EXPRESSWALA ACADEMY

# Maths 1A: Multiple and Submultiple Angles

## Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

**1.** If $\tan A = \frac{4}{5}$, find $\cos 2A$. [1 mark]

**2.** If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

**3.** Using $\cos 2A$, find the exact value of $\cos^2 195^\circ$. [2 marks]

**4.** If $\sin A = -\frac{3}{5}$, find $\sin 3A$. [2 marks]

**5.** If $\cos A = \frac{4}{5}$ and $\pi < A < 2\pi$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3 marks]

**6.** If $\tan \frac{A}{2} = -\frac{3}{2}$, find $\cos A$. [1 mark]

**7.** Prove that $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$. [5 marks]

**8.** Write the exact value of $\sin 216^\circ$. [1 mark]

**9.** Show that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$. Hence find $\cos^2 36^\circ + \cos^2 72^\circ$. [5 marks]

**10.** Find the exact value of $\cot^2 67\frac{1}{2}^\circ - \tan^2 67\frac{1}{2}^\circ$. [2 marks]

**11.** If $\sin 2\theta = -\frac{3}{5}$, find $\sin^2\theta + \cos^4\theta$. [1 mark]

**12.** Find the value of $\operatorname{cosec} 45^\circ - \cot 45^\circ$. [2 marks]

**13.** Show that $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$. [5 marks]

**14.** Find the value of $\left(2\cos\frac{8\pi}{9} - 1\right)\left(2\cos\frac{16\pi}{9} - 1\right)\left(2\cos\frac{32\pi}{9} - 1\right)$. [3 marks]

**15.** Find the value of $\sin 60^\circ + \sin 90^\circ + \sin 120^\circ + \sin 150^\circ + \sin 180^\circ$. [3 marks]

**16.** Find the value of $4\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}$. [1 mark]

**17.** Prove that $\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta$ and hence find $\cos 25^\circ \cos 35^\circ \cos 85^\circ$. [5 marks]

**18.** If $\frac{x}{y} = \frac{\tan 135^\circ}{\tan 15^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

**19.** For $A = 120^\circ$, is $\cos\frac{A}{2} + \sin\frac{A}{2}$ positive or negative? [1 mark]

**20.** Simplify $\sqrt{1 - \sin A}$ for $A = 330^\circ$ and find its value. [2 marks]

## Key sheet

| Q | Answer |
|---|---|
| 1 | $\frac{9}{41}$ |
| 2 | $\sin 2A = -\frac{24}{25},\ \cos 2A = \frac{7}{25},\ \tan 2A = -\frac{24}{7}$ |
| 3 | $\frac{2+\sqrt{3}}{4}$ |
| 4 | $-\frac{117}{125}$ |
| 5 | $\sin\frac{A}{2} = \frac{\sqrt{10}}{10},\ \cos\frac{A}{2} = -\frac{3\sqrt{10}}{10},\ \tan\frac{A}{2} = -\frac{1}{3}$ |
| 6 | $-\frac{5}{13}$ |
| 7 | Proved: $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ |
| 8 | $-\frac{\sqrt{10-2\sqrt{5}}}{4}$ |
| 9 | $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos^2 36^\circ + \cos^2 72^\circ = \frac{3}{4}$ |
| 10 | $-4\sqrt{2}$ |
| 11 | $\frac{91}{100}$ |
| 12 | $\sqrt{2} - 1$ |
| 13 | Proved: $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$ |
| 14 | $1$ |
| 15 | $\frac{3}{2} + \sqrt{3}$ |
| 16 | $-\frac{1}{2}$ |
| 17 | Proved; $\cos 25^\circ \cos 35^\circ \cos 85^\circ = \frac{\sqrt{6}}{16} - \frac{\sqrt{2}}{16}$ |
| 18 | $\frac{\sqrt{3}}{3}$ |
| 19 | Positive |
| 20 | $-(\cos 165^\circ - \sin 165^\circ) = \frac{\sqrt{6}}{2}$ |

## Solutions

**1.** If $\tan A = \frac{4}{5}$, find $\cos 2A$. [1]

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$= \frac{9}{41}$

**Ans:** $\frac{9}{41}$

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**2.** If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\cos A = -\sqrt{1 - \sin^2 A} = -\frac{4}{5}$ ($\cos A$ is negative in this quadrant)

$\sin 2A = 2\sin A \cos A = 2(\frac{3}{5})(-\frac{4}{5}) = -\frac{24}{25}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = -\frac{24}{7}$

