EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 09

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The range (principal values) of $\tan^{-1} x$ is

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **B.** $[0, \pi]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **D.** $(0, \pi)$

2. The principal value of $\sin^{-1} \frac{1}{2}$ is

A. $\frac{5\pi}{6}$ **B.** $\frac{\pi}{6}$ **C.** $\frac{\pi}{3}$ **D.** $-\frac{\pi}{6}$

3. The principal value of $\cot^{-1} 0$ is

A. $\frac{\pi}{2}$ **B.** $-\frac{\pi}{2}$ **C.** $\frac{3\pi}{2}$ **D.** 0

4. $\tan\left(\tan^{-1}\left(-\frac{11}{4}\right)\right) =$

A. $\frac{15}{4}$ **B.** $-\frac{4}{11}$ **C.** $\frac{11}{4}$ **D.** $-\frac{11}{4}$

5. $\cot^{-1}\left(\cot\frac{6\pi}{7}\right) =$

A. $-\frac{5\pi}{14}$ **B.** $\frac{6\pi}{7}$ **C.** $\frac{\pi}{7}$ **D.** $-\frac{6\pi}{7}$

6. If $\cot^{-1} 6 = \alpha$, then $\cot^{-1}(-6) =$

A. α **B.** $\pi - \alpha$ **C.** $\frac{\pi}{2} - \alpha$ **D.** $\pi + \alpha$

7. $\cot^{-1}(-x) =$

A. $\cot^{-1} x$ **B.** $2\pi - \cot^{-1} x$

C. $\pi - \cot^{-1} x$ **D.** $\pi + \cot^{-1} x$

8. $\sec^{-1} 6$ written as an inverse cosine is

A. $\sin^{-1}\frac{1}{6}$ **B.** $\pi - \cos^{-1}\frac{1}{6}$

C. $\cos^{-1}\frac{1}{6}$ **D.** $\cos^{-1}\frac{5}{6}$

9. If $\cos^{-1} x = \frac{\pi}{10}$, then $\sin^{-1} x =$

A. $\frac{2\pi}{5}$ **B.** $\frac{\pi}{10}$ **C.** $-\frac{\pi}{10}$ **D.** $\frac{9\pi}{10}$

10. $\sec^{-1}(-5) + \operatorname{cosec}^{-1}(-5) =$

A. 0 **B.** π **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{4}$

11. $\sin^{-1}\frac{9}{41}+\sin^{-1}\frac{3}{5}=$

A. $\sin^{-1}\left(-\frac{84}{205}\right)$ **B.** $\pi-\sin^{-1}\frac{156}{205}$

C. $\cos^{-1}\frac{156}{205}$ **D.** $\sin^{-1}\frac{156}{205}$

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12. $\tan^{-1}3+\tan^{-1}3=$

A. $\pi+\tan^{-1}\left(-\frac{3}{4}\right)$ **B.** $\tan^{-1}(-0.75)$

C. $\tan^{-1}6$ **D.** $-\pi+\tan^{-1}(-0.75)$

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13. $\tan^{-1}\frac{1}{9}+\tan^{-1}\frac{4}{5}=$

A. $\frac{3\pi}{4}$ **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{3}$ **D.** $-\frac{\pi}{4}$

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14. $\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{9}=$

A. $\tan^{-1}\frac{173}{455}$ **B.** $\tan^{-1}\frac{29}{72}$

C. $\tan^{-1}\frac{174}{409}$ **D.** $\tan^{-1}\frac{173}{409}$

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15. $\sin\left(\cos^{-1}\frac{5}{13}\right)=$

A. $\frac{12}{5}$ **B.** $\frac{5}{12}$ **C.** $\frac{5}{13}$ **D.** $\frac{12}{13}$

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16. $\cos\left(\tan^{-1}\frac{21}{20}\right)=$

A. $\frac{21}{29}$ **B.** $\frac{21}{20}$ **C.** $\frac{20}{29}$ **D.** $\frac{20}{21}$

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17. $2\tan^{-1}\frac{4}{9}$ written as a single inverse sine is

A. $\sin^{-1}\frac{65}{97}$ **B.** $\pi-\sin^{-1}\frac{72}{97}$

C. $\sin^{-1}\frac{72}{97}$ **D.** $\cos^{-1}\frac{72}{97}$

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18. $2\tan^{-1}\frac{9}{2}$ written as a single inverse cosine is

A. $\cos^{-1}\frac{77}{85}$ **B.** $\cos^{-1}\left(-\frac{77}{85}\right)$

C. $\cos^{-1}\frac{36}{85}$ **D.** $-\cos^{-1}\left(-\frac{77}{85}\right)$

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19. For $x\le-1$, the graph of $y=\sec^{-1}x$ ends at the point

