EXPRESSWALA ACADEMY

Maths 1A: Linear Equations

MCQ Test, Set 08

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If the matrix $\begin{bmatrix} 1 & -3 \\ -2 & k \end{bmatrix}$ has rank 1, then $k =$

A. 6 **B.** $\frac{3}{2}$ **C.** -6 **D.** $\frac{2}{3}$

2. The rank of $\begin{bmatrix} 4 & -2 & 0 \\ 2 & -2 & -1 \\ -1 & 2 & 2 \end{bmatrix}$ is

A. 3 **B.** 0 **C.** 1 **D.** 2

3. Is the matrix $\begin{bmatrix} -3 & 5 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ in echelon form?

A. No: a row begins with fewer zeros than the row above it

B. No: a zero row lies above a non-zero row

C. No: two non-zero rows begin with the same number of zeros

D. Yes, and its rank is 2

4. The rank of the echelon matrix $\begin{bmatrix} -4 & -3 & 1 & 7 & 2 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is

A. 2 **B.** 5 **C.** 1 **D.** 3

5. To reduce $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 5 & 2 \\ -3 & 0 & 3 \end{bmatrix}$ to echelon form, which of the following operations is NOT an elementary transformation?

A. $R_1 \to R_1 + 4R_3$ **B.** $C_3 \leftrightarrow C_1$

C. $R_3 \to 4R_3$ **D.** $R_3 \to R_3^2$

6. Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & 3 & -1 & -1 \\ 1 & -3 & 1 & 1 \end{bmatrix}$ is found to be

A. 0 **B.** 1 **C.** 3 **D.** 2

7. If $\text{rank}(A) = 5$, $\text{rank}(B) = 2$ and AB is defined, the greatest possible value of $\text{rank}(AB)$ is

A. 7 **B.** 2 **C.** 5 **D.** 10

8. A matrix A of order 3×5 has rank 3. Then $\text{rank}(A^T) =$

A. 3 **B.** 4 **C.** 0 **D.** 5

9. The system with coefficient matrix $\begin{bmatrix} 0 & 2 & 6 \\ -1 & -5 & -4 \\ -1 & -2 & 0 \end{bmatrix}$ and constant matrix $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is

A. inconsistent **B.** non-homogeneous

C. homogeneous **D.** always without a solution

10. The system $-4x - 3y = 31$ and $5x - y = -15$ has

A. exactly two solutions **B.** infinitely many solutions

C. a unique solution **D.** no solution

11. The system $2x - y = -3$ and $4x - 2y = -6$ has

A. a unique solution **B.** infinitely many solutions

C. exactly two solutions **D.** no solution

12. The homogeneous system $-3x + y = 0$, $-6x - 2y = 0$ has

A. exactly one non-zero solution **B.** no solution

C. only the trivial solution **D.** infinitely many solutions

13. The system $x - 4y = 0$, $4x + ky = 0$ has non-trivial solutions when $k =$

A. 16 **B.** -4 **C.** -16 **D.** -1

14. The system $kx + z = 0$, $-6x - 4y - z = 0$ and $-4x - 3y - z = 0$ has a non-trivial solution when $k =$

A. -3 **B.** 2 **C.** -1 **D.** -2

15. The system $kx - 2y + 3z = 0$, $-4x - 6y - z = 0$ and $x + z = 0$ has a non-trivial solution when $k =$

A. -2 **B.** 3 **C.** 2 **D.** 1

16. For the system $x + y - 2z = 8$, $-2x = 4$ and $-5x - y + 2z = 6$, the pair $(\text{rank}(A), \text{rank}[A \mid B])$ is

A. $(2, 2)$ **B.** $(2, 3)$ **C.** $(1, 2)$ **D.** $(3, 3)$

17. A system $AX = B$ in 4 unknowns has $\text{rank}(A) = 4$ and $\text{rank}[A \mid B] = 5$. The system

A. has exactly two solutions **B.** has infinitely many solutions

C. has a unique solution **D.** is inconsistent

18. For the system $3x + 3y + z = 13$, $-x + 4y + 4z = -8$ and $3y + z = 1$, the determinant Δ_1 used in Cramer's rule is

A. -24 **B.** 48 **C.** 96 **D.** -96

19. For the system $3x + 3y + 2z = 3$, $-3y - 3z = -3$ and $2x + 3y + 2z = 2$, the determinant Δ_3 used in Cramer's rule is

