EXPRESSWALA ACADEMY

# Maths 1A: Inverse Trigonometric Functions

## Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

**1.** Find the domain of $\sin^{-1}(5x - 3)$. [1 mark]

**2.** Find the principal value of $\cot^{-1}\frac{1}{\sqrt{3}}$. [1 mark]

**3.** Evaluate $\tan^{-1}\sqrt{3} - \sin^{-1}1$. [2 marks]

**4.** Find $\cos^{-1}\left(\cos\frac{3\pi}{8}\right)$. [1 mark]

**5.** Using the rules for negative arguments, evaluate $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) + \cos^{-1}(-1)$. [2 marks]

**6.** Express $\operatorname{cosec}^{-1}\frac{8}{5}$ as an inverse sine. [1 mark]

**7.** If $\sec^{-1}x = \frac{\pi}{8}$, find $\operatorname{cosec}^{-1}x$. [2 marks]

**8.** Prove that $\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{56}{65}$. [5 marks]

**9.** Find $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$. [3 marks]

**10.** Solve $\tan^{-1}5x + \tan^{-1}12x = \frac{\pi}{4}$. [5 marks]

**11.** Prove that $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$. [5 marks]

**12.** Express $\sin^{-1}\frac{20}{29}$ as an inverse cosine and as an inverse tangent. [2 marks]

**13.** Express $2\tan^{-1}\frac{1}{5}$ as a single inverse tangent, a single inverse sine and a single inverse cosine. [3 marks]

**14.** Write the horizontal asymptotes of the graph of $y = \cot^{-1}x$. [1 mark]

**15.** Find $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$. [2 marks]

**16.** Find the value of $\tan^{-1}\frac{2x}{1-x^2}$ when $x = \sqrt{3} + 2$. [3 marks]

**17.** If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{5}$, find $\cos^{-1}x + \cos^{-1}y$. [3 marks]

**18.** Prove that $\tan^{-1}\frac{1}{1+3\cdot4} + \tan^{-1}\frac{1}{1+4\cdot5} + \cdots + \tan^{-1}\frac{1}{1+(n+2)(n+3)} = \tan^{-1}\frac{n}{1+3(n+3)}$. [5 marks]

**19.** Which substitution removes the square root in $\sqrt{x^2 - 16}$? What does the root become? [1 mark]

**20.** Simplify $\sin^{-1}\frac{x}{\sqrt{25+x^2}}$ for $x \in \mathbb{R}$. [2 marks]

## Key sheet

| Q | Answer |
|---|---|
| 1 | $\left[\frac{2}{5}, \frac{4}{5}\right]$ |
| 2 | $\frac{\pi}{3}$ |
| 3 | $-\frac{\pi}{6}$ |
| 4 | $\frac{3\pi}{8}$ |
| 5 | $\frac{2\pi}{3}$ |
| 6 | $\sin^{-1}\frac{5}{8}$ |
| 7 | $\frac{3\pi}{8}$ |
| 8 | Proved: $\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{56}{65}$ |
| 9 | $\frac{\pi}{4}$ |
| 10 | $x = \frac{1}{20}$ |
| 11 | Proved |
| 12 | $\cos^{-1}\frac{21}{29}$, $\tan^{-1}\frac{20}{21}$ |
| 13 | $\tan^{-1}\frac{5}{12}$, $\sin^{-1}\frac{5}{13}$, $\cos^{-1}\frac{12}{13}$ |
| 14 | $y = 0$ and $y = \pi$ |
| 15 | $\frac{\pi}{4}$ |
| 16 | $-\frac{\pi}{6}$ |
| 17 | $\frac{3\pi}{5}$ |
| 18 | Proved |
| 19 | $x = 4\sec\theta$; the root becomes $4\lvert\tan\theta\rvert$ |
| 20 | $\tan^{-1}\frac{x}{5}$ |

## Solutions

**1.** Find the domain of $\sin^{-1}(5x-3)$. [1]

$-1 \le 5x-3 \le 1$

$2 \le 5x \le 4$

$\frac{2}{5} \le x \le \frac{4}{5}$

**Ans:** $[\frac{2}{5}, \frac{4}{5}]$

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**2.** Find the principal value of $\cot^{-1}\frac{1}{\sqrt{3}}$. [1]

We need $\theta \in (0,\pi)$ with $\cot\theta = \frac{1}{\sqrt{3}}$.

