EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = -\frac{3}{4}$, find $\sin 2A$. [1 mark]

2. If $\cos A = -\frac{12}{37}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. If $\cos 2A = \frac{1}{4}$, find $\tan^2 A$. [2 marks]

4. If $\cos A = \frac{1}{5}$, find $\cos 3A$. [2 marks]

5. If $\cos A = \frac{6}{7}$ and $\pi < A < 2\pi$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3 marks]

6. If $\tan \frac{A}{2} = \frac{3}{8}$, find $\tan A$. [1 mark]

7. Show that $\sin 3A = 3\sin A - 4\sin^3 A$. [5 marks]

8. Write the exact value of $\cos 234^\circ$. [1 mark]

9. Find the exact value of $\sin 18^\circ$ and hence find $\cos 36^\circ \cos 72^\circ$. [5 marks]

10. Find the exact value of $\tan^2 22\frac{1}{2}^\circ + \cot^2 22\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = \frac{1}{6}$, find $\sin^2\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\operatorname{cosec} 105^\circ + \cot 105^\circ$. [2 marks]

13. Prove that $\operatorname{cosec} 2A - \cot 2A = \tan A$. [5 marks]

14. Find the value of $\tan \frac{5\pi}{48}\left(1 + \sec \frac{5\pi}{24}\right)\left(1 + \sec \frac{5\pi}{12}\right)$. [3 marks]

15. Find the value of $\sin 45^\circ + \sin 105^\circ + \sin 165^\circ + \cdots + \sin 345^\circ$. [3 marks]

16. Find the value of $\sin \frac{\pi}{7} \sin \frac{2\pi}{7} \sin \frac{3\pi}{7} - \left(\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7}\right)$. [1 mark]

17. Show that $\cos\theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4}\cos 3\theta$. Hence evaluate $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\tan 127\frac{1}{2}^\circ}{\tan 7\frac{1}{2}^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 580^\circ$, is $\cos \frac{A}{2} - \sin \frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1+\sin A}$ for $A = 210^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$-\frac{24}{25}$
2	$\sin 2A = \frac{840}{1369},\ \cos 2A = -\frac{1081}{1369},\ \tan 2A = -\frac{840}{1081}$
3	$\frac{3}{5}$
4	$-\frac{71}{125}$
5	$\sin\frac{A}{2} = \frac{\sqrt{14}}{14},\ \cos\frac{A}{2} = -\frac{\sqrt{182}}{14},\ \tan\frac{A}{2} = -\frac{\sqrt{13}}{13}$
6	$\frac{48}{55}$
7	Proved: $\sin 3A = 3\sin A - 4\sin^3 A$
8	$-\frac{\sqrt{10-2\sqrt{5}}}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos 36^\circ \cos 72^\circ = \frac{1}{4}$
10	6
11	$\frac{143}{144}$
12	$\sqrt{3}+\sqrt{6}-2-\sqrt{2}$
13	Proved: $\operatorname{cosec} 2A - \cot 2A = \tan A$
14	$2+\sqrt{3}$
15	0
16	$\frac{\sqrt{7}-4}{8}$
17	Proved; $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$
18	$\frac{\sqrt{6}}{3}$
19	Positive
20	$\cos 105^\circ + \sin 105^\circ = \frac{\sqrt{2}}{2}$

Solutions

1. If $\tan A = -\frac{3}{4}$, find $\sin 2A$. [1]

$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$= -\frac{24}{25}$

Ans: $-\frac{24}{25}$

---

2. If $\cos A = -\frac{12}{37}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{35}{37}$ ($\sin A$ is negative in this quadrant)

$\sin 2A = 2\sin A\cos A = 2(-\frac{35}{37})(-\frac{12}{37}) = \frac{840}{1369}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{144}{1369} - \frac{1225}{1369} = -\frac{1081}{1369}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = -\frac{840}{1081}$

Ans: $\sin 2A = \frac{840}{1369},\ \cos 2A = -\frac{1081}{1369},\ \tan 2A = -\frac{840}{1081}$

---

3. If $\cos 2A = \frac{1}{4}$, find $\tan^2 A$. [2]

$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

$= \frac{3}{5}$

Ans: $\frac{3}{5}$

---

4. If $\cos A = \frac{1}{5}$, find $\cos 3A$. [2]

$\cos 3A = 4\cos^3 A - 3\cos A$

$= 4(\frac{1}{5})^3 - 3(\frac{1}{5})$

$= -\frac{71}{125}$

Ans: $-\frac{71}{125}$

---

5. If $\cos A = \frac{6}{7}$ and $\pi < A < 2\pi$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3]

Here $\frac{\pi}{2} < \frac{A}{2} < \pi$, so $\sin\frac{A}{2}$ is positive and $\cos\frac{A}{2}$ is negative.

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \frac{\sqrt{14}}{14}$

$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\frac{\sqrt{182}}{14}$

$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = -\frac{\sqrt{13}}{13}$

Ans: $\sin\frac{A}{2} = \frac{\sqrt{14}}{14},\ \cos\frac{A}{2} = -\frac{\sqrt{182}}{14},\ \tan\frac{A}{2} = -\frac{\sqrt{13}}{13}$

---

6. If $\tan\frac{A}{2} = \frac{3}{8}$, find $\tan A$. [1]

$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$= \frac{48}{55}$

Ans: $\frac{48}{55}$

---

7. Show that $\sin 3A = 3\sin A - 4\sin^3 A$. [5]

$\sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos^2 A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

Ans: Proved: $\sin 3A = 3\sin A - 4\sin^3 A$

---

8. Write the exact value of $\cos 234°$. [1]

$\cos 234° = -\cos 54°$

$\cos 54° = \frac{\sqrt{10-2\sqrt{5}}}{4}$

Ans: $-\frac{\sqrt{10-2\sqrt{5}}}{4}$

---

9. Find the exact value of $\sin 18°$ and hence find $\cos 36° \cos 72°$. [5]

Let $\theta = 18°$. Then $5\theta = 90°$, so $2\theta = 90° - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1 - \sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

$18°$ is acute, so $\sin 18° > 0$ and $\sin 18° = \frac{\sqrt{5}-1}{4}$.

