EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 07

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The range (principal values) of $\operatorname{cosec}^{-1} x$ is

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **B.** $[0, \pi]$

C. $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ **D.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

A. $\frac{5\pi}{6}$ **B.** $\frac{4\pi}{3}$ **C.** $\frac{\pi}{3}$ **D.** $-\frac{\pi}{3}$

3. The principal value of $\cot^{-1} 1$ is

A. $\frac{\pi}{4}$ **B.** $\frac{3\pi}{4}$ **C.** $-\frac{\pi}{4}$ **D.** $\frac{5\pi}{4}$

4. $\cos\left(\cos^{-1}\left(-\frac{4}{9}\right)\right) =$

A. $\frac{4}{9}$ **B.** $\frac{13}{9}$ **C.** $-\frac{4}{9}$ **D.** $-\frac{9}{4}$

5. $\sin^{-1}\left(\sin\frac{2\pi}{9}\right) =$

A. $\frac{7\pi}{9}$ **B.** $-\frac{2\pi}{9}$ **C.** $\frac{2\pi}{9}$ **D.** $\frac{5\pi}{18}$

6. If $\cos^{-1}\frac{4}{9} = \alpha$, then $\cos^{-1}\left(-\frac{4}{9}\right) =$

A. α **B.** $\pi - \alpha$ **C.** $\pi + \alpha$ **D.** $\frac{\pi}{2} - \alpha$

7. If $\sec^{-1}\frac{5}{3} = \alpha$, then $\sec^{-1}\left(-\frac{5}{3}\right) =$

A. α **B.** $\pi + \alpha$ **C.** $\frac{\pi}{2} - \alpha$ **D.** $\pi - \alpha$

8. $\operatorname{cosec}^{-1}\frac{7}{6}$ written as an inverse sine is

A. $\sin^{-1}\frac{6}{7}$ **B.** $\pi - \sin^{-1}\frac{6}{7}$

C. $\cos^{-1}\frac{6}{7}$ **D.** $\sin^{-1}\frac{36}{49}$

9. If $\sin^{-1} x = -\frac{\pi}{8}$, then $\cos^{-1} x =$

A. $\frac{5\pi}{8}$ **B.** $\frac{9\pi}{8}$ **C.** $-\frac{\pi}{8}$ **D.** $\frac{\pi}{8}$

10. $\tan^{-1}(-6) + \cot^{-1}(-6) =$

A. $\frac{\pi}{4}$ **B.** π **C.** 0 **D.** $\frac{\pi}{2}$

11. $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=$

A. $\sin^{-1}\frac{36}{325}$ **B.** $\cos^{-1}\frac{204}{325}$

C. $\pi-\sin^{-1}\frac{204}{325}$ **D.** $\sin^{-1}\frac{204}{325}$

12. $\tan^{-1}\frac{2}{5}+\tan^{-1}\frac{3}{7}=$

A. $\frac{\pi}{4}$ **B.** $-\frac{\pi}{4}$ **C.** $\frac{\pi}{3}$ **D.** $\frac{3\pi}{4}$

13. $\tan^{-1}4-\tan^{-1}(-3)=$

A. $\tan^{-1}1$ **B.** $\tan^{-1}(-0.636363636363636)$

C. $-\pi+\tan^{-1}(-0.636363636363636)$ **D.** $\pi+\tan^{-1}\left(-\frac{7}{11}\right)$

14. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{9}=$

A. $\tan^{-1}\frac{61}{57}$ **B.** $\tan^{-1}\frac{62}{57}$

C. $\tan^{-1}\frac{31}{36}$ **D.** $\tan^{-1}\frac{61}{87}$

15. $\sin\left(\cos^{-1}\frac{40}{41}\right)=$

A. $\frac{9}{41}$ **B.** $\frac{40}{41}$ **C.** $\frac{9}{40}$ **D.** $\frac{40}{9}$

16. $\cos\left(\tan^{-1}\frac{7}{24}\right)=$

A. $\frac{24}{7}$ **B.** $\frac{24}{25}$ **C.** $\frac{7}{25}$ **D.** $\frac{7}{24}$

17. $2\tan^{-1}\frac{1}{9}$ written as a single inverse tangent is

A. $\tan^{-1}\frac{9}{41}$ **B.** $\tan^{-1}\frac{9}{40}$

C. $\tan^{-1}\frac{2}{9}$ **D.** $\pi+\tan^{-1}\frac{9}{40}$

18. $2\tan^{-1}6$ written as a single inverse sine is

