EXPRESSWALA ACADEMY

Maths 1A: Multiple and Submultiple Angles

Question Paper, Set 10

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = \frac{7}{6}$, find $\tan 2A$. [1 mark]

2. If $\cos A = \frac{20}{29}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3 marks]

3. If $\cos 2A = \frac{3}{4}$ and A is acute, find $\sin A$. [2 marks]

4. If $\tan A = \frac{6}{7}$, find $\tan 3A$. [2 marks]

5. If $\cos A = -\frac{1}{6}$ and $-\pi < A < 0$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3 marks]

6. If $\tan\frac{A}{2} = \frac{3}{2}$, find $\tan A$. [1 mark]

7. Prove that $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$. [5 marks]

8. Write the exact value of $\sin 234^\circ$. [1 mark]

9. Find $\sin 18^\circ$ from $\sin 2\theta = \cos 3\theta$ with $\theta = 18^\circ$, and hence evaluate $\cos 108^\circ$. [5 marks]

10. Find the exact value of $\tan 7\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ$. [2 marks]

11. If $\sin 2\theta = \frac{5}{6}$, find $\sin^2\theta + \cos^4\theta$. [1 mark]

12. Find the value of $\tan 82\frac{1}{2}^\circ + \tan 7\frac{1}{2}^\circ$. [2 marks]

13. Prove that $\frac{1+\sin 2A}{1-\sin 2A} = \tan^2\left(\frac{\pi}{4} + A\right)$. [5 marks]

14. Find the value of $\left(2\cos\frac{8\pi}{17} - 1\right)\left(2\cos\frac{16\pi}{17} - 1\right)\left(2\cos\frac{32\pi}{17} - 1\right)\left(2\cos\frac{64\pi}{17} - 1\right)$. [3 marks]

15. Find the value of $\sin 60^\circ + \sin 120^\circ + \sin 180^\circ + \sin 240^\circ + \sin 300^\circ$. [3 marks]

16. Find the value of $\left(\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}\right)^2$. [1 mark]

17. Show that $\sin\theta\sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$. Hence evaluate $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\sin 45^\circ}{\sin 15^\circ}$, find $\frac{x+y}{x-y}$. [2 marks]

19. For $A = 100^\circ$, is $\cos\frac{A}{2} - \sin\frac{A}{2}$ positive or negative? [1 mark]

20. Simplify $\sqrt{1-\sin A}$ for $A = 600^\circ$ and find its value. [2 marks]

Key sheet

Q	Answer
1	$-\frac{84}{13}$
2	$\sin 2A = -\frac{840}{841},\ \cos 2A = -\frac{41}{841},\ \tan 2A = \frac{840}{41}$
3	$\sin A = \frac{\sqrt{2}}{4}$
4	$-\frac{666}{413}$
5	$\sin \frac{A}{2} = -\frac{\sqrt{21}}{6},\ \cos \frac{A}{2} = \frac{\sqrt{15}}{6},\ \tan \frac{A}{2} = -\frac{\sqrt{35}}{5}$
6	$-\frac{12}{5}$
7	Proved: $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$
8	$-\frac{\sqrt{5}+1}{4}$
9	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos 108^\circ = -\frac{\sqrt{5}-1}{4}$
10	$4 + 3\sqrt{3} - 3\sqrt{2} - 2\sqrt{6}$
11	$\frac{119}{144}$
12	$2\sqrt{2} + 2\sqrt{6}$
13	Proved: $\frac{1+\sin 2A}{1-\sin 2A} = \tan^2\left(\frac{\pi}{4} + A\right)$
14	1
15	0
16	$\frac{1}{64}$
17	Proved; $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$
18	$1 + \frac{2\sqrt{3}}{3}$
19	Negative
20	$\cos 300^\circ - \sin 300^\circ = \frac{1+\sqrt{3}}{2}$

Solutions

1. If $\tan A = \frac{7}{6}$, find $\tan 2A$. [1]

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$= -\frac{84}{13}$

Ans: $-\frac{84}{13}$

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2. If $\cos A = \frac{20}{29}$ and $\frac{3\pi}{2} < A < 2\pi$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$. [3]

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{21}{29}$ ($\sin A$ is negative in this quadrant)

$\sin 2A = 2\sin A\cos A = 2(-\frac{21}{29})(\frac{20}{29}) = -\frac{840}{841}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{400}{841} - \frac{441}{841} = -\frac{41}{841}$

$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{840}{41}$

Ans: $\sin 2A = -\frac{840}{841},\ \cos 2A = -\frac{41}{841},\ \tan 2A = \frac{840}{41}$

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3. If $\cos 2A = \frac{3}{4}$ and A is acute, find $\sin A$. [2]

$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$

$\sqrt{\frac{1}{8}} = \frac{\sqrt{2}}{4}$

In this quadrant $\sin A$ is positive, so $\sin A = \frac{\sqrt{2}}{4}$.

