EXPRESSWALA ACADEMY

# Maths 1A: Linear Equations

## MCQ Test, Set 09

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

**1.** For $A = \begin{bmatrix} 3 & 2 & -4 \\ 4 & 6 & 2 \\ 1 & -4 & -1 \end{bmatrix}$, the determinant of the submatrix obtained by deleting row 1 and column 3 is

**A.** $22$ **B.** $-22$ **C.** $-10$ **D.** $-14$

**2.** The rank of $\begin{bmatrix} 1 & 3 & -1 \\ -3 & 2 & 4 \\ -1 & 8 & 2 \end{bmatrix}$ is

**A.** $1$ **B.** $0$ **C.** $3$ **D.** $2$

**3.** The rank of the echelon matrix $\begin{bmatrix} 0 & 4 & 7 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is

**A.** $3$ **B.** $4$ **C.** $1$ **D.** $2$

**4.** The rank of the echelon matrix $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ is

**A.** $3$ **B.** $2$ **C.** $4$ **D.** $1$

**5.** Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & 3 & 0 \\ -3 & 1 & -1 & 4 \\ 2 & -2 & 2 & -2 \end{bmatrix}$ is found to be

**A.** $4$ **B.** $3$ **C.** $1$ **D.** $2$

**6.** Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & -2 & -1 \\ 4 & -5 & 0 & -3 \\ 3 & -2 & 2 & -2 \end{bmatrix}$ is found to be

**A.** $2$ **B.** $4$ **C.** $3$ **D.** $1$

**7.** $A$ and $B$ are square matrices of order 6 with $\text{rank}(A) = 1$ and $\text{rank}(B) = 2$. The greatest possible value of $\text{rank}(A + B)$ is

**A.** $3$ **B.** $1$ **C.** $6$ **D.** $2$

**8.** The rank of the identity matrix $I_5$ is

**A.** $0$ **B.** $25$ **C.** $5$ **D.** $1$

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**9.** The augmented matrix of the system $5y - 4z = -5$, $6x - 3y + 4z = -8$ and $4x - 4y + 5z = -1$ is

**A.** $\left[\begin{array}{ccc|c} 5 & -4 & 0 & -5 \\ 6 & -3 & 4 & -8 \\ 4 & -4 & 5 & -1 \end{array}\right]$

**B.** $\left[\begin{array}{ccc|c} 0 & 5 & -4 & 5 \\ 6 & -3 & 4 & 8 \\ 4 & -4 & 5 & 1 \end{array}\right]$

**C.** $\left[\begin{array}{ccc|c} -5 & 5 & -4 & 0 \\ -8 & -3 & 4 & 6 \\ -1 & -4 & 5 & 4 \end{array}\right]$

**D.** $\left[\begin{array}{ccc|c} 0 & 5 & -4 & -5 \\ 6 & -3 & 4 & -8 \\ 4 & -4 & 5 & -1 \end{array}\right]$

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**10.** The solution of $5x + 2y = 25$ and $-2x - 4y = -26$ is

**A.** $x = 5,\ y = 3$ **B.** $x = 3,\ y = 5$ **C.** $x = -3,\ y = 5$ **D.** $x = 3,\ y = -5$

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**11.** The system $6x - y = 21$ and $2x - 6y = -10$ has

**A.** infinitely many solutions **B.** a unique solution

**C.** no solution **D.** exactly two solutions

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**12.** The system $6x + 2y = 0$, $-12x + ky = 0$ has non-trivial solutions when $k =$

**A.** $-4$ **B.** $-1$ **C.** $2$ **D.** $4$

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**13.** The system $8x - 7y = 0$, $-16x + ky = 0$ has non-trivial solutions when $k =$

**A.** $-14$ **B.** $-7$ **C.** $14$ **D.** $\frac{7}{2}$

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**14.** The system $-2x + 5y - z = 0$, $y - z = 0$ and $x + y + kz = 0$ has a non-trivial solution when $k =$

