$ for $A = 690^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$\frac{20}{29}$
2	$\sin 2A = -\frac{240}{289},\ \cos 2A = \frac{161}{289},\ \tan 2A = -\frac{240}{161}$
3	$\sin A = -\frac{\sqrt{10}}{5}$
4	$\frac{107}{198}$
5	$\sin \frac{A}{2} = \frac{\sqrt{34}}{6},\ \cos \frac{A}{2} = \frac{\sqrt{2}}{6},\ \tan \frac{A}{2} = \sqrt{17}$
6	$\frac{5}{13}$
7	Proved: $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$
8	$-\frac{\sqrt{5}-1}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\sec 36^\circ = \sqrt{5} - 1$
10	$\sqrt{2} + \sqrt{3}$
11	$\frac{17}{25}$
12	$\frac{2\sqrt{3}}{3}$
13	Proved: $\cot A - \tan A - 2\tan 2A = 4\cot 4A$
14	1
15	$\frac{1}{2}$
16	32
17	Proved; $\cos 5^\circ \cos 55^\circ \cos 65^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$
18	$\frac{\sqrt{2}}{2}$
19	Positive
20	$\cos 345^\circ + \sin 345^\circ = \frac{\sqrt{2}}{2}$

Solutions

1. If $\tan A = \frac{2}{5}$, find $\sin 2A$. [1]

$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$= \frac{20}{29}$

Ans: $\frac{20}{29}$

---

2. If $\cos A = -\frac{15}{17}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\sin A = \sqrt{1-\cos^2 A} = \frac{8}{17}$ ($\sin A$ is positive in this quadrant)

$\sin 2A = 2\sin A\cos A = 2(\frac{8}{17})(-\frac{15}{17}) = -\frac{240}{289}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{225}{289} - \frac{64}{289} = \frac{161}{289}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = -\frac{240}{161}$

Ans: $\sin 2A = -\frac{240}{289},\ \cos 2A = \frac{161}{289},\ \tan 2A = -\frac{240}{161}$

---

3. If $\cos 2A = \frac{1}{5}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin A$. [2]

$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$

$\sqrt{\frac{2}{5}} = \frac{\sqrt{10}}{5}$

In this quadrant $\sin A$ is negative, so $\sin A = -\frac{\sqrt{10}}{5}$.

Ans: $\sin A = -\frac{\sqrt{10}}{5}$

---

4. If $\tan A = \frac{1}{6}$, find $\tan 3A$. [2]

$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$= \frac{3(\frac{1}{6}) - (\frac{1}{6})^3}{1-3(\frac{1}{6})^2}$

$= \frac{107}{198}$

Ans: $\frac{107}{198}$

---

5. If $\cos A = -\frac{8}{9}$ and $0 < A < \pi$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3]

Here $0 < \frac{A}{2} < \frac{\pi}{2}$, so $\sin\frac{A}{2}$ is positive and $\cos\frac{A}{2}$ is positive.

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \frac{\sqrt{34}}{6}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \frac{\sqrt{2}}{6}$

$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \sqrt{17}$

Ans: $\sin\frac{A}{2} = \frac{\sqrt{34}}{6},\ \cos\frac{A}{2} = \frac{\sqrt{2}}{6},\ \tan\frac{A}{2} = \sqrt{17}$

---

6. If $\tan\frac{A}{2} = -\frac{2}{3}$, find $\cos A$. [1]

$\cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$= \frac{5}{13}$

Ans: $\frac{5}{13}$

---

7. Show that $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A} = \tan A$. [5]

Numerator $= \sin A + 2\sin A\cos A = \sin A(1+2\cos A)$

Denominator $= \cos A + 2\cos^2 A = \cos A(1+2\cos A)$

LHS $= \frac{\sin A}{\cos A} = \tan A$ = RHS

Ans: Proved: $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A} = \tan A$

---

8. Write the exact value of $\sin 198^\circ$. [1]

$\sin 198^\circ = -\sin 18^\circ$

$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Ans: $-\frac{\sqrt{5}-1}{4}$

---

9. Find $\sin 18^\circ$ from $\sin 2\theta = \cos 3\theta$ with $\theta = 18^\circ$, and hence evaluate $\sec 36^\circ$. [5]

Let $\theta = 18^\circ$. Then $5\theta = 90^\circ$, so $2\theta = 90^\circ - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1-\sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

18° is acute, so $\sin 18^\circ > 0$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

$\cos 36^\circ = 1 - 2\sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$

$\sec 36^\circ = \frac{4}{\sqrt{5}+1} = \frac{4(\sqrt{5}-1)}{4} = \sqrt{5}-1$

Ans: $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\sec 36^\circ = \sqrt{5}-1$

---

10. Find the exact value of $\tan 82\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ$. [2]

$\tan 82\frac{1}{2}^\circ = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1),\ \tan 22\frac{1}{2}^\circ = \sqrt{2}-1$

