EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = -\frac{7}{4}$, find $\sin 2A$. [1 mark]

2. If $\cos A = \frac{8}{17}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. If $\cos 2A = -\frac{3}{7}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin A$. [2 marks]

4. If $\sin A = \frac{3}{5}$, find $\sin 3A$. [2 marks]

5. If $\cos A = \frac{1}{4}$ and $\pi < A < 2\pi$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3 marks]

6. If $\tan\frac{A}{2} = \frac{3}{2}$, find $\sin A$. [1 mark]

7. Show that $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$. [5 marks]

8. Write the exact value of $\cos 144^\circ$. [1 mark]

9. Find the exact value of $\sin 18^\circ$ and hence find $\sin 54^\circ$. [5 marks]

10. Find the exact value of $\tan 52\frac{1}{2}^\circ + \cot 52\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = -\frac{3}{7}$, find $\sin^2\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\cot 7\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$. [2 marks]

13. Show that $\cot A - \tan A = 2\cot 2A$. [5 marks]

14. Find the value of $(2\cos 45^\circ - 1)(2\cos 45^\circ + 1)(2\cos 90^\circ - 1)$. [3 marks]

15. Find the value of $\sin 45^\circ + \sin 90^\circ + \sin 135^\circ$. [3 marks]

16. Find the value of $\frac{\tan\frac{\pi}{7} + \tan\frac{2\pi}{7} + \tan\frac{4\pi}{7}}{\tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{4\pi}{7}}$. [1 mark]

17. Show that $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$. Hence evaluate $\sin 25^\circ \sin 35^\circ \sin 85^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\sin 120^\circ}{\sin 30^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 340^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1 + \sin A}$ for $A = 600^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$-\frac{56}{65}$
2	$\sin 2A = -\frac{240}{289},\ \cos 2A = -\frac{161}{289},\ \tan 2A = \frac{240}{161}$
3	$\sin A = -\frac{\sqrt{35}}{7}$
4	$\frac{117}{125}$
5	$\sin \frac{A}{2} = \frac{\sqrt{6}}{4},\ \cos \frac{A}{2} = -\frac{\sqrt{10}}{4},\ \tan \frac{A}{2} = -\frac{\sqrt{15}}{5}$
6	$\frac{12}{13}$
7	Proved: $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$
8	$-\frac{\sqrt{5}+1}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\sin 54^\circ = \frac{\sqrt{5}+1}{4}$
10	$2\sqrt{6} - 2\sqrt{2}$
11	$\frac{187}{196}$
12	$4 + 2\sqrt{3}$
13	Proved: $\cot A - \tan A = 2\cot 2A$
14	-1
15	$\cot 22\frac{1}{2}^\circ$
16	1
17	Proved; $\sin 25^\circ \sin 35^\circ \sin 85^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$
18	$2 + \sqrt{3}$
19	Negative
20	$-(\cos 300^\circ + \sin 300^\circ) = \frac{\sqrt{3}-1}{2}$

Solutions

1. If $\tan A = -\frac{7}{4}$, find $\sin 2A$. [1]

$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$

$= -\frac{56}{65}$

Ans: $-\frac{56}{65}$

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2. If $\cos A = \frac{8}{17}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{15}{17}$ ($\sin A$ is negative in this quadrant)

$\sin 2A = 2\sin A\cos A = 2(-\frac{15}{17})(\frac{8}{17}) = -\frac{240}{289}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{64}{289} - \frac{225}{289} = -\frac{161}{289}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{240}{161}$

Ans: $\sin 2A = -\frac{240}{289},\ \cos 2A = -\frac{161}{289},\ \tan 2A = \frac{240}{161}$

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3. If $\cos 2A = -\frac{3}{7}$ and $\pi < A < \frac{3\pi}{2}$, find $\sin A$. [2]

$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$

$\sqrt{\frac{5}{7}} = \frac{\sqrt{35}}{7}$

In this quadrant $\sin A$ is negative, so $\sin A = -\frac{\sqrt{35}}{7}$.

Ans: $\sin A = -\frac{\sqrt{35}}{7}$

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4. If $\sin A = \frac{3}{5}$, find $\sin 3A$. [2]

$\sin 3A = 3\sin A - 4\sin^3 A$

$= 3(\frac{3}{5}) - 4(\frac{3}{5})^3$

$= \frac{117}{125}$

Ans: $\frac{117}{125}$

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5. If $\cos A = \frac{1}{4}$ and $\pi < A < 2\pi$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3]

Here $\frac{\pi}{2} < \frac{A}{2} < \pi$, so $\sin\frac{A}{2}$ is positive and $\cos\frac{A}{2}$ is negative.

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \frac{\sqrt{6}}{4}$

$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\frac{\sqrt{10}}{4}$

$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = -\frac{\sqrt{15}}{5}$

Ans: $\sin\frac{A}{2} = \frac{\sqrt{6}}{4},\ \cos\frac{A}{2} = -\frac{\sqrt{10}}{4},\ \tan\frac{A}{2} = -\frac{\sqrt{15}}{5}$

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6. If $\tan\frac{A}{2} = \frac{3}{2}$, find $\sin A$. [1]

$\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$= \frac{12}{13}$

Ans: $\frac{12}{13}$

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7. Show that $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$. [5]

$\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$

$\frac{\cos 3A}{\cos A} = 4\cos^2 A - 3$

LHS $= 3 - 4\sin^2 A - 4\cos^2 A + 3 = 6 - 4(\sin^2 A + \cos^2 A)$

$= 6 - 4 = 2$ = RHS

Ans: Proved: $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$

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8. Write the exact value of $\cos 144°$. [1]

$\cos 144° = -\cos 36°$

$\cos 36° = \frac{\sqrt{5}+1}{4}$

Ans: $-\frac{\sqrt{5}+1}{4}$

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9. Find the exact value of $\sin 18°$ and hence find $\sin 54°$. [5]

Let $\theta = 18°$. Then $5\theta = 90°$, so $2\theta = 90° - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1 - \sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

$18°$ is acute, so $\sin 18° > 0$ and $\sin 18° = \frac{\sqrt{5}-1}{4}$.