A. 9 **B.** -6 **C.** -9 **D.** 3

20. The image of $(8, 6)$ under reflection in the x-axis is

A. $(-6, -8)$ **B.** $(6, 8)$ **C.** $(-8, -6)$ **D.** $(8, -6)$

21. The matrix of reflection in the line $y = -x$ is

A. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

22. For reflection in the line $y = -2x$, the entry $\cos 2\theta$ of the reflection matrix is

A. $-\frac{3}{5}$ **B.** $\frac{3}{5}$ **C.** $-\frac{4}{5}$ **D.** $\frac{4}{5}$

23. The matrix of reflection in the line $y = -\frac{4}{3}x$ is

A. $\begin{bmatrix} -\frac{7}{25} & \frac{24}{25} \\ -\frac{24}{25} & -\frac{7}{25} \end{bmatrix}$

B. $\begin{bmatrix} -\frac{24}{25} & -\frac{7}{25} \\ -\frac{7}{25} & \frac{24}{25} \end{bmatrix}$

C. $\begin{bmatrix} \frac{7}{25} & -\frac{24}{25} \\ -\frac{24}{25} & -\frac{7}{25} \end{bmatrix}$

D. $\begin{bmatrix} -\frac{7}{25} & -\frac{24}{25} \\ -\frac{24}{25} & \frac{7}{25} \end{bmatrix}$

24. When $(2, 9)$ is rotated anticlockwise about the origin through 210°, its image is

A. $(-\frac{9\sqrt{3}}{2} - 1, \frac{9}{2} - \sqrt{3})$ **B.** $(-\frac{9}{2} - \sqrt{3}, 1 - \frac{9\sqrt{3}}{2})$

C. $(\frac{9}{2} - \sqrt{3}, -\frac{9\sqrt{3}}{2} - 1)$ **D.** $(-\frac{9}{2} + \sqrt{3}, 1 + \frac{9\sqrt{3}}{2})$

25. The matrix that rotates every point anticlockwise about the origin through 135° is

A. $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

B. $\begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

C. $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

Key sheet

1. A	**2.** A	**3.** B	**4.** A	**5.** D
6. B	**7.** B	**8.** A	**9.** C	**10.** C
11. B	**12.** C	**13.** C	**14.** D	**15.** C
16. B	**17.** D	**18.** D	**19.** A	**20.** D
21. B	**22.** A	**23.** D	**24.** C	**25.** C

Solutions

1. If the matrix $\begin{bmatrix} 1 & -3 \\ -2 & k \end{bmatrix}$ has rank 1, then $k =$

Rank 1 means the matrix is singular: $k - (-3)(-2) = 0$

$k = 6$

Ans: (A) 6

---

2. The rank of $\begin{bmatrix} 4 & -2 & 0 \\ 2 & -2 & -1 \\ -1 & 2 & 2 \end{bmatrix}$ is

$|A| = -2$

$|A| \neq 0$, so the rank is 3.

Ans: (A) 3

---

3. Is the matrix $\begin{bmatrix} -3 & 5 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ in echelon form?

Zeros before the first non-zero entry, row by row: 0, none (zero row), 2.

Echelon form needs this count to increase down the rows, with zero rows at the bottom. No: a zero row lies above a non-zero row.

Ans: (B) No: a zero row lies above a non-zero row

---

4. The rank of the echelon matrix $\begin{bmatrix} -4 & -3 & 1 & 7 & 2 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is

The matrix is in echelon form, so its rank is the number of non-zero rows.

$\text{rank} = 2$

Ans: (A) 2

---

5. To reduce $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 5 & 2 \\ -3 & 0 & 3 \end{bmatrix}$ to echelon form, which of the following operations is NOT an elementary transformation?

The elementary transformations are: interchange two rows (or columns), multiply a row by a non-zero number, add a multiple of another row.

$R_3 \to R_3^2$ is none of these, so it can change the rank.

Ans: (D) $R_3 \to R_3^2$

---

6. Using elementary transformations, the rank of $\begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & 3 & -1 & -1 \\ 1 & -3 & 1 & 1 \end{bmatrix}$ is found to be

Apply $R_2 \to R_2 + R_1$, $R_3 \to R_3 - R_1$: $A \sim \begin{bmatrix} 1 & -3 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Number of non-zero rows $= 1$, so the rank is 1.

Ans: (B) 1

---

7. If $\text{rank}(A) = 5$, $\text{rank}(B) = 2$ and AB is defined, the greatest possible value of $\text{rank}(AB)$ is

$\text{rank}(AB)$ cannot exceed the rank of either A or B.

$\text{rank}(AB) \leq \min(5, 2) = 2$

Ans: (B) 2

---

8. A matrix A of order 3×5 has rank 3. Then $\text{rank}(A^T) =$

$\text{rank}(A^T) = \text{rank}(A)$.

$\text{rank}(A^T) = 3$

Ans: (A) 3

---

9. The system with coefficient matrix $\begin{bmatrix} 0 & 2 & 6 \\ -1 & -5 & -4 \\ -1 & -2 & 0 \end{bmatrix}$ and constant matrix $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is

A system is homogeneous when every constant is zero, that is $B = O$.

Here $B = O$, so the system is homogeneous.

Ans: (C) homogeneous

---

10. The system $-4x - 3y = 31$ and $5x - y = -15$ has

$\frac{a_1}{a_2} = \frac{-4}{5}$, $\frac{b_1}{b_2} = \frac{-3}{-1}$, $\frac{c_1}{c_2} = \frac{31}{-15}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: the lines meet in one point.

