EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = -\frac{1}{5}$, find $\cos 2A$. [1 mark]

2. If $\sin A = \frac{15}{17}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. If $\cos 2A = \frac{2}{7}$, find $\cos^2 A$. [2 marks]

4. If $\sin A = \frac{2}{5}$, find $\sin 3A$. [2 marks]

5. If $\cos A = -\frac{8}{9}$ and $-\pi < A < 0$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3 marks]

6. If $\tan\frac{A}{2} = \frac{5}{2}$, find $\sin A$. [1 mark]

7. Show that $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} = \tan\frac{A}{2}$. [5 marks]

8. Write the exact value of $\cos 108^\circ$. [1 mark]

9. Find the exact value of $\sin 18^\circ$ and hence find $\sin 18^\circ + \cos 36^\circ$. [5 marks]

10. Find the exact value of $\cot 7\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = -\frac{5}{7}$, find $\sin^4\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\cot 67\frac{1}{2}^\circ - \tan 67\frac{1}{2}^\circ$. [2 marks]

13. Show that $\operatorname{cosec} 2A + \cot 2A = \cot A$. [5 marks]

14. Find the value of $\cos\frac{17\pi}{33}\cos\frac{34\pi}{33}\cos\frac{68\pi}{33}\cos\frac{136\pi}{33}\cos\frac{272\pi}{33}$. [3 marks]

15. Find the value of $\sin 45^\circ + \sin 75^\circ + \sin 105^\circ + \sin 135^\circ$. [3 marks]

16. Find the value of $\left(\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}\right)^2$. [1 mark]

17. Prove that $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$ and hence find $\cos 15^\circ \cos 45^\circ \cos 75^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\sin 75^\circ}{\sin 45^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 120^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1-\sin A}$ for $A = 510^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$\frac{12}{13}$
2	$\sin 2A = -\frac{240}{289},\ \cos 2A = -\frac{161}{289},\ \tan 2A = \frac{240}{161}$
3	$\frac{9}{14}$
4	$\frac{118}{125}$
5	$\sin \frac{A}{2} = -\frac{\sqrt{34}}{6},\ \cos \frac{A}{2} = \frac{\sqrt{2}}{6},\ \tan \frac{A}{2} = -\sqrt{17}$
6	$\frac{20}{29}$
7	Proved: $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} = \tan \frac{A}{2}$
8	$-\frac{\sqrt{5}-1}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\sin 18^\circ + \cos 36^\circ = \frac{\sqrt{5}}{2}$
10	$4 + 2\sqrt{3}$
11	$\frac{73}{98}$
12	-2
13	Proved: $\operatorname{cosec} 2A + \cot 2A = \cot A$
14	$\frac{1}{32}$
15	$\frac{\sqrt{6}}{2} + \frac{3\sqrt{2}}{2}$
16	$\frac{1}{64}$
17	Proved; $\cos 15^\circ \cos 45^\circ \cos 75^\circ = \frac{\sqrt{2}}{8}$
18	$3 + 2\sqrt{3}$
19	Negative
20	$\cos 255^\circ - \sin 255^\circ = \frac{\sqrt{2}}{2}$

Solutions

1. If $\tan A = -\frac{1}{5}$, find $\cos 2A$. [1]

$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$= \frac{12}{13}$

Ans: $\frac{12}{13}$

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2. If $\sin A = \frac{15}{17}$ and $\frac{\pi}{2} < A < \pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\cos A = -\sqrt{1 - \sin^2 A} = -\frac{8}{17}$ ($\cos A$ is negative in this quadrant)

$\sin 2A = 2\sin A \cos A = 2(\frac{15}{17})(-\frac{8}{17}) = -\frac{240}{289}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{64}{289} - \frac{225}{289} = -\frac{161}{289}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{240}{161}$

Ans: $\sin 2A = -\frac{240}{289},\ \cos 2A = -\frac{161}{289},\ \tan 2A = \frac{240}{161}$

