EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = \frac{4}{3}$, find $\tan 2A$. [1 mark]

2. If $\cos A = -\frac{3}{5}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. Using $\cos 2A$, find the exact value of $\sin^2 15^\circ$. [2 marks]

4. If $\sin A = -\frac{2}{5}$, find $\sin 3A$. [2 marks]

5. If $\cos A = \frac{2}{7}$ and $-\pi < A < 0$, find $\sin \frac{A}{2}$, $\cos \frac{A}{2}$ and $\tan \frac{A}{2}$. [3 marks]

6. If $\tan \frac{A}{2} = -\frac{4}{3}$, find $\sin A$. [1 mark]

7. Prove that $\cos^3 A \sin 3A + \sin^3 A \cos 3A = \frac{3}{4}\sin 4A$. [5 marks]

8. Write the exact value of $\cos 342^\circ$. [1 mark]

9. Find the exact value of $\sin 18^\circ$ and hence find $\cos 144^\circ$. [5 marks]

10. Find the exact value of $\tan 52\frac{1}{2}^\circ + \tan 37\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = \frac{4}{5}$, find $\sin^2\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\cot 22\frac{1}{2}^\circ + \tan 22\frac{1}{2}^\circ$. [2 marks]

13. Show that $\cot A + \tan A = 2\operatorname{cosec} 2A$. [5 marks]

14. Find the value of $\cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \cos \frac{8\pi}{7}$. [3 marks]

15. Find the value of $\sin 12^\circ + \sin 24^\circ + \sin 36^\circ + \cdots + \sin 168^\circ$. [3 marks]

16. Find the value of $\left(\sin \frac{\pi}{7} \sin \frac{2\pi}{7} \sin \frac{3\pi}{7}\right)^2$. [1 mark]

17. Prove that $\tan\theta \tan(60^\circ - \theta) \tan(60^\circ + \theta) = \tan 3\theta$ and hence find $\tan 10^\circ \tan 30^\circ \tan 50^\circ \tan 70^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\sin 90^\circ}{\sin 30^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 320^\circ$, is $\cos \frac{A}{2} - \sin \frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1+\sin A}$ for $A = 510^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$-\frac{24}{7}$
2	$\sin 2A = \frac{24}{25},\ \cos 2A = -\frac{7}{25},\ \tan 2A = -\frac{24}{7}$
3	$\frac{2-\sqrt{3}}{4}$
4	$-\frac{118}{125}$
5	$\sin\frac{A}{2} = -\frac{\sqrt{70}}{14},\ \cos\frac{A}{2} = \frac{3\sqrt{14}}{14},\ \tan\frac{A}{2} = -\frac{\sqrt{5}}{3}$
6	$-\frac{24}{25}$
7	Proved: $\cos^3 A \sin 3A + \sin^3 A \cos 3A = \frac{3}{4}\sin 4A$
8	$\frac{\sqrt{10+2\sqrt{5}}}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos 144^\circ = -\frac{\sqrt{5}+1}{4}$
10	$2\sqrt{6} - 2\sqrt{2}$
11	$\frac{21}{25}$
12	$2\sqrt{2}$
13	Proved: $\cot A + \tan A = 2\operatorname{cosec} 2A$
14	$\frac{1}{8}$
15	$\cot 6^\circ$
16	$\frac{7}{64}$
17	Proved; $\tan 10^\circ \tan 30^\circ \tan 50^\circ \tan 70^\circ = \frac{1}{3}$
18	3
19	Negative
20	$-(\cos 255^\circ + \sin 255^\circ) = \frac{\sqrt{6}}{2}$

Solutions

1. If $\tan A = \frac{4}{3}$, find $\tan 2A$. [1]

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= -\frac{24}{7}$

Ans: $-\frac{24}{7}$

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2. If $\cos A = -\frac{3}{5}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{4}{5}$ ($\sin A$ is negative in this quadrant)

$\sin 2A = 2\sin A\cos A = 2(-\frac{4}{5})(-\frac{3}{5}) = \frac{24}{25}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = -\frac{24}{7}$