Ans: (C) a unique solution

---

11. The system $2x - y = -3$ and $4x - 2y = -6$ has

$\frac{a_1}{a_2} = \frac{2}{4}$, $\frac{b_1}{b_2} = \frac{-1}{-2}$, $\frac{c_1}{c_2} = \frac{-3}{-6}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: the lines coincide.

Ans: (B) infinitely many solutions

---

12. The homogeneous system $-3x + y = 0$, $-6x - 2y = 0$ has

$|A| = 12$

$|A| \neq 0$, so only $x = y = 0$.

Ans: (C) only the trivial solution

---

13. The system $x - 4y = 0$, $4x + ky = 0$ has non-trivial solutions when $k =$

$\frac{1}{4} = \frac{-4}{k}$

$k = -16$

Ans: (C) -16

---

14. The system $kx + z = 0$, $-6x - 4y - z = 0$ and $-4x - 3y - z = 0$ has a non-trivial solution when $k =$

Non-trivial solutions need $|A| = 0$: $k + 2 = 0$

$k = -2$

Ans: (D) -2

---

15. The system $kx - 2y + 3z = 0$, $-4x - 6y - z = 0$ and $x + z = 0$ has a non-trivial solution when $k =$

Non-trivial solutions need $|A| = 0$: $12 - 6k = 0$

$k = 2$

Ans: (C) 2

---

16. For the system $x + y - 2z = 8$, $-2x = 4$ and $-5x - y + 2z = 6$, the pair $(\text{rank}(A), \text{rank}[A \mid B])$ is

Apply $R_3 \to R_3 - 2R_2$: $\sim \left[\begin{array}{ccc|c} 1 & 1 & -2 & 8 \\ 0 & 2 & -4 & 20 \\ 0 & 0 & 0 & 6 \end{array}\right]$

Coefficient part: 2 non-zero rows; whole matrix: 3.

Ans: (B) $(2, 3)$

---

17. A system $AX = B$ in 4 unknowns has $\text{rank}(A) = 4$ and $\text{rank}[A \mid B] = 5$. The system

Unequal ranks: inconsistent. Equal ranks $= n$: unique solution. Equal ranks $< n$: infinitely many. Here the system is inconsistent.

Ans: (D) is inconsistent

---

18. For the system $3x + 3y + z = 13$, $-x + 4y + 4z = -8$ and $3y + z = 1$, the determinant Δ_1 used in Cramer's rule is

Replace column 1 of A by the constants: $\Delta_1 = \begin{vmatrix} 13 & 3 & 1 \\ -8 & 4 & 4 \\ 1 & 3 & 1 \end{vmatrix}$

$= 13(4 - 12) - 3(-8 - 4) + (-24 - 4) = -96$

Ans: (D) -96

---

19. For the system $3x + 3y + 2z = 3$, $-3y - 3z = -3$ and $2x + 3y + 2z = 2$, the determinant Δ_3 used in Cramer's rule is

Replace column 3 of A by the constants: $\Delta_3 = \begin{vmatrix} 3 & 3 & 3 \\ 0 & -3 & -3 \\ 2 & 3 & 2 \end{vmatrix}$

$= 3(-6 + 9) - 3(0 + 6) + 3(0 + 6) = 9$

Ans: (A) 9

---

20. The image of $(8, 6)$ under reflection in the x-axis is

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$

Ans: (D) $(8, -6)$

---

21. The matrix of reflection in the line $y = -x$ is

Reflection in the line $y = -x$ sends $(x, y) \mapsto (-y, -x)$, so the matrix is $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

Ans: (B) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

---

22. For reflection in the line $y = -2x$, the entry $\cos 2\theta$ of the reflection matrix is

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-(-2)^2}{1+(-2)^2} = -\frac{3}{5}$

Ans: (A) $-\frac{3}{5}$

---

23. The matrix of reflection in the line $y = -\frac{4}{3}x$ is

$m = -\frac{4}{3}$: $\frac{1-m^2}{1+m^2} = -\frac{7}{25}$, $\frac{2m}{1+m^2} = -\frac{24}{25}$

$\begin{bmatrix} -\frac{7}{25} & -\frac{24}{25} \\ -\frac{24}{25} & \frac{7}{25} \end{bmatrix}$

Ans: (D) $\begin{bmatrix} -\frac{7}{25} & -\frac{24}{25} \\ -\frac{24}{25} & \frac{7}{25} \end{bmatrix}$

---

24. When $(2, 9)$ is rotated anticlockwise about the origin through 210°, its image is

$$\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} - \sqrt{3} \\ -\frac{9\sqrt{3}}{2} - 1 \end{bmatrix}$$

Ans: (C) $(\frac{9}{2} - \sqrt{3}, -\frac{9\sqrt{3}}{2} - 1)$

---

25. The matrix that rotates every point anticlockwise about the origin through 135° is

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

Ans: (C) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$