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3. If $\cos 2A = \frac{2}{7}$, find $\cos^2 A$. [2]

$\cos^2 A = \frac{1+\cos 2A}{2}$

$= \frac{9}{14}$

Ans: $\frac{9}{14}$

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4. If $\sin A = \frac{2}{5}$, find $\sin 3A$. [2]

$\sin 3A = 3\sin A - 4\sin^3 A$

$= 3(\frac{2}{5}) - 4(\frac{2}{5})^3$

$= \frac{118}{125}$

Ans: $\frac{118}{125}$

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5. If $\cos A = -\frac{8}{9}$ and $-\pi < A < 0$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3]

Here $-\frac{\pi}{2} < \frac{A}{2} < 0$, so $\sin \frac{A}{2}$ is negative and $\cos \frac{A}{2}$ is positive.

$\sin \frac{A}{2} = -\sqrt{\frac{1-\cos A}{2}} = -\frac{\sqrt{34}}{6}$

$\cos \frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \frac{\sqrt{2}}{6}$

$\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = -\sqrt{17}$

Ans: $\sin \frac{A}{2} = -\frac{\sqrt{34}}{6},\ \cos \frac{A}{2} = \frac{\sqrt{2}}{6},\ \tan \frac{A}{2} = -\sqrt{17}$

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6. If $\tan \frac{A}{2} = \frac{5}{2}$, find $\sin A$. [1]

$\sin A = \frac{2\tan \frac{A}{2}}{1+\tan^2 \frac{A}{2}}$

$= \frac{20}{29}$

Ans: $\frac{20}{29}$

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7. Show that $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} = \tan \frac{A}{2}$. [5]

$\frac{\sin 2A}{1+\cos 2A} = \frac{2\sin A \cos A}{2\cos^2 A} = \tan A$

LHS $= \tan A \cdot \frac{\cos A}{1+\cos A} = \frac{\sin A}{1+\cos A}$

$= \frac{2\sin \frac{A}{2}\cos \frac{A}{2}}{2\cos^2 \frac{A}{2}} = \tan \frac{A}{2}$ = RHS

Ans: Proved: $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} = \tan \frac{A}{2}$

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8. Write the exact value of $\cos 108^\circ$. [1]

$\cos 108^\circ = -\cos 72^\circ$

$\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

Ans: $-\frac{\sqrt{5}-1}{4}$

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9. Find the exact value of $\sin 18^\circ$ and hence find $\sin 18^\circ + \cos 36^\circ$. [5]

Let $\theta = 18^\circ$. Then $5\theta = 90^\circ$, so $2\theta = 90^\circ - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1-\sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

18° is acute, so $\sin 18^\circ > 0$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

$\cos 36^\circ = 1 - 2\sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$

Sum $= \frac{\sqrt{5}-1+\sqrt{5}+1}{4} = \frac{\sqrt{5}}{2}$

Ans: $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\sin 18^\circ + \cos 36^\circ = \frac{\sqrt{5}}{2}$

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10. Find the exact value of $\cot 7\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$. [2]

$\cot 7\frac{1}{2}^\circ = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1),\ \tan 7\frac{1}{2}^\circ = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$

$\cot 7\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ = 4 + 2\sqrt{3}$

Ans: $4 + 2\sqrt{3}$

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11. If $\sin 2\theta = -\frac{5}{7}$, find $\sin^4\theta + \cos^4\theta$. [1]

$\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$

$= 1 - \frac{1}{2}\cdot\frac{25}{49} = \frac{73}{98}$

Ans: $\frac{73}{98}$

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12. Find the value of $\cot 67\frac{1}{2}^\circ - \tan 67\frac{1}{2}^\circ$. [2]

$\cot A - \tan A = 2\cot 2A$

With $A = 67\frac{1}{2}^\circ$: $\cot 67\frac{1}{2}^\circ - \tan 67\frac{1}{2}^\circ = -2$