Ans: $\sin 2A = \frac{24}{25},\ \cos 2A = -\frac{7}{25},\ \tan 2A = -\frac{24}{7}$

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3. Using $\cos 2A$, find the exact value of $\sin^2 15°$. [2]

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\sin^2 15° = \frac{1-\cos 30°}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$= \frac{2-\sqrt{3}}{4}$

Ans: $\frac{2-\sqrt{3}}{4}$

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4. If $\sin A = -\frac{2}{5}$, find $\sin 3A$. [2]

$\sin 3A = 3\sin A - 4\sin^3 A$

$= 3(-\frac{2}{5}) - 4(-\frac{2}{5})^3$

$= -\frac{118}{125}$

Ans: $-\frac{118}{125}$

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5. If $\cos A = \frac{2}{7}$ and $-\pi < A < 0$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3]

Here $-\frac{\pi}{2} < \frac{A}{2} < 0$, so $\sin\frac{A}{2}$ is negative and $\cos\frac{A}{2}$ is positive.

$\sin\frac{A}{2} = -\sqrt{\frac{1-\cos A}{2}} = -\frac{\sqrt{70}}{14}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \frac{3\sqrt{14}}{14}$

$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = -\frac{\sqrt{5}}{3}$

Ans: $\sin\frac{A}{2} = -\frac{\sqrt{70}}{14},\ \cos\frac{A}{2} = \frac{3\sqrt{14}}{14},\ \tan\frac{A}{2} = -\frac{\sqrt{5}}{3}$

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6. If $\tan\frac{A}{2} = -\frac{4}{3}$, find $\sin A$. [1]

$\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$= -\frac{24}{25}$

Ans: $-\frac{24}{25}$

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7. Prove that $\cos^3 A \sin 3A + \sin^3 A \cos 3A = \frac{3}{4}\sin 4A$. [5]

$4\cos^3 A = \cos 3A + 3\cos A$ and $4\sin^3 A = 3\sin A - \sin 3A$

$4\cdot\text{LHS} = (\cos 3A + 3\cos A)\sin 3A + (3\sin A - \sin 3A)\cos 3A$

$= 3(\sin 3A \cos A + \cos 3A \sin A) = 3\sin 4A$

So LHS $= \frac{3}{4}\sin 4A$ = RHS

Ans: Proved: $\cos^3 A \sin 3A + \sin^3 A \cos 3A = \frac{3}{4}\sin 4A$

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8. Write the exact value of $\cos 342^\circ$. [1]

$\cos 342^\circ = \cos 18^\circ$

$\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

Ans: $\frac{\sqrt{10+2\sqrt{5}}}{4}$

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9. Find the exact value of $\sin 18^\circ$ and hence find $\cos 144^\circ$. [5]

Let $\theta = 18^\circ$. Then $5\theta = 90^\circ$, so $2\theta = 90^\circ - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1-\sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

18° is acute, so $\sin 18^\circ > 0$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

$\cos 144^\circ = -\cos 36^\circ$

$\cos 36^\circ = 1 - 2\sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$

So $\cos 144^\circ = -\frac{\sqrt{5}+1}{4}$

Ans: $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos 144^\circ = -\frac{\sqrt{5}+1}{4}$

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10. Find the exact value of $\tan 52\frac{1}{2}^\circ + \tan 37\frac{1}{2}^\circ$. [2]

$\tan 52\frac{1}{2}^\circ = (\sqrt{3}+\sqrt{2})(\sqrt{2}-1),\ \tan 37\frac{1}{2}^\circ = (\sqrt{3}-\sqrt{2})(\sqrt{2}+1)$

$\tan 52\frac{1}{2}^\circ + \tan 37\frac{1}{2}^\circ = 2\sqrt{6} - 2\sqrt{2}$

Ans: $2\sqrt{6} - 2\sqrt{2}$

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11. If $\sin 2\theta = \frac{4}{5}$, find $\sin^2\theta + \cos^4\theta$. [1]