$\therefore \cot^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{3}$

**Ans:** $\frac{\pi}{3}$

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**3.** Evaluate $\tan^{-1}\sqrt{3} - \sin^{-1}1$. [2]

$\tan^{-1}\sqrt{3} = \frac{\pi}{3},\ \sin^{-1}1 = \frac{\pi}{2}$

$\frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$

**Ans:** $-\frac{\pi}{6}$

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**4.** Find $\cos^{-1}\left(\cos\frac{3\pi}{8}\right)$. [1]

$\frac{3\pi}{8}$ lies in the principal range $[0,\pi]$.

So $\cos^{-1}(\cos\theta) = \theta$ gives $\frac{3\pi}{8}$.

**Ans:** $\frac{3\pi}{8}$

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**5.** Using the rules for negative arguments, evaluate $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) + \cos^{-1}(-1)$. [2]

$\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) = -\operatorname{cosec}^{-1}\frac{2}{\sqrt{3}} = -\frac{\pi}{3}$

$\cos^{-1}(-1) = \pi - \cos^{-1}1 = \pi - 0 = \pi$

$\text{Sum} = \frac{2\pi}{3}$

**Ans:** $\frac{2\pi}{3}$

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**6.** Express $\operatorname{cosec}^{-1}\frac{8}{5}$ as an inverse sine. [1]

$\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x},\ |x| \ge 1$

$\operatorname{cosec}^{-1}\frac{8}{5} = \sin^{-1}\frac{5}{8}$

**Ans:** $\sin^{-1}\frac{5}{8}$

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**7.** If $\sec^{-1}x = \frac{\pi}{8}$, find $\operatorname{cosec}^{-1}x$. [2]

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}$

**Ans:** $\frac{3\pi}{8}$

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**8.** Prove that $\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{56}{65}$. [5]

Let $x = \frac{5}{13}$, $y = \frac{4}{5}$.

$\sqrt{1-x^2} = \frac{12}{13},\ \sqrt{1-y^2} = \frac{3}{5}$

Here $x \le y$.

$xy + \sqrt{1-x^2}\sqrt{1-y^2} = \frac{5}{13}\cdot\frac{4}{5} + \frac{12}{13}\cdot\frac{3}{5} = \frac{56}{65}$

So the formula gives $\cos^{-1}\frac{56}{65}$.

Hence $\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{56}{65}$.

**Ans: Proved:** $\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} = \cos^{-1}\frac{56}{65}$

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**9.** Find $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$. [3]

Here $xy = \frac{3}{28} < 1$.

$\frac{x+y}{1-xy} = \frac{\frac{25}{28}}{\frac{25}{28}} = 1$

$\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}1 = \frac{\pi}{4}$

**Ans:** $\frac{\pi}{4}$

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**10.** Solve $\tan^{-1}5x + \tan^{-1}12x = \frac{\pi}{4}$. [5]

$\tan^{-1}\frac{17x}{1-60x^2} = \frac{\pi}{4} \Rightarrow \frac{17x}{1-60x^2} = 1$

$60x^2 + 17x - 1 = 0$

$x = -\frac{1}{3}$ or $x = \frac{1}{20}$

For $x = -\frac{1}{3}$ both inverse tangents are negative, so the left side is negative and cannot equal $\frac{\pi}{4}$; reject it.

$\therefore x = \frac{1}{20}$

**Ans:** $x = \frac{1}{20}$

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**11.** Prove that $\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$. [5]

Let $x = \frac{1}{7}$, $y = \frac{2}{11}$, $z = \frac{1}{2}$, all positive.

$S_1 = x + y + z = \frac{127}{154},\ S_2 = xy + yz + zx = \frac{29}{154} < 1,\ S_3 = xyz = \frac{1}{77}$

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\frac{S_1 - S_3}{1 - S_2} = \tan^{-1}\frac{\frac{125}{154}}{\frac{125}{154}}$

$= \tan^{-1}1 = \frac{\pi}{4}$

**Ans: Proved**

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**12.** Express $\sin^{-1}\frac{20}{29}$ as an inverse cosine and as an inverse tangent. [2]

$x = \frac{20}{29} > 0$, $\sqrt{1-x^2} = \frac{21}{29}$.