$\sin 54° = \cos 36° = 1 - 2\sin^2 18° = \frac{\sqrt{5}+1}{4}$

Ans: $\sin 18° = \frac{\sqrt{5}-1}{4}$; $\sin 54° = \frac{\sqrt{5}+1}{4}$

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10. Find the exact value of $\tan 52\frac{1}{2}° + \cot 52\frac{1}{2}°$. [2]

$\tan 52\frac{1}{2}° = (\sqrt{3}+\sqrt{2})(\sqrt{2}-1)$, $\cot 52\frac{1}{2}° = (\sqrt{3}-\sqrt{2})(\sqrt{2}+1)$

$\tan 52\frac{1}{2}° + \cot 52\frac{1}{2}° = 2\sqrt{6} - 2\sqrt{2}$

Ans: $2\sqrt{6} - 2\sqrt{2}$

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11. If $\sin 2\theta = -\frac{3}{7}$, find $\sin^2\theta + \cos^4\theta$. [1]

$\sin^2\theta + \cos^4\theta = 1 - \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{4} \cdot \frac{9}{49} = \frac{187}{196}$

Ans: $\frac{187}{196}$

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12. Find the value of $\cot 7\frac{1}{2}° - \tan 7\frac{1}{2}°$. [2]

$\cot A - \tan A = 2\cot 2A$

With $A = 7\frac{1}{2}°$: $\cot 7\frac{1}{2}° - \tan 7\frac{1}{2}° = 4 + 2\sqrt{3}$

Ans: $4 + 2\sqrt{3}$

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13. Show that $\cot A - \tan A = 2\cot 2A$. [5]

LHS $= \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} = \frac{\cos^2 A - \sin^2 A}{\sin A\cos A}$

$= \frac{\cos 2A}{\frac{1}{2}\sin 2A} = 2\cot 2A$ = RHS

Ans: Proved: $\cot A - \tan A = 2\cot 2A$

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14. Find the value of $(2\cos 45^\circ - 1)(2\cos 45^\circ + 1)(2\cos 90^\circ - 1)$. [3]

$(2\cos\theta - 1)(2\cos\theta + 1) = 2\cos 2\theta + 1$, then $(2\cos 2\theta + 1)(2\cos 2\theta - 1) = 2\cos 4\theta + 1$

$= 2\cos 180^\circ + 1 = -1$

Ans: -1

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15. Find the value of $\sin 45^\circ + \sin 90^\circ + \sin 135^\circ$. [3]

$a_1 = 45^\circ$, $d = 45^\circ$, $n = 3$, $a_n = 135^\circ$

$\text{Sum} = \frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}} = \frac{\sin 90^\circ \sin 67\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ}$

$= \cot 22\frac{1}{2}^\circ$

Ans: $\cot 22\frac{1}{2}^\circ$

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16. Find the value of $\frac{\tan\frac{\pi}{7}+\tan\frac{2\pi}{7}+\tan\frac{4\pi}{7}}{\tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{4\pi}{7}}$. [1]

Known results: $\tan\frac{\pi}{7} + \tan\frac{2\pi}{7} + \tan\frac{4\pi}{7} = -\sqrt{7}$ and $\tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{4\pi}{7} = -\sqrt{7}$

$\text{Value} = 1$

Ans: 1

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17. Show that $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$. Hence evaluate $\sin 25^\circ \sin 35^\circ \sin 85^\circ$. [5]

$\sin(60^\circ + \theta)\sin(60^\circ - \theta) = \sin^2 60^\circ - \sin^2\theta = \frac{3}{4} - \sin^2\theta$

$\text{LHS} = \sin\theta\left(\frac{3}{4} - \sin^2\theta\right) = \frac{1}{4}(3\sin\theta - 4\sin^3\theta)$

$= \frac{1}{4}\sin 3\theta$ = RHS

With $\theta = 25^\circ$: $\sin 25^\circ \sin 35^\circ \sin 85^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$

Ans: Proved; $\sin 25^\circ \sin 35^\circ \sin 85^\circ = \frac{\sqrt{2}}{16} + \frac{\sqrt{6}}{16}$

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18. If $\frac{x}{y} = \frac{\sin 120^\circ}{\sin 30^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y} = \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

$= \frac{\tan 75^\circ}{\tan 45^\circ} = 2 + \sqrt{3}$

Ans: $2 + \sqrt{3}$

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19. For $A = 340^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2} = 170^\circ$

$c - s > 0$ for $\frac{A}{2}$ in $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ and < 0 in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ (adding 2π where needed).

So $\cos 170^\circ - \sin 170^\circ$ is negative.

Ans: Negative

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20. Simplify $\sqrt{1 + \sin A}$ for $A = 600^\circ$ and find its value. [2]

$\sqrt{1 + \sin A} = \left|\cos\frac{A}{2} + \sin\frac{A}{2}\right|$

$\frac{A}{2} = 300^\circ$, where $\cos\frac{A}{2} + \sin\frac{A}{2}$ is negative.

$\sqrt{1 + \sin 600^\circ} = -(\cos 300^\circ + \sin 300^\circ) = \frac{\sqrt{3}-1}{2}$

Ans: $-(\cos 300^\circ + \sin 300^\circ) = \frac{\sqrt{3}-1}{2}$