**Ans:** $\sin 2A = -\frac{24}{25},\ \cos 2A = \frac{7}{25},\ \tan 2A = -\frac{24}{7}$

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**3.** Using $\cos 2A$, find the exact value of $\cos^2 195^\circ$. [2]

$\cos^2 A = \frac{1+\cos 2A}{2}$

$\cos^2 195^\circ = \frac{1+\cos 390^\circ}{2}$

$\cos 390^\circ = \frac{\sqrt{3}}{2}$

$= \frac{2+\sqrt{3}}{4}$

**Ans:** $\frac{2+\sqrt{3}}{4}$

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**4.** If $\sin A = -\frac{3}{5}$, find $\sin 3A$. [2]

$\sin 3A = 3\sin A - 4\sin^3 A$

$= 3(-\frac{3}{5}) - 4(-\frac{3}{5})^3$

$= -\frac{117}{125}$

**Ans:** $-\frac{117}{125}$

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**5.** If $\cos A = \frac{4}{5}$ and $\pi < A < 2\pi$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3]

Here $\frac{\pi}{2} < \frac{A}{2} < \pi$, so $\sin \frac{A}{2}$ is positive and $\cos \frac{A}{2}$ is negative.

$\sin \frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \frac{\sqrt{10}}{10}$

$\cos \frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\frac{3\sqrt{10}}{10}$

$\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = -\frac{1}{3}$

**Ans:** $\sin \frac{A}{2} = \frac{\sqrt{10}}{10},\ \cos \frac{A}{2} = -\frac{3\sqrt{10}}{10},\ \tan \frac{A}{2} = -\frac{1}{3}$

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**6.** If $\tan \frac{A}{2} = -\frac{3}{2}$, find $\cos A$. [1]

$\cos A = \frac{1-\tan^2 \frac{A}{2}}{1+\tan^2 \frac{A}{2}}$

$= -\frac{5}{13}$

**Ans:** $-\frac{5}{13}$

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**7.** Prove that $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$. [5]

Let $t = \tan A$, so $\tan 2A = \frac{2t}{1-t^2}$.

$\tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

$= \frac{\frac{2t}{1-t^2} + t}{1 - \frac{2t^2}{1-t^2}} = \frac{2t + t - t^3}{1 - t^2 - 2t^2}$

$= \frac{3t - t^3}{1 - 3t^2}$

**Ans: Proved:** $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

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**8.** Write the exact value of $\sin 216°$. [1]

$\sin 216° = -\sin 36°$

$\sin 36° = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

**Ans:** $-\frac{\sqrt{10 - 2\sqrt{5}}}{4}$

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**9.** Show that $\sin 18° = \frac{\sqrt{5}-1}{4}$. Hence find $\cos^2 36° + \cos^2 72°$. [5]

Let $\theta = 18°$. Then $5\theta = 90°$, so $2\theta = 90° - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1 - \sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-1 \pm \sqrt{5}}{4}$

$18°$ is acute, so $\sin 18° > 0$ and $\sin 18° = \frac{\sqrt{5}-1}{4}$.

$\cos 72° = \sin 18°$, $\cos 36° = \frac{\sqrt{5}+1}{4}$

Sum $= \frac{6 + 2\sqrt{5} + 6 - 2\sqrt{5}}{16} = \frac{12}{16} = \frac{3}{4}$

**Ans:** $\sin 18° = \frac{\sqrt{5}-1}{4}$; $\cos^2 36° + \cos^2 72° = \frac{3}{4}$

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**10.** Find the exact value of $\cot^2 67\frac{1}{2}° - \tan^2 67\frac{1}{2}°$. [2]

$\cot 67\frac{1}{2}° = \sqrt{2} - 1,\ \tan 67\frac{1}{2}° = \sqrt{2} + 1$

$\cot^2 67\frac{1}{2}° - \tan^2 67\frac{1}{2}° = -4\sqrt{2}$

**Ans:** $-4\sqrt{2}$

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**11.** If $\sin 2\theta = -\frac{3}{5}$, find $\sin^2\theta + \cos^4\theta$. [1]

$\sin^2\theta + \cos^4\theta = 1 - \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{4} \cdot \frac{9}{25} = \frac{91}{100}$

**Ans:** $\frac{91}{100}$

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**12.** Find the value of $\operatorname{cosec} 45° - \cot 45°$. [2]

$\operatorname{cosec} 2A - \cot 2A = \tan A$

With $A = 22\frac{1}{2}°$: $\operatorname{cosec} 45° - \cot 45° = \sqrt{2} - 1$

**Ans:** $\sqrt{2} - 1$

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**13.** Show that $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$. [5]

