EXPRESSWALA ACADEMY

Maths 1A: Linear Equations

Question Paper, Set 01

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the rank of $A = \begin{bmatrix} -3 & 4 \\ -4 & -1 \end{bmatrix}$. [1 mark]

2. Find the rank of $A = \begin{bmatrix} -2 & -2 & 2 \\ -1 & 1 & 1 \\ 0 & -3 & -1 \end{bmatrix}$ by finding the largest non-singular square submatrix. [2 marks]

3. Find the rank of the matrix $\begin{bmatrix} 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. [1 mark]

4. Find the rank of $A = \begin{bmatrix} 1 & 3 & 4 \\ -3 & -8 & -10 \\ -1 & -2 & -2 \end{bmatrix}$ using elementary row transformations. [2 marks]

5. If $\text{rank}(A) = 5$, $\text{rank}(B) = 1$ and AB is defined, what is the greatest possible value of $\text{rank}(AB)$? [1 mark]

6. Write down the system of equations whose augmented matrix is $\left[\begin{array}{ccc|c} 4 & -4 & 2 & -1 \\ 0 & -5 & -5 & 6 \\ -3 & 0 & 4 & -3 \end{array}\right]$. [1 mark]

7. Examine whether the system $-4x + 4y = -7$ and $-8x + 8y = -10$ is consistent. [2 marks]

8. For what value of k does the system $kx - y = -2$, $-8x + 2y = 3$ have no solution? [2 marks]

9. Find k so that the system $4x + ky = 0$, $16x + 16y = 0$ has a non-trivial solution. [2 marks]

10. Does the system $2x + y - z = 0$, $-x - 3y + 2z = 0$ and $4x - 2y + 2z = 0$ have a non-trivial solution? [3 marks]

11. Show that the system $-4x - 3y - z = 0$, $3x + 6y + 2z = 0$ and $-x = 0$ has a non-trivial solution, and find all its solutions. [5 marks]

12. Show that the system $x + 4y + 4z = -2$, $-y + 4z = -4$ and $2x + 9y + 4z = -5$ is inconsistent, using the rank method. [3 marks]

13. Examine the consistency of the system $x + y + 2z = -2$, $-x + 3y - 3z = 5$ and $2x + 4y + z = 5$ by the rank method and solve it if consistent. [5 marks]

14. Solve $x - y - z = -2$, $3x - 2y = 5$ and $2x - y + z = 7$ by the rank method. [5 marks]

15. Using Cramer's rule, solve the system $-2x + z = -8$, $x + y - 2z = 10$ and $4x + y + 4z = 7$. [5 marks]

16. Find the image of the point $(-9, 7)$ under reflection in the y-axis. [1 mark]

17. The point $(4, -2)$ is reflected in the line $y = x$, and the image is then reflected in the x-axis. Find the final image. [2 marks]

18. The point $(4, 2)$ is reflected in the line $y = -\frac{2}{3}x$. Find the coordinates of its image. [3 marks]

19. Find the image of $(-6, 5)$ when it is rotated anticlockwise about the origin through $90°$. [1 mark]

20. The point $(7, -6)$ is rotated anticlockwise about the origin through $135°$. Find its image. [3 marks]

Key sheet

Q	Answer
1	2
2	3
3	2
4	2
5	1
6	$4x - 4y + 2z = -1$, $-5y - 5z = 6$ and $-3x + 4z = -3$
7	Inconsistent: no solution
8	$k = 4$
9	$k = 4$
10	No: only the trivial solution ($\lvert A\rvert = -8 \neq 0$)
11	$(x, y, z) = (0, k, -3k)$, k any number
12	Inconsistent: $\text{rank}(A) = 2 \neq \text{rank}[A\lvert B] = 3$
13	$x = 4,\ y = 0,\ z = -3$
14	$(x, y, z) = (9 - 2k, 11 - 3k, k)$, k any number
15	$x = 3,\ y = 3,\ z = -2$
16	$(9, 7)$
17	$(-2, -4)$
18	$(-\frac{4}{13}, -\frac{58}{13})$
19	$(-5, -6)$
20	$(-\frac{\sqrt{2}}{2}, \frac{13\sqrt{2}}{2})$

Solutions

1. Find the rank of $A = \begin{bmatrix} -3 & 4 \\ -4 & -1 \end{bmatrix}$. [1]

$\det A = (-3)(-1) - (4)(-4) = 19$

$\det A \neq 0$, so A itself is a non-singular submatrix of order 2.

$\text{rank}(A) = 2$

Ans: 2

---

2. Find the rank of $A = \begin{bmatrix} -2 & -2 & 2 \\ -1 & 1 & 1 \\ 0 & -3 & -1 \end{bmatrix}$ by finding the largest non-singular square submatrix. [2]

$|A| = -2(-1+3) + 2(1-0) + 2(3-0) = 4$

$|A| \neq 0$, so A is non-singular.

$\text{rank}(A) = 3$

Ans: 3

---

3. Find the rank of the matrix $\begin{bmatrix} 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. [1]

The non-zero rows begin with 2, 3 zeros: the count increases row by row, and the zero rows are at the bottom.