Ans: $\sin A = \frac{\sqrt{2}}{4}$

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4. If $\tan A = \frac{6}{7}$, find $\tan 3A$. [2]

$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$= \frac{3(\frac{6}{7}) - (\frac{6}{7})^3}{1-3(\frac{6}{7})^2}$

$= -\frac{666}{413}$

Ans: $-\frac{666}{413}$

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5. If $\cos A = -\frac{1}{6}$ and $-\pi < A < 0$, find $\sin\frac{A}{2}$, $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$. [3]

Here $-\frac{\pi}{2} < \frac{A}{2} < 0$, so $\sin\frac{A}{2}$ is negative and $\cos\frac{A}{2}$ is positive.

$\sin\frac{A}{2} = -\sqrt{\frac{1-\cos A}{2}} = -\frac{\sqrt{21}}{6}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \frac{\sqrt{15}}{6}$

$\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = -\frac{\sqrt{35}}{5}$

Ans: $\sin\frac{A}{2} = -\frac{\sqrt{21}}{6},\ \cos\frac{A}{2} = \frac{\sqrt{15}}{6},\ \tan\frac{A}{2} = -\frac{\sqrt{35}}{5}$

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6. If $\tan\frac{A}{2} = \frac{3}{2}$, find $\tan A$. [1]

$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$= -\frac{12}{5}$

Ans: $-\frac{12}{5}$

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7. Prove that $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$. [5]

Numerator $= 2\sin^2 A + 2\sin A\cos A = 2\sin A(\sin A + \cos A)$

Denominator $= 2\cos^2 A + 2\sin A\cos A = 2\cos A(\cos A + \sin A)$

LHS $= \tan A$ = RHS

Ans: Proved: $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A} = \tan A$

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8. Write the exact value of $\sin 234^\circ$. [1]

$\sin 234^\circ = -\sin 54^\circ$

$\sin 54^\circ = \frac{\sqrt{5}+1}{4}$

Ans: $-\frac{\sqrt{5}+1}{4}$

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9. Find $\sin 18^\circ$ from $\sin 2\theta = \cos 3\theta$ with $\theta = 18^\circ$, and hence evaluate $\cos 108^\circ$. [5]

Let $\theta = 18^\circ$. Then $5\theta = 90^\circ$, so $2\theta = 90^\circ - 3\theta$ and $\sin 2\theta = \cos 3\theta$.

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$; divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1-\sin^2\theta) - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-1\pm\sqrt{5}}{4}$

18° is acute, so $\sin 18^\circ > 0$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

$\cos 108^\circ = \cos(90^\circ + 18^\circ) = -\sin 18^\circ$

$= -\frac{\sqrt{5}-1}{4}$

Ans: $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$; $\cos 108^\circ = -\frac{\sqrt{5}-1}{4}$

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10. Find the exact value of $\tan 7\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ$. [2]

$\tan 7\frac{1}{2}^\circ = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1),\ \tan 22\frac{1}{2}^\circ = \sqrt{2}-1$

$\tan 7\frac{1}{2}^\circ \tan 22\frac{1}{2}^\circ = 4 + 3\sqrt{3} - 3\sqrt{2} - 2\sqrt{6}$

Ans: $4 + 3\sqrt{3} - 3\sqrt{2} - 2\sqrt{6}$

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11. If $\sin 2\theta = \frac{5}{6}$, find $\sin^2\theta + \cos^4\theta$. [1]

$\sin^2\theta + \cos^4\theta = 1 - \frac{1}{4}\sin^2 2\theta$

$= 1 - \frac{1}{4}\cdot\frac{25}{36} = \frac{119}{144}$

Ans: $\frac{119}{144}$

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12. Find the value of $\tan 82\frac{1}{2}^\circ + \tan 7\frac{1}{2}^\circ$. [2]

$\tan(45^\circ + A) + \tan(45^\circ - A) = 2\sec 2A$

With $A = 37\frac{1}{2}^\circ$: $\tan 82\frac{1}{2}^\circ + \tan 7\frac{1}{2}^\circ = 2\sqrt{2} + 2\sqrt{6}$