$\sin^{-1}\frac{20}{29} = \cos^{-1}\sqrt{1-x^2} = \cos^{-1}\frac{21}{29}$

$\sin^{-1}\frac{20}{29} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \tan^{-1}\frac{20}{21}$

**Ans:** $\cos^{-1}\frac{21}{29}$, $\tan^{-1}\frac{20}{21}$

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**13.** Express $2\tan^{-1}\frac{1}{5}$ as a single inverse tangent, a single inverse sine and a single inverse cosine. [3]

$0 < x = \frac{1}{5} < 1$.

$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}} = \tan^{-1}\frac{5}{12}$

$2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\frac{\frac{2}{5}}{\frac{26}{25}} = \sin^{-1}\frac{5}{13}$

$2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2} = \cos^{-1}\frac{\frac{24}{25}}{\frac{26}{25}} = \cos^{-1}\frac{12}{13}$

**Ans:** $\tan^{-1}\frac{5}{12}$, $\sin^{-1}\frac{5}{13}$, $\cos^{-1}\frac{12}{13}$

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**14.** Write the horizontal asymptotes of the graph of $y = \cot^{-1}x$. [1]

$\cot^{-1}x \to 0$ as $x \to \infty$ and $\to \pi$ as $x \to -\infty$.

**Ans:** $y = 0$ **and** $y = \pi$

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**15.** Find $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$. [2]

$\frac{7\pi}{4}$ is not in the principal range $[0, \pi]$.

Using the periodic graph of $\cos^{-1}(\cos x)$, shift or reflect it into the range: the value is $\frac{\pi}{4}$.

Check: $\frac{\pi}{4}$ is in $[0, \pi]$ and $\cos(\frac{\pi}{4}) = \cos(\frac{7\pi}{4})$.

**Ans:** $\frac{\pi}{4}$

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**16.** Find the value of $\tan^{-1}\frac{2x}{1-x^2}$ when $x = \sqrt{3}+2$. [3]

$x = \sqrt{3}+2$ lies in $x > 1$, where $\tan^{-1}\frac{2x}{1-x^2} = -\pi + 2\tan^{-1}x$.

$= -\frac{\pi}{6}$

**Ans:** $-\frac{\pi}{6}$

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**17.** If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{5}$, find $\cos^{-1}x + \cos^{-1}y$. [3]

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x,\ \cos^{-1}y = \frac{\pi}{2} - \sin^{-1}y$

$\cos^{-1}x + \cos^{-1}y = \pi - (\sin^{-1}x + \sin^{-1}y) = \pi - \frac{2\pi}{5} = \frac{3\pi}{5}$

**Ans:** $\frac{3\pi}{5}$

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**18.** Prove that $\tan^{-1}\frac{1}{1+3\cdot4} + \tan^{-1}\frac{1}{1+4\cdot5} + \cdots + \tan^{-1}\frac{1}{1+(n+2)(n+3)} = \tan^{-1}\frac{n}{1+3(n+3)}$. [5]

The $k$th term is $\tan^{-1}\frac{1}{1+(k+2)(k+3)}$.

$\tan^{-1}(k+3) - \tan^{-1}(k+2) = \tan^{-1}\frac{(k+3)-(k+2)}{1+(k+3)(k+2)} = \tan^{-1}\frac{1}{1+(k+2)(k+3)}$

Adding for $k = 1, 2, \ldots, n$, all middle terms cancel:

$\text{Sum} = \tan^{-1}(n+3) - \tan^{-1}3$

$= \tan^{-1}\frac{(n+3)-3}{1+3(n+3)} = \tan^{-1}\frac{n}{1+3(n+3)}$

**Ans: Proved**

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**19.** Which substitution removes the square root in $\sqrt{x^2-16}$? What does the root become? [1]

For $\sqrt{x^2-16}$ put $x = 4\sec\theta$.

Then the root becomes $4|\tan\theta|$.

**Ans:** $x = 4\sec\theta$**; the root becomes** $4|\tan\theta|$

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**20.** Simplify $\sin^{-1}\frac{x}{\sqrt{25+x^2}}$ for $x \in \mathbb{R}$. [2]

Put $x = 5\tan\theta$; then $\sqrt{25+x^2} = 5\sec\theta$.

$\sin^{-1}\frac{5\tan\theta}{5\sec\theta} = \sin^{-1}(\sin\theta) = \theta = \tan^{-1}\frac{x}{5}$

**Ans:** $\tan^{-1}\frac{x}{5}$

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