So the matrix is in echelon form, and its rank is the number of non-zero rows.

$\text{rank} = 2$

Ans: 2

---

4. Find the rank of $A = \begin{bmatrix} 1 & 3 & 4 \\ -3 & -8 & -10 \\ -1 & -2 & -2 \end{bmatrix}$ using elementary row transformations. [2]

Apply $R_2 \to R_2 + 3R_1$, $R_3 \to R_3 + R_1$: $A \sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

Apply $R_3 \to R_3 - R_2$: $\sim \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

This is in echelon form with 2 non-zero rows.

$\therefore \text{rank}(A) = 2$

Ans: 2

---

5. If $\text{rank}(A) = 5$, $\text{rank}(B) = 1$ and AB is defined, what is the greatest possible value of $\text{rank}(AB)$? [1]

$\text{rank}(AB)$ cannot exceed the rank of either A or B.

$\text{rank}(AB) \leq \min(5, 1) = 1$

Ans: 1

---

6. Write down the system of equations whose augmented matrix is $\left[\begin{array}{ccc|c} 4 & -4 & 2 & -1 \\ 0 & -5 & -5 & 6 \\ -3 & 0 & 4 & -3 \end{array}\right]$. [1]

Each row gives one equation: the first three entries are the coefficients of x, y, z, the last is the constant.

$4x - 4y + 2z = -1$, $-5y - 5z = 6$ and $-3x + 4z = -3$

Ans: $4x - 4y + 2z = -1$, $-5y - 5z = 6$ **and** $-3x + 4z = -3$

---

7. Examine whether the system $-4x + 4y = -7$ and $-8x + 8y = -10$ is consistent. [2]

$\frac{a_1}{a_2} = \frac{-4}{-8}$, $\frac{b_1}{b_2} = \frac{4}{8}$, $\frac{c_1}{c_2} = \frac{-7}{-10}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: the lines are parallel.

The system is inconsistent: no solution.

Ans: Inconsistent: no solution

---

8. For what value of k does the system $kx - y = -2$, $-8x + 2y = 3$ have no solution? [2]

No solution needs $\frac{k}{-8} = \frac{-1}{2} \neq \frac{-2}{3}$

$\frac{k}{-8} = \frac{1}{-2} \Rightarrow k = 4$

With $k = 4$ the constants are not in the same ratio, so the lines are parallel.

Ans: $k = 4$

---

9. Find k so that the system $4x + ky = 0$, $16x + 16y = 0$ has a non-trivial solution. [2]

A non-trivial solution needs $\frac{a_1}{a_2} = \frac{b_1}{b_2}$: $\frac{4}{16} = \frac{k}{16}$

$k = 4$

Ans: $k = 4$

---

10. Does the system $2x + y - z = 0$, $-x - 3y + 2z = 0$ and $4x - 2y + 2z = 0$ have a non-trivial solution? [3]

$|A| = 2(-6 + 4) - (-2 - 8) - (2 + 12) = -8$

$|A| \neq 0$, so the system has only the trivial solution $x = y = z = 0$.

Ans: No: only the trivial solution ($|A| = -8 \neq 0$)

---

11. Show that the system $-4x - 3y - z = 0$, $3x + 6y + 2z = 0$ and $-x = 0$ has a non-trivial solution, and find all its solutions. [5]

$|A| = -4(0 - 0) + 3(0 + 2) - (0 + 6) = 0$

$|A| = 0$, so there are non-trivial solutions.

Put $z = -3k$ in the first two equations:

$-4x - 3y = -3k$, $3x + 6y = 6k$

Solving (determinant $-15 \neq 0$): $x = 0,\ y = k$

These values also satisfy the third equation.

$(x, y, z) = (0, k, -3k)$, k any number

Ans: $(x, y, z) = (0, k, -3k)$**,** k **any number**

---

12. Show that the system $x + 4y + 4z = -2$, $-y + 4z = -4$ and $2x + 9y + 4z = -5$ is inconsistent, using the rank method. [3]

Apply $R_3 \to R_3 - 2R_1$: $[A \mid B] \sim \left[\begin{array}{ccc|c} 1 & 4 & 4 & -2 \\ 0 & -1 & 4 & -4 \\ 0 & 1 & -4 & -1 \end{array}\right]$

Apply $R_3 \to R_3 + R_2$: $\sim \left[\begin{array}{ccc|c} 1 & 4 & 4 & -2 \\ 0 & -1 & 4 & -4 \\ 0 & 0 & 0 & -5 \end{array}\right]$

$\text{rank}(A) = 2$ (non-zero rows of the coefficient part), $\text{rank}[A \mid B] = 3$.

The ranks differ, so the system is inconsistent: no solution.