A. $\left(-1,\frac{\pi}{2}\right)$ **B.** $\left(-1,-\frac{\pi}{2}\right)$

C. $(-1,\pi)$ **D.** $(-1,0)$

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20. $\cos^{-1}\left(\cos\frac{13\pi}{7}\right)=$

A. $\frac{6\pi}{7}$ **B.** $\frac{13\pi}{7}$ **C.** $-\frac{\pi}{7}$ **D.** $\frac{\pi}{7}$

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21. $\sin^{-1}\left(\sin\frac{8\pi}{7}\right)=$

A. $\frac{6\pi}{7}$ **B.** $\frac{8\pi}{7}$ **C.** $\frac{\pi}{7}$ **D.** $-\frac{\pi}{7}$

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22. If $x=\frac{1}{2}$, then $\sin^{-1}(3x-4x^3)=$

A. 0 **B.** $\frac{\pi}{2}$ **C.** π **D.** $-\frac{\pi}{2}$

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23. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{5}$, then $\cos^{-1}x+\cos^{-1}y=$

A. $\frac{4\pi}{5}$ **B.** $\frac{9\pi}{5}$ **C.** $\frac{\pi}{5}$ **D.** $\frac{3\pi}{10}$

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24. $\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{7}{9}=$

A. $\frac{\pi}{3}$ **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{6}$

25. To remove the root in $\sqrt{81-x^2}$, use

A. the secant substitution $x=9\sec\theta$

B. the cosine substitution $x=9\cos\theta$

C. the tangent substitution $x=9\tan\theta$

D. the sine substitution $x=9\sin\theta$

Key sheet

1. A	**2.** B	**3.** A	**4.** D	**5.** B
6. B	**7.** C	**8.** C	**9.** A	**10.** C
11. D	**12.** A	**13.** B	**14.** D	**15.** D
16. C	**17.** C	**18.** B	**19.** C	**20.** D
21. D	**22.** B	**23.** A	**24.** B	**25.** D

Solutions

1. The range (principal values) of $\tan^{-1} x$ is

The range of $\tan^{-1} x$ is the interval on which $\tan x$ was restricted: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Ans: (A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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2. The principal value of $\sin^{-1} \frac{1}{2}$ is

The angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with $\sin\theta = \frac{1}{2}$ is $\frac{\pi}{6}$.

Ans: (B) $\frac{\pi}{6}$

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3. The principal value of $\cot^{-1} 0$ is

The angle in $(0, \pi)$ with $\cot\theta = 0$ is $\frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

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4. $\tan\left(\tan^{-1}\left(-\frac{11}{4}\right)\right) =$

$\tan(\tan^{-1} x) = x$ on the domain of $\tan^{-1}$.

Ans: (D) $-\frac{11}{4}$

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5. $\cot^{-1}\left(\cot\frac{6\pi}{7}\right) =$

$\frac{6\pi}{7}$ is in $(0, \pi)$, so the value is $\frac{6\pi}{7}$ itself.

Ans: (B) $\frac{6\pi}{7}$

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6. If $\cot^{-1} 6 = \alpha$, then $\cot^{-1}(-6) =$

$\cot^{-1}(-x) = \pi - \cot^{-1} x$

$\cot^{-1}(-6) = \pi - \alpha$

Ans: (B) $\pi - \alpha$

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7. $\cot^{-1}(-x) =$

The rule for negative arguments: $\cot^{-1}(-x) = \pi - \cot^{-1} x$.

Ans: (C) $\pi - \cot^{-1} x$

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8. $\sec^{-1} 6$ written as an inverse cosine is

$\sec^{-1} x = \cos^{-1}\frac{1}{x},\ |x| \geq 1$

$\sec^{-1} 6 = \cos^{-1}\frac{1}{6}$

Ans: (C) $\cos^{-1}\frac{1}{6}$

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9. If $\cos^{-1} x = \frac{\pi}{10}$, then $\sin^{-1} x =$

$\sin^{-1} x = \frac{\pi}{2} - \frac{\pi}{10} = \frac{2\pi}{5}$

Ans: (A) $\frac{2\pi}{5}$

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10. $\sec^{-1}(-5) + \operatorname{cosec}^{-1}(-5) =$

$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$ for every x in the common domain.