**A.** $3$ **B.** $-2$ **C.** $-4$ **D.** $-3$

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**15.** The system $-x + ky + 2z = 0$, $-4x - 3y + 5z = 0$ and $-5x - 2y + z = 0$ has a non-trivial solution when $k =$

**A.** $1$ **B.** $-1$ **C.** $-2$ **D.** $0$

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**16.** For the system $x - 3z = 9$, $x + y = 0$ and $-x - 2y - 3z = 9$, the pair $(\mathrm{rank}(A), \mathrm{rank}[A \mid B])$ is

**A.** $(2, 3)$ **B.** $(2, 2)$ **C.** $(1, 2)$ **D.** $(3, 3)$

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**17.** A system $AX = B$ in 3 unknowns has $\mathrm{rank}(A) = 2$ and $\mathrm{rank}[A \mid B] = 2$. The system

**A.** has infinitely many solutions **B.** is inconsistent

**C.** has exactly two solutions **D.** has a unique solution

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**18.** For the system $x + 2z = 0$, $-2x + 2y + 2z = -4$ and $x + 2y - 3z = 7$, the determinant $\Delta_3$ used in Cramer's rule is

**A.** $24$ **B.** $22$ **C.** $-44$ **D.** $-22$

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**19.** For the system $3x + 2y + 2z = -3$, $4x + 3z = 2$ and $x - y = 1$, the determinant $\Delta_1$ used in Cramer's rule is

**A.** $-14$ **B.** $7$ **C.** $14$ **D.** $-7$

**20.** The image of $(5, 3)$ under reflection in the line $y = x$ is

**A.** $(3, 5)$ **B.** $(-3, -5)$ **C.** $(-5, 3)$ **D.** $(-5, -3)$

**21.** The image of $(-3, -1)$ under reflection in the y-axis is

**A.** $(-3, 1)$ **B.** $(1, 3)$ **C.** $(3, -1)$ **D.** $(-1, -3)$

**22.** The image of $(7, 6)$ under reflection in the line $y = \frac{4}{3}x$ is

**A.** $(-\frac{193}{25}, \frac{126}{25})$ **B.** $(\frac{42}{5}, \frac{19}{5})$
**C.** $(\frac{19}{5}, \frac{42}{5})$ **D.** $(-\frac{19}{5}, -\frac{42}{5})$

**23.** The image of $(-2, -3)$ under reflection in the line $y = -\frac{4}{3}x$ is

**A.** $(-\frac{86}{25}, -\frac{27}{25})$ **B.** $(\frac{27}{25}, \frac{86}{25})$
**C.** $(-\frac{58}{25}, \frac{69}{25})$ **D.** $(\frac{86}{25}, \frac{27}{25})$

**24.** When $(10, -3)$ is rotated anticlockwise about the origin through $270^\circ$, its image is

**A.** $(-3, 10)$ **B.** $(3, 10)$ **C.** $(-3, -10)$ **D.** $(-10, -3)$

**25.** The matrix that rotates every point anticlockwise about the origin through $60^\circ$ is

**A.** $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$

**B.** $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

**C.** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

**D.** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

## Key sheet

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|---|---|---|---|---|
| **1.** B | **2.** D | **3.** D | **4.** A | **5.** D |
| **6.** A | **7.** A | **8.** C | **9.** D | **10.** B |
| **11.** B | **12.** A | **13.** C | **14.** D | **15.** B |
| **16.** B | **17.** A | **18.** B | **19.** D | **20.** A |
| **21.** C | **22.** C | **23.** D | **24.** C | **25.** C |

## Solutions

**1.** For $A = \begin{bmatrix} 3 & 2 & -4 \\ 4 & 6 & 2 \\ 1 & -4 & -1 \end{bmatrix}$, the determinant of the submatrix obtained by deleting row 1 and column 3 is

Deleting row 1 and column 3 leaves $\begin{bmatrix} 4 & 6 \\ 1 & -4 \end{bmatrix}$.