Ans: -2

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13. Show that $\operatorname{cosec} 2A + \cot 2A = \cot A$. [5]

LHS $= \frac{1+\cos 2A}{\sin 2A}$

$= \frac{2\cos^2 A}{2\sin A\cos A} = \frac{\cos A}{\sin A} = \cot A$ = RHS

Ans: Proved: $\operatorname{cosec} 2A + \cot 2A = \cot A$

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14. Find the value of $\cos\frac{17\pi}{33}\cos\frac{34\pi}{33}\cos\frac{68\pi}{33}\cos\frac{136\pi}{33}\cos\frac{272\pi}{33}$. [3]

This is $\cos\theta\cos 2\theta\cdots\cos 2^4\theta = \frac{\sin 2^5\theta}{2^5\sin\theta}$ with $\theta = \frac{17\pi}{33}$.

$\sin 2^5\theta = \sin\theta$

Value $= \frac{1}{32}$

Ans: $\frac{1}{32}$

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15. Find the value of $\sin 45^\circ + \sin 75^\circ + \sin 105^\circ + \sin 135^\circ$. [3]

$a_1 = 45^\circ$, $d = 30^\circ$, $n = 4$, $a_n = 135^\circ$

$\text{Sum} = \frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}} = \frac{\sin 90^\circ \sin 60^\circ}{\sin 15^\circ}$

$= \frac{\sqrt{6}}{2} + \frac{3\sqrt{2}}{2}$

Ans: $\frac{\sqrt{6}}{2} + \frac{3\sqrt{2}}{2}$

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16. Find the value of $\left(\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}\right)^2$. [1]

Known result: $\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7} = -\frac{1}{8}$

$\text{Square} = \frac{1}{64}$

Ans: $\frac{1}{64}$

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17. Prove that $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$ and hence find $\cos 15^\circ \cos 45^\circ \cos 75^\circ$. [5]

$\cos(60^\circ+\theta)\cos(60^\circ-\theta) = \cos^2 60^\circ - \sin^2\theta = \cos^2\theta - \frac{3}{4}$

LHS $= \cos\theta\left(\cos^2\theta - \frac{3}{4}\right) = \frac{1}{4}(4\cos^3\theta - 3\cos\theta)$

$= \frac{1}{4}\cos 3\theta$ = RHS

With $\theta = 15^\circ$: $\cos 15^\circ \cos 45^\circ \cos 75^\circ = \frac{\sqrt{2}}{8}$

Ans: Proved; $\cos 15^\circ \cos 45^\circ \cos 75^\circ = \frac{\sqrt{2}}{8}$

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18. If $\frac{x}{y} = \frac{\sin 75^\circ}{\sin 45^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y} = \frac{\sin A+\sin B}{\sin A-\sin B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

$= \frac{\tan 60^\circ}{\tan 15^\circ} = 3 + 2\sqrt{3}$

Ans: $3 + 2\sqrt{3}$

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19. For $A = 120^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2} = 60^\circ$

$c - s > 0$ for $\frac{A}{2}$ in $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ and < 0 in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ (adding 2π where needed).

So $\cos 60^\circ - \sin 60^\circ$ is negative.

Ans: Negative

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20. Simplify $\sqrt{1-\sin A}$ for $A = 510^\circ$ and find its value. [2]

$\sqrt{1-\sin A} = \left|\cos\frac{A}{2} - \sin\frac{A}{2}\right|$

$\frac{A}{2} = 255^\circ$, where $\cos\frac{A}{2} - \sin\frac{A}{2}$ is positive.

$\sqrt{1-\sin 510^\circ} = \cos 255^\circ - \sin 255^\circ = \frac{\sqrt{2}}{2}$

Ans: $\cos 255^\circ - \sin 255^\circ = \frac{\sqrt{2}}{2}$