$\sin^2\theta + \cos^4\theta = 1 - \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{4}\cdot\frac{16}{25} = \frac{21}{25}$

Ans: $\frac{21}{25}$

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12. Find the value of $\cot 22\frac{1}{2}^\circ + \tan 22\frac{1}{2}^\circ$. [2]

$\cot A + \tan A = 2\operatorname{cosec} 2A$

With $A = 22\frac{1}{2}^\circ$: $\cot 22\frac{1}{2}^\circ + \tan 22\frac{1}{2}^\circ = 2\sqrt{2}$

Ans: $2\sqrt{2}$

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13. Show that $\cot A + \tan A = 2\operatorname{cosec} 2A$. [5]

LHS $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A}$

$= \frac{2}{\sin 2A} = 2\operatorname{cosec} 2A$ = RHS

Ans: Proved: $\cot A + \tan A = 2\operatorname{cosec} 2A$

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14. Find the value of $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{8\pi}{7}$. [3]

This is $\cos\theta\cos 2\theta\cdots\cos 2^2\theta = \frac{\sin 2^3\theta}{2^3\sin\theta}$ with $\theta = \frac{2\pi}{7}$.

$\sin 2^3\theta = \sin\theta$

$\text{Value} = \frac{1}{8}$

Ans: $\frac{1}{8}$

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15. Find the value of $\sin 12^\circ + \sin 24^\circ + \sin 36^\circ + \cdots + \sin 168^\circ$. [3]

$a_1 = 12^\circ$, $d = 12^\circ$, $n = 14$, $a_n = 168^\circ$

$\text{Sum} = \frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}} = \frac{\sin 90^\circ\sin 84^\circ}{\sin 6^\circ}$

$= \cot 6^\circ$

Ans: $\cot 6^\circ$

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16. Find the value of $\left(\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}\right)^2$. [1]

Known result: $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7} = \frac{\sqrt{7}}{8}$

$\text{Square} = \frac{7}{64}$

Ans: $\frac{7}{64}$

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17. Prove that $\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$ and hence find $\tan 10^\circ\tan 30^\circ\tan 50^\circ\tan 70^\circ$. [5]

$\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$ (using $\sin^2 60^\circ = \frac{3}{4}$)

$\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$ (using $\cos^2 60^\circ = \frac{1}{4}$)

Dividing: $\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$

With $\theta = 10^\circ$: $\tan 10^\circ\tan 50^\circ\tan 70^\circ = \frac{\sqrt{3}}{3}$

Multiply by $\tan 30^\circ = \frac{\sqrt{3}}{3}$: the value is $\frac{1}{3}$.

Ans: Proved; $\tan 10^\circ\tan 30^\circ\tan 50^\circ\tan 70^\circ = \frac{1}{3}$

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18. If $\frac{x}{y} = \frac{\sin 90^\circ}{\sin 30^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y} = \frac{\sin A+\sin B}{\sin A-\sin B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

$= \frac{\tan 60^\circ}{\tan 30^\circ} = 3$

Ans: 3

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19. For $A = 320^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2} = 160^\circ$

$c - s > 0$ for $\frac{A}{2}$ in $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ and < 0 in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ (adding 2π where needed).

So $\cos 160^\circ - \sin 160^\circ$ is negative.

Ans: Negative

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20. Simplify $\sqrt{1+\sin A}$ for $A = 510^\circ$ and find its value. [2]

$\sqrt{1+\sin A} = \left|\cos\frac{A}{2} + \sin\frac{A}{2}\right|$

$\frac{A}{2} = 255^\circ$, where $\cos\frac{A}{2} + \sin\frac{A}{2}$ is negative.

$\sqrt{1+\sin 510^\circ} = -(\cos 255^\circ + \sin 255^\circ) = \frac{\sqrt{6}}{2}$

Ans: $-(\cos 255^\circ + \sin 255^\circ) = \frac{\sqrt{6}}{2}$