$\det = (4)(-4) - (6)(1) = -22$

**Ans: (B)** $-22$

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**2.** The rank of $\begin{bmatrix} 1 & 3 & -1 \\ -3 & 2 & 4 \\ -1 & 8 & 2 \end{bmatrix}$ is

$|A| = 0$

A $2 \times 2$ submatrix $\begin{bmatrix} 1 & 3 \\ -3 & 2 \end{bmatrix}$ has determinant $11$, so the rank is $2$.

**Ans: (D)** $2$

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**3.** The rank of the echelon matrix $\begin{bmatrix} 0 & 4 & 7 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is

The matrix is in echelon form, so its rank is the number of non-zero rows.

$\text{rank} = 2$

**Ans: (D)** $2$

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**4.** The rank of the echelon matrix $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ is

The matrix is in echelon form, so its rank is the number of non-zero rows.

$\text{rank} = 3$

**Ans: (A)** $3$

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**5.** Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & 3 & 0 \\ -3 & 1 & -1 & 4 \\ 2 & -2 & 2 & -2 \end{bmatrix}$ is found to be

Apply $R_2 \to R_2 + 3R_1$, $R_3 \to R_3 - 2R_1$: $A \sim \begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & -8 & 8 & 4 \\ 0 & 4 & -4 & -2 \end{bmatrix}$

Apply $R_2 \leftrightarrow R_3$: $\sim \begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 4 & -4 & -2 \\ 0 & -8 & 8 & 4 \end{bmatrix}$

Apply $R_3 \to R_3 + 2R_2$: $\sim \begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 4 & -4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Number of non-zero rows $= 2$, so the rank is $2$.

**Ans: (D)** $2$

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**6.** Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & -2 & -1 \\ 4 & -5 & 0 & -3 \\ 3 & -2 & 2 & -2 \end{bmatrix}$ is found to be

Apply $R_2 \to R_2 - 4R_1$, $R_3 \to R_3 - 3R_1$: $A \sim \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & 7 & 8 & 1 \\ 0 & 7 & 8 & 1 \end{bmatrix}$

Apply $R_3 \to R_3 - R_2$: $\sim \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Number of non-zero rows $= 2$, so the rank is $2$.

**Ans: (A)** $2$

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**7.** $A$ and $B$ are square matrices of order 6 with $\text{rank}(A) = 1$ and $\text{rank}(B) = 2$. The greatest possible value of $\text{rank}(A + B)$ is

$\text{rank}(A + B) \le \text{rank}(A) + \text{rank}(B)$.

$1 + 2 = 3$

**Ans: (A)** $3$

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**8.** The rank of the identity matrix $I_5$ is

$I_5$ is in echelon form with 5 non-zero rows.

$\text{rank}(I_5) = 5$

**Ans: (C)** $5$

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**9.** The augmented matrix of the system $5y - 4z = -5$, $6x - 3y + 4z = -8$ and $4x - 4y + 5z = -1$ is

Write the coefficients of $x, y, z$ in order (a missing variable has coefficient $0$), then the constant.

$[A \mid B] = \left[\begin{array}{ccc|c} 0 & 5 & -4 & -5 \\ 6 & -3 & 4 & -8 \\ 4 & -4 & 5 & -1 \end{array}\right]$

**Ans: (D)** $\left[\begin{array}{ccc|c} 0 & 5 & -4 & -5 \\ 6 & -3 & 4 & -8 \\ 4 & -4 & 5 & -1 \end{array}\right]$

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**10.** The solution of $5x + 2y = 25$ and $-2x - 4y = -26$ is

Eliminate one variable and substitute back.

$x = 3,\ y = 5$

**Ans: (B)** $x = 3,\ y = 5$

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**11.** The system $6x - y = 21$ and $2x - 6y = -10$ has

$\frac{a_1}{a_2} = \frac{6}{2},\ \frac{b_1}{b_2} = \frac{-1}{-6},\ \frac{c_1}{c_2} = \frac{21}{-10}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: the lines meet in one point.