Ans: (C) $\frac{\pi}{2}$

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11. $\sin^{-1}\frac{9}{41}+\sin^{-1}\frac{3}{5}=$

$\sqrt{1-x^2}=\frac{40}{41},\ \sqrt{1-y^2}=\frac{4}{5}$

$x^2+y^2=\frac{17154}{42025}\le 1.$

$\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)=\sin^{-1}\left(\frac{9}{41}\cdot\frac{4}{5}+\frac{3}{5}\cdot\frac{40}{41}\right)=\sin^{-1}\frac{156}{205}$

Ans: (D) $\sin^{-1}\frac{156}{205}$

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12. $\tan^{-1}3+\tan^{-1}3=$

Here $x,y>0,\ xy=9>1.$

$\frac{x+y}{1-xy}=\frac{6}{-8}=-\frac{3}{4}$

$=\pi+\tan^{-1}\left(-\frac{3}{4}\right)$

Ans: (A) $\pi+\tan^{-1}\left(-\frac{3}{4}\right)$

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13. $\tan^{-1}\frac{1}{9}+\tan^{-1}\frac{4}{5}=$

Here $xy=\frac{4}{45}<1.$

$\frac{x+y}{1-xy}=\frac{\frac{41}{45}}{\frac{41}{45}}=1$

$=\tan^{-1}1=\frac{\pi}{4}$

Ans: (B) $\frac{\pi}{4}$

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14. $\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{9}=$

$S_1=\frac{29}{72},\ S_2=\frac{23}{432},\ S_3=\frac{1}{432}$

$\tan^{-1}\frac{S_1-S_3}{1-S_2}=\tan^{-1}\frac{173}{409}$

Ans: (D) $\tan^{-1}\frac{173}{409}$

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15. $\sin\left(\cos^{-1}\frac{5}{13}\right)=$

$\cos\theta=\frac{5}{13}$ with $\theta\in[0,\pi]$, so $\sin\theta=\frac{12}{13}$.

Ans: (D) $\frac{12}{13}$

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16. $\cos\left(\tan^{-1}\frac{21}{20}\right)=$

$\tan\theta=\frac{21}{20}$: opposite 21, adjacent 20, hypotenuse 29.

$\cos\theta=\frac{20}{29}$.

Ans: (C) $\frac{20}{29}$

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17. $2\tan^{-1}\frac{4}{9}$ written as a single inverse sine is

$\frac{2x}{1+x^2}=\frac{72}{97}$

For $x\le 1$ the formula is $\sin^{-1}\frac{2x}{1+x^2}$.

Ans: (C) $\sin^{-1}\frac{72}{97}$

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18. $2\tan^{-1}\frac{9}{2}$ written as a single inverse cosine is

$\frac{1-x^2}{1+x^2}=-\frac{77}{85}$

For $x\ge 0$, $2\tan^{-1}x=\cos^{-1}\frac{1-x^2}{1+x^2}$.

Ans: (B) $\cos^{-1}\left(-\frac{77}{85}\right)$

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19. For $x \le -1$, the graph of $y = \sec^{-1} x$ ends at the point

This follows from the shape of the graph described in the lesson on graphs.

Ans: (C) $(-1, \pi)$

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20. $\cos^{-1}\left(\cos \frac{13\pi}{7}\right) =$

$\frac{13\pi}{7}$ lies outside $[0, \pi]$; bring it into the range using the periodic graph.

The value is $\frac{\pi}{7}$.

Ans: (D) $\frac{\pi}{7}$

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21. $\sin^{-1}\left(\sin \frac{8\pi}{7}\right) =$

$\frac{8\pi}{7}$ lies outside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; bring it into the range using the periodic graph.

The value is $-\frac{\pi}{7}$.

Ans: (D) $-\frac{\pi}{7}$

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22. If $x = \frac{1}{2}$, then $\sin^{-1}(3x - 4x^3) =$

$x = \frac{1}{2}$ lies in $-\frac{1}{2} \le x \le \frac{1}{2}$, where $\sin^{-1}(3x - 4x^3) = 3\sin^{-1} x$.

$= \frac{\pi}{2}$

Ans: (B) $\frac{\pi}{2}$

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23. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{5}$, then $\cos^{-1} x + \cos^{-1} y =$

$\pi - \frac{\pi}{5} = \frac{4\pi}{5}$

Ans: (A) $\frac{4\pi}{5}$

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24. $\tan^{-1}\frac{1}{8} + \tan^{-1}\frac{7}{9} =$

Standard result: $\tan^{-1}\frac{p}{q} + \tan^{-1}\frac{q-p}{q+p} = \frac{\pi}{4}$.

Ans: (B) $\frac{\pi}{4}$

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25. To remove the root in $\sqrt{81 - x^2}$, use

$81 - 81\sin^2\theta = 81\cos^2\theta$.

Ans: (D) the sine substitution $x = 9\sin\theta$