$\tan 82\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}$

Ans: $\sqrt{2}+\sqrt{3}$

---

11. If $\sin 2\theta = \frac{4}{5}$, find $\sin^4\theta + \cos^4\theta$. [1]

$\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$

$= 1 - \frac{1}{2}\cdot\frac{16}{25} = \frac{17}{25}$

Ans: $\frac{17}{25}$

---

12. Find the value of $\cot 30^\circ - \tan 30^\circ$. [2]

$\cot A - \tan A = 2\cot 2A$

With $A = 30^\circ$: $\cot 30^\circ - \tan 30^\circ = \frac{2\sqrt{3}}{3}$

Ans: $\frac{2\sqrt{3}}{3}$

---

13. Prove that $\cot A - \tan A - 2\tan 2A = 4\cot 4A$. [5]

$\cot A - \tan A = 2\cot 2A$

LHS $= 2\cot 2A - 2\tan 2A = 2(\cot 2A - \tan 2A)$

$= 2\cdot 2\cot 4A = 4\cot 4A$ = RHS

Ans: Proved: $\cot A - \tan A - 2\tan 2A = 4\cot 4A$

---

14. Find the value of $(2\cos 67\frac{1}{2}^\circ - 1)(2\cos 67\frac{1}{2}^\circ + 1)(2\cos 135^\circ - 1)$. [3]

$(2\cos\theta - 1)(2\cos\theta + 1) = 2\cos 2\theta + 1$, then $(2\cos 2\theta + 1)(2\cos 2\theta - 1) = 2\cos 4\theta + 1$

$= 2\cos 270^\circ + 1 = 1$

Ans: 1

---

15. Find the value of $\cos 20^\circ + \cos 60^\circ + \cos 100^\circ + \cos 140^\circ$. [3]

$a_1 = 20^\circ$, $d = 40^\circ$, $n = 4$, $a_n = 140^\circ$

$\text{Sum} = \frac{\cos\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}} = \frac{\cos 80^\circ \sin 80^\circ}{\sin 20^\circ}$

The angles in the numerator add to π, so $\cos x \sin x = \frac{1}{2}\sin 2x$ and $\sin(\pi - y) = \sin y$ cancel the denominator.

$= \frac{1}{2}$

Ans: $\frac{1}{2}$

---

16. Find the value of $4\left(\text{cosec}^2\frac{\pi}{7} + \text{cosec}^2\frac{2\pi}{7} + \text{cosec}^2\frac{3\pi}{7}\right)$. [1]

Known result: $\text{cosec}^2\frac{\pi}{7} + \text{cosec}^2\frac{2\pi}{7} + \text{cosec}^2\frac{3\pi}{7} = 8$

$4\left(\text{cosec}^2\frac{\pi}{7} + \text{cosec}^2\frac{2\pi}{7} + \text{cosec}^2\frac{3\pi}{7}\right) = 4\,(8) = 32$

Ans: 32

---

17. Show that $\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta$. Hence evaluate $\cos 5^\circ \cos 55^\circ \cos 65^\circ$. [5]

$\cos(60^\circ + \theta)\cos(60^\circ - \theta) = \cos^2 60^\circ - \sin^2\theta = \cos^2\theta - \frac{3}{4}$

LHS $= \cos\theta\left(\cos^2\theta - \frac{3}{4}\right) = \frac{1}{4}(4\cos^3\theta - 3\cos\theta)$

$= \frac{1}{4}\cos 3\theta$ = RHS

With $\theta = 5^\circ$: $\cos 5^\circ \cos 55^\circ \cos 65^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$

Ans: Proved; $\cos 5^\circ \cos 55^\circ \cos 65^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$

---

18. If $\frac{x}{y} = \frac{\tan 142\frac{1}{2}^\circ}{\tan 7\frac{1}{2}^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y} = \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$

$= \frac{\sin 150^\circ}{\sin 135^\circ} = \frac{\sqrt{2}}{2}$

Ans: $\frac{\sqrt{2}}{2}$

---

19. For $A = 60^\circ$, is $\cos\frac{A}{2} + \sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2} = 30^\circ$

$c + s > 0$ for $\frac{A}{2}$ in $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and < 0 in $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ (adding 2π where needed).

So $\cos 30^\circ + \sin 30^\circ$ is positive.

Ans: Positive

---

20. Simplify $\sqrt{1 + \sin A}$ for $A = 690^\circ$ and find its value. [2]

$\sqrt{1 + \sin A} = \left|\cos\frac{A}{2} + \sin\frac{A}{2}\right|$

$\frac{A}{2} = 345^\circ$, where $\cos\frac{A}{2} + \sin\frac{A}{2}$ is positive.

$\sqrt{1 + \sin 690^\circ} = \cos 345^\circ + \sin 345^\circ = \frac{\sqrt{2}}{2}$

Ans: $\cos 345^\circ + \sin 345^\circ = \frac{\sqrt{2}}{2}$