$\tan\theta = \cot\theta - 2\cot 2\theta$ (from $\cot\theta - \tan\theta = 2\cot 2\theta$)

$\tan A = \cot A - 2\cot 2A$, $2\tan 2A = 2\cot 2A - 4\cot 4A$, $4\tan 4A = 4\cot 4A - 8\cot 8A$

Adding, the middle terms cancel: $\tan A + 2\tan 2A + 4\tan 4A = \cot A - 8\cot 8A$

So LHS $= \cot A$ = RHS

**Ans: Proved:** $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$

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**14.** Find the value of $\left(2\cos\frac{8\pi}{9}-1\right)\left(2\cos\frac{16\pi}{9}-1\right)\left(2\cos\frac{32\pi}{9}-1\right)$. [3]

$(2\cos\theta-1)(2\cos 2\theta-1)\cdots(2\cos 2^2\theta-1)=\frac{2\cos 2^3\theta+1}{2\cos\theta+1}$ with $\theta=\frac{8\pi}{9}$

$2^3\theta$ differs from $\pm\theta$ by a multiple of $2\pi$, so $\cos 2^3\theta=\cos\theta$ and the value is $1$.

**Ans:** $1$

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**15.** Find the value of $\sin 60^\circ+\sin 90^\circ+\sin 120^\circ+\sin 150^\circ+\sin 180^\circ$. [3]

$a_1=60^\circ$, $d=30^\circ$, $n=5$, $a_n=180^\circ$

$\text{Sum}=\frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}}=\frac{\sin 120^\circ\ \sin 75^\circ}{\sin 15^\circ}$

$=\frac{3}{2}+\sqrt{3}$

**Ans:** $\frac{3}{2}+\sqrt{3}$

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**16.** Find the value of $4\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}$. [1]

Known result: $\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}=-\frac{1}{8}$

$4\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}=4\left(-\frac{1}{8}\right)=-\frac{1}{2}$

**Ans:** $-\frac{1}{2}$

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**17.** Prove that $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)=\frac{1}{4}\cos 3\theta$ and hence find $\cos 25^\circ\cos 35^\circ\cos 85^\circ$. [5]

$\cos(60^\circ+\theta)\cos(60^\circ-\theta)=\cos^2 60^\circ-\sin^2\theta=\cos^2\theta-\frac{3}{4}$

LHS $=\cos\theta\left(\cos^2\theta-\frac{3}{4}\right)=\frac{1}{4}(4\cos^3\theta-3\cos\theta)$

$=\frac{1}{4}\cos 3\theta$ = RHS

With $\theta=25^\circ$: $\cos 25^\circ\cos 35^\circ\cos 85^\circ=\frac{\sqrt{6}}{16}-\frac{\sqrt{2}}{16}$

**Ans: Proved;** $\cos 25^\circ\cos 35^\circ\cos 85^\circ=\frac{\sqrt{6}}{16}-\frac{\sqrt{2}}{16}$

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**18.** If $\frac{x}{y}=\frac{\tan 135^\circ}{\tan 15^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y}=\frac{\tan A+\tan B}{\tan A-\tan B}=\frac{\sin(A+B)}{\sin(A-B)}$

$=\frac{\sin 150^\circ}{\sin 120^\circ}=\frac{\sqrt{3}}{3}$

**Ans:** $\frac{\sqrt{3}}{3}$

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**19.** For $A=120^\circ$, is $\cos\frac{A}{2}+\sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2}=60^\circ$

$c+s>0$ for $\frac{A}{2}$ in $\left(-\frac{\pi}{4},\frac{3\pi}{4}\right)$ and $<0$ in $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$ (adding $2\pi$ where needed).

So $\cos 60^\circ+\sin 60^\circ$ is positive.

**Ans: Positive**

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**20.** Simplify $\sqrt{1-\sin A}$ for $A=330^\circ$ and find its value. [2]

$\sqrt{1-\sin A}=\left|\cos\frac{A}{2}-\sin\frac{A}{2}\right|$

$\frac{A}{2}=165^\circ$, where $\cos\frac{A}{2}-\sin\frac{A}{2}$ is negative.

$\sqrt{1-\sin 330^\circ}=-(\cos 165^\circ-\sin 165^\circ)=\frac{\sqrt{6}}{2}$

**Ans:** $-(\cos 165^\circ-\sin 165^\circ)=\frac{\sqrt{6}}{2}$

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