$\cos 72° = \sin 18°$ and $\cos 36° = \frac{\sqrt{5}+1}{4}$

Product $= \frac{(\sqrt{5}+1)(\sqrt{5}-1)}{16} = \frac{4}{16} = \frac{1}{4}$

Ans: $\sin 18° = \frac{\sqrt{5}-1}{4}$; $\cos 36° \cos 72° = \frac{1}{4}$

---

10. Find the exact value of $\tan^2 22\frac{1}{2}° + \cot^2 22\frac{1}{2}°$. [2]

$\tan 22\frac{1}{2}° = \sqrt{2} - 1,\ \cot 22\frac{1}{2}° = \sqrt{2} + 1$

$\tan^2 22\frac{1}{2}° + \cot^2 22\frac{1}{2}° = 6$

Ans: 6

---

11. If $\sin 2\theta = \frac{1}{6}$, find $\sin^2\theta + \cos^4\theta$. [1]

$\sin^2\theta + \cos^4\theta = 1 - \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{4} \cdot \frac{1}{36} = \frac{143}{144}$

Ans: $\frac{143}{144}$

---

12. Find the value of $\operatorname{cosec} 105° + \cot 105°$. [2]

$\operatorname{cosec} 2A + \cot 2A = \cot A$

With $A = 52\frac{1}{2}°$: $\operatorname{cosec} 105° + \cot 105° = \sqrt{3} + \sqrt{6} - 2 - \sqrt{2}$

Ans: $\sqrt{3} + \sqrt{6} - 2 - \sqrt{2}$

---

13. Prove that $\operatorname{cosec} 2A - \cot 2A = \tan A$. [5]

LHS $= \frac{1-\cos 2A}{\sin 2A}$

$= \frac{2\sin^2 A}{2\sin A\cos A} = \tan A$ = RHS

Ans: Proved: $\operatorname{cosec} 2A - \cot 2A = \tan A$

---

14. Find the value of $\tan\frac{5\pi}{48}\left(1+\sec\frac{5\pi}{24}\right)\left(1+\sec\frac{5\pi}{12}\right)$. [3]

$\tan\frac{\theta}{2}(1+\sec\theta)=\tan\theta$, and repeating, the product is $\tan 2\theta$ with $\theta=\frac{5\pi}{24}$.

$\tan\frac{5\pi}{12}=2+\sqrt{3}$

Ans: $2+\sqrt{3}$

---

15. Find the value of $\sin 45^\circ+\sin 105^\circ+\sin 165^\circ+\cdots+\sin 345^\circ$. [3]

$a_1=45^\circ$, $d=60^\circ$, $n=6$, $a_n=345^\circ$

$\text{Sum}=\frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}}=\frac{\sin 195^\circ\ \sin 180^\circ}{\sin 30^\circ}$

$=0$

Ans: 0

---

16. Find the value of $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}-\left(\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}\right)$. [1]

Known results: $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\frac{\sqrt{7}}{8}$ and $\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}=\frac{1}{2}$

$\text{Value}=\frac{\sqrt{7}-4}{8}$

Ans: $\frac{\sqrt{7}-4}{8}$

---

17. Show that $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)=\frac{1}{4}\cos 3\theta$. Hence evaluate $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ$. [5]

$\cos(60^\circ+\theta)\cos(60^\circ-\theta)=\cos^2 60^\circ-\sin^2\theta=\cos^2\theta-\frac{3}{4}$

$\text{LHS}=\cos\theta\left(\cos^2\theta-\frac{3}{4}\right)=\frac{1}{4}(4\cos^3\theta-3\cos\theta)$

$=\frac{1}{4}\cos 3\theta$ = RHS

With $\theta=20^\circ$: $\cos 20^\circ\cos 40^\circ\cos 80^\circ=\frac{1}{8}$

Multiply by $\cos 60^\circ=\frac{1}{2}$: the value is $\frac{1}{16}$.

Ans: Proved; $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=\frac{1}{16}$

---

18. If $\frac{x}{y}=\frac{\tan 127\frac{1}{2}^\circ}{\tan 7\frac{1}{2}^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y}=\frac{\tan A+\tan B}{\tan A-\tan B}=\frac{\sin(A+B)}{\sin(A-B)}$

$=\frac{\sin 135^\circ}{\sin 120^\circ}=\frac{\sqrt{6}}{3}$

Ans: $\frac{\sqrt{6}}{3}$

---

19. For $A=580^\circ$, is $\cos\frac{A}{2}-\sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2}=290^\circ$

$c-s>0$ for $\frac{A}{2}$ in $\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$ and <0 in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ (adding 2π where needed).

So $\cos 290^\circ-\sin 290^\circ$ is positive.

Ans: Positive

---

20. Simplify $\sqrt{1+\sin A}$ for $A=210^\circ$ and find its value. [2]

$\sqrt{1+\sin A}=\left|\cos\frac{A}{2}+\sin\frac{A}{2}\right|$

$\frac{A}{2}=105^\circ$, where $\cos\frac{A}{2}+\sin\frac{A}{2}$ is positive.

$\sqrt{1+\sin 210^\circ}=\cos 105^\circ+\sin 105^\circ=\frac{\sqrt{2}}{2}$

Ans: $\cos 105^\circ+\sin 105^\circ=\frac{\sqrt{2}}{2}$