Ans: Inconsistent: $\text{rank}(A) = 2 \neq \text{rank}[A|B] = 3$

---

13. Examine the consistency of the system $x + y + 2z = -2$, $-x + 3y - 3z = 5$ and $2x + 4y + z = 5$ by the rank method and solve it if consistent. [5]

Apply $R_2 \to R_2 + R_1$, $R_3 \to R_3 - 2R_1$: $[A \mid B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 4 & -1 & 3 \\ 0 & 2 & -3 & 9 \end{array}\right]$

Apply $R_2 \leftrightarrow R_3$: $\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 2 & -3 & 9 \\ 0 & 4 & -1 & 3 \end{array}\right]$

Apply $R_3 \to R_3 - 2R_2$: $\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 2 & -3 & 9 \\ 0 & 0 & 5 & -15 \end{array}\right]$

$\text{rank}(A) = \text{rank}[A \mid B] = 3$ = number of unknowns: consistent, unique solution.

From row 3: $5z = -15$ gives $z = -3$

From row 2: $2y - 3z = 9$, so $y = 0$

From row 1: $x + y + 2z = -2$, so $x = 4$

$x = 4,\ y = 0,\ z = -3$

Ans: $x = 4,\ y = 0,\ z = -3$

---

14. Solve $x - y - z = -2$, $3x - 2y = 5$ and $2x - y + z = 7$ by the rank method. [5]

Apply $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$: $[A \mid B] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 1 & 3 & 11 \end{array}\right]$

Apply $R_3 \to R_3 - R_2$: $\sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\text{rank}(A) = \text{rank}[A \mid B] = 2 < 3$: consistent, infinitely many solutions.

Put $z = k$. From row 2: $y + 3z = 11$, so $y = 11 - 3k$

From row 1: $x - y - z = -2$, so $x = 9 - 2k$

$(x, y, z) = (9 - 2k, 11 - 3k, k)$, k any number

Ans: $(x, y, z) = (9 - 2k, 11 - 3k, k)$, k **any number**

---

15. Using Cramer's rule, solve the system $-2x + z = -8$, $x + y - 2z = 10$ and $4x + y + 4z = 7$. [5]

$$\Delta = \begin{vmatrix} -2 & 0 & 1 \\ 1 & 1 & -2 \\ 4 & 1 & 4 \end{vmatrix} = -2(4+2) + 0(4+8) + (1-4) = -15$$

$$\Delta_1 = \begin{vmatrix} -8 & 0 & 1 \\ 10 & 1 & -2 \\ 7 & 1 & 4 \end{vmatrix} = -8(4+2) + 0(40+14) + (10-7) = -45$$

$$\Delta_2 = \begin{vmatrix} -2 & -8 & 1 \\ 1 & 10 & -2 \\ 4 & 7 & 4 \end{vmatrix} = -2(40+14) + 8(4+8) + (7-40) = -45$$

$$\Delta_3 = \begin{vmatrix} -2 & 0 & -8 \\ 1 & 1 & 10 \\ 4 & 1 & 7 \end{vmatrix} = -2(7-10) + 0(7-40) - 8(1-4) = 30$$

$x = \frac{\Delta_1}{\Delta} = \frac{-45}{-15} = 3,\ y = \frac{\Delta_2}{\Delta} = \frac{-45}{-15} = 3,\ z = \frac{\Delta_3}{\Delta} = \frac{30}{-15} = -2$

Ans: $x = 3,\ y = 3,\ z = -2$

---

16. Find the image of the point $(-9, 7)$ under reflection in the y-axis. [1]

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -9 \\ 7 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

Ans: $(9, 7)$

---

17. The point $(4, -2)$ is reflected in the line $y = x$, and the image is then reflected in the x-axis. Find the final image. [2]

Reflection in the line $y = x$: $(4, -2) \mapsto (-2, 4)$

Reflection in the x-axis: $(-2, 4) \mapsto (-2, -4)$

In matrix form: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$

Ans: $(-2, -4)$

---

18. The point $(4, 2)$ is reflected in the line $y = -\frac{2}{3}x$. Find the coordinates of its image. [3]

$\tan\theta = m = -\frac{2}{3}$, so $\cos 2\theta = \frac{1-m^2}{1+m^2} = \frac{5}{13},\ \sin 2\theta = \frac{2m}{1+m^2} = -\frac{12}{13}$

Reflection matrix: $\begin{bmatrix} \frac{5}{13} & -\frac{12}{13} \\ -\frac{12}{13} & -\frac{5}{13} \end{bmatrix}$

$$\begin{bmatrix} \frac{5}{13} & -\frac{12}{13} \\ -\frac{12}{13} & -\frac{5}{13} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{4}{13} \\ -\frac{58}{13} \end{bmatrix}$$

Image: $(-\frac{4}{13}, -\frac{58}{13})$

Ans: $(-\frac{4}{13}, -\frac{58}{13})$

---

19. Find the image of $(-6, 5)$ when it is rotated anticlockwise about the origin through $90°$. [1]

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -6 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ -6 \end{bmatrix}$$

Ans: $(-5, -6)$

---

20. The point $(7, -6)$ is rotated anticlockwise about the origin through 135°. Find its image. [3]

Rotation matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 7 \\ -6 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{13\sqrt{2}}{2} \end{bmatrix}$$

Image: $(-\frac{\sqrt{2}}{2}, \frac{13\sqrt{2}}{2})$

Ans: $(-\frac{\sqrt{2}}{2}, \frac{13\sqrt{2}}{2})$