**Ans: (B) a unique solution**

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**12.** The system $6x + 2y = 0$, $-12x + ky = 0$ has non-trivial solutions when $k =$

$\frac{6}{-12} = \frac{2}{k}$

$k = -4$

**Ans: (A)** $-4$

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**13.** The system $8x - 7y = 0$, $-16x + ky = 0$ has non-trivial solutions when $k =$

$\frac{8}{-16} = \frac{-7}{k}$

$k = 14$

**Ans: (C)** $14$

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**14.** The system $-2x + 5y - z = 0$, $y - z = 0$ and $x + y + kz = 0$ has a non-trivial solution when $k =$

Non-trivial solutions need $|A| = 0$: $-2k - 6 = 0$

$k = -3$

**Ans: (D)** $-3$

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**15.** The system $-x + ky + 2z = 0$, $-4x - 3y + 5z = 0$ and $-5x - 2y + z = 0$ has a non-trivial solution when $k =$

Non-trivial solutions need $|A| = 0$: $-21k - 21 = 0$

$k = -1$

**Ans: (B)** $-1$

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**16.** For the system $x - 3z = 9$, $x + y = 0$ and $-x - 2y - 3z = 9$, the pair $(\text{rank}(A), \text{rank}[A \mid B])$ is

Apply $R_3 \to R_3 + 2R_2$: $\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 9 \\ 0 & 1 & 3 & -9 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Coefficient part: 2 non-zero rows; whole matrix: 2.

**Ans: (B)** $(2, 2)$

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**17.** A system $AX = B$ in 3 unknowns has $\text{rank}(A) = 2$ and $\text{rank}[A \mid B] = 2$. The system

Unequal ranks: inconsistent. Equal ranks $= n$: unique solution. Equal ranks $< n$: infinitely many.

Here the system has infinitely many solutions.

**Ans: (A) has infinitely many solutions**

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**18.** For the system $x + 2z = 0$, $-2x + 2y + 2z = -4$ and $x + 2y - 3z = 7$, the determinant $\Delta_3$ used in Cramer's rule is

Replace column 3 of $A$ by the constants: $\Delta_3 = \begin{vmatrix} 1 & 0 & 0 \\ -2 & 2 & -4 \\ 1 & 2 & 7 \end{vmatrix}$

$= (14 + 8) + 0(-14 + 4) + 0(-4 - 2) = 22$

**Ans: (B)** $22$

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**19.** For the system $3x + 2y + 2z = -3$, $4x + 3z = 2$ and $x - y = 1$, the determinant $\Delta_1$ used in Cramer's rule is

Replace column 1 of $A$ by the constants: $\Delta_1 = \begin{vmatrix} -3 & 2 & 2 \\ 2 & 0 & 3 \\ 1 & -1 & 0 \end{vmatrix}$

$= -3(0 + 3) - 2(0 - 3) + 2(-2 - 0) = -7$

**Ans: (D)** $-7$

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**20.** The image of $(5, 3)$ under reflection in the line $y = x$ is

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

**Ans: (A)** $(3, 5)$

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**21.** The image of $(-3, -1)$ under reflection in the y-axis is

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

**Ans: (C)** $(3, -1)$

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**22.** The image of $(7, 6)$ under reflection in the line $y = \frac{4}{3}x$ is

Reflection matrix $\begin{bmatrix} -\frac{7}{25} & \frac{24}{25} \\ \frac{24}{25} & \frac{7}{25} \end{bmatrix}$

Image $(\frac{19}{5}, \frac{42}{5})$

**Ans: (C)** $(\frac{19}{5}, \frac{42}{5})$

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**23.** The image of $(-2, -3)$ under reflection in the line $y = -\frac{4}{3}x$ is

Reflection matrix $\begin{bmatrix} -\frac{7}{25} & -\frac{24}{25} \\ -\frac{24}{25} & \frac{7}{25} \end{bmatrix}$

Image $(\frac{86}{25}, \frac{27}{25})$

**Ans: (D)** $(\frac{86}{25}, \frac{27}{25})$

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**24.** When $(10, -3)$ is rotated anticlockwise about the origin through $270°$, its image is

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ -10 \end{bmatrix}$$

**Ans: (C)** $(-3, -10)$

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**25.** The matrix that rotates every point anticlockwise about the origin through $60°$ is

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

**Ans: (C)** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

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