Ans: $2\sqrt{2} + 2\sqrt{6}$

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13. Prove that $\frac{1+\sin 2A}{1-\sin 2A} = \tan^2\left(\frac{\pi}{4} + A\right)$. [5]

$1 + \sin 2A = (\cos A + \sin A)^2$, $1 - \sin 2A = (\cos A - \sin A)^2$

LHS $= \left(\frac{\cos A+\sin A}{\cos A-\sin A}\right)^2 = \left(\frac{1+\tan A}{1-\tan A}\right)^2$

$= \tan^2\left(\frac{\pi}{4} + A\right)$ = RHS

Ans: Proved: $\frac{1+\sin 2A}{1-\sin 2A} = \tan^2\left(\frac{\pi}{4} + A\right)$

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14. Find the value of $\left(2\cos\frac{8\pi}{17}-1\right)\left(2\cos\frac{16\pi}{17}-1\right)\left(2\cos\frac{32\pi}{17}-1\right)\left(2\cos\frac{64\pi}{17}-1\right)$. [3]

$(2\cos\theta-1)(2\cos 2\theta-1)\cdots(2\cos 2^3\theta-1)=\frac{2\cos 2^4\theta+1}{2\cos\theta+1}$ with $\theta=\frac{8\pi}{17}$

$2^4\theta$ differs from $\pm\theta$ by a multiple of 2π, so $\cos 2^4\theta=\cos\theta$ and the value is 1.

Ans: 1

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15. Find the value of $\sin 60^\circ+\sin 120^\circ+\sin 180^\circ+\sin 240^\circ+\sin 300^\circ$. [3]

$a_1=60^\circ$, $d=60^\circ$, $n=5$, $a_n=300^\circ$

$\text{Sum}=\frac{\sin\left(\frac{a_1+a_n}{2}\right)\sin\frac{nd}{2}}{\sin\frac{d}{2}}=\frac{\sin 180^\circ\ \sin 150^\circ}{\sin 30^\circ}$

$=0$

Ans: 0

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16. Find the value of $\left(\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}\right)^2$. [1]

Known result: $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}=\frac{1}{8}$

$\text{Square}=\frac{1}{64}$

Ans: $\frac{1}{64}$

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17. Show that $\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta)=\frac{1}{4}\sin 3\theta$. Hence evaluate $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ$. [5]

$\sin(60^\circ+\theta)\sin(60^\circ-\theta)=\sin^2 60^\circ-\sin^2\theta=\frac{3}{4}-\sin^2\theta$

LHS $=\sin\theta\left(\frac{3}{4}-\sin^2\theta\right)=\frac{1}{4}(3\sin\theta-4\sin^3\theta)$

$=\frac{1}{4}\sin 3\theta$ = RHS

With $\theta=10^\circ$: $\sin 10^\circ\sin 50^\circ\sin 70^\circ=\frac{1}{8}$

Multiply by $\sin 30^\circ=\frac{1}{2}$: the value is $\frac{1}{16}$.

Ans: Proved; $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\frac{1}{16}$

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18. If $\frac{x}{y}=\frac{\sin 45^\circ}{\sin 15^\circ}$, find $\frac{x+y}{x-y}$. [2]

$\frac{x+y}{x-y}=\frac{\sin A+\sin B}{\sin A-\sin B}=\frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

$=\frac{\tan 30^\circ}{\tan 15^\circ}=1+\frac{2\sqrt{3}}{3}$

Ans: $1+\frac{2\sqrt{3}}{3}$

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19. For $A=100^\circ$, is $\cos\frac{A}{2}-\sin\frac{A}{2}$ positive or negative? [1]

$\frac{A}{2}=50^\circ$

$c-s>0$ for $\frac{A}{2}$ in $\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$ and <0 in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ (adding 2π where needed).

So $\cos 50^\circ-\sin 50^\circ$ is negative.

Ans: Negative

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20. Simplify $\sqrt{1-\sin A}$ for $A=600^\circ$ and find its value. [2]

$\sqrt{1-\sin A}=\left|\cos\frac{A}{2}-\sin\frac{A}{2}\right|$

$\frac{A}{2}=300^\circ$, where $\cos\frac{A}{2}-\sin\frac{A}{2}$ is positive.

$\sqrt{1-\sin 600^\circ}=\cos 300^\circ-\sin 300^\circ=\frac{1+\sqrt{3}}{2}$

Ans: $\cos 300^\circ-\sin 300^\circ=\frac{1+\sqrt{3}}{2}$