A. $\cos^{-1}\frac{12}{37}$ **B.** $\sin^{-1}\frac{12}{37}$

C. $\sin^{-1}\left(-\frac{35}{37}\right)$ **D.** $\pi-\sin^{-1}\frac{12}{37}$

19. Which graph is decreasing over its whole domain?

A. the inverse tangent, $y=\tan^{-1}x$

B. the inverse cotangent, $y=\cot^{-1}x$

C. the inverse sine, $y=\sin^{-1}x$

D. the inverse secant, $y=\sec^{-1}x$

20. $\cos^{-1}(\cos 8)=$

A. $-8+2\pi$ **B.** 8 **C.** $8-2\pi$ **D.** $-8+3\pi$

21. $\sin^{-1}(\sin -4)=$

A. $-4+\pi$ **B.** $-4+2\pi$ **C.** -4 **D.** $4-\pi$

22. If $x=\sqrt{3}+2$, then $\cos^{-1}\frac{1-x^2}{1+x^2}=$

A. $\frac{4\pi}{3}$ **B.** $\frac{\pi}{6}$ **C.** $-\frac{5\pi}{6}$ **D.** $\frac{5\pi}{6}$

23. If $\cos^{-1} x + \cos^{-1} y = \frac{2\pi}{3}$, then $\sin^{-1} x + \sin^{-1} y =$

A. $\frac{\pi}{3}$ **B.** $\frac{2\pi}{3}$ **C.** $\frac{4\pi}{3}$ **D.** $-\frac{\pi}{6}$

24. $\tan^{-1} \frac{4}{9} + \tan^{-1} \frac{5}{13} =$

A. $\frac{\pi}{3}$ **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{6}$

25. To remove the root in $\sqrt{64 - x^2}$, use

A. the cosine substitution $x = 8\cos\theta$ **B.** the sine substitution $x = 8\sin\theta$

C. the secant substitution $x = 8\sec\theta$ **D.** the tangent substitution $x = 8\tan\theta$

Key sheet

1. C	**2.** D	**3.** A	**4.** C	**5.** C
6. B	**7.** D	**8.** A	**9.** A	**10.** D
11. D	**12.** A	**13.** D	**14.** A	**15.** A
16. B	**17.** B	**18.** D	**19.** B	**20.** C
21. D	**22.** D	**23.** A	**24.** C	**25.** B

Solutions

1. The range (principal values) of $\operatorname{cosec}^{-1} x$ is

The range of $\operatorname{cosec}^{-1} x$ is the interval on which $\operatorname{cosec} x$ was restricted: $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$.

Ans: (C) $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

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2. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

The angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with $\sin\theta = -\frac{\sqrt{3}}{2}$ is $-\frac{\pi}{3}$.

Ans: (D) $-\frac{\pi}{3}$

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3. The principal value of $\cot^{-1} 1$ is

The angle in $(0, \pi)$ with $\cot\theta = 1$ is $\frac{\pi}{4}$.

Ans: (A) $\frac{\pi}{4}$

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4. $\cos\left(\cos^{-1}\left(-\frac{4}{9}\right)\right) =$

$\cos(\cos^{-1} x) = x$ on the domain of $\cos^{-1}$.

Ans: (C) $-\frac{4}{9}$

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5. $\sin^{-1}\left(\sin\frac{2\pi}{9}\right) =$

$\frac{2\pi}{9}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so the value is $\frac{2\pi}{9}$ itself.

Ans: (C) $\frac{2\pi}{9}$

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6. If $\cos^{-1}\frac{4}{9} = \alpha$, then $\cos^{-1}\left(-\frac{4}{9}\right) =$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\cos^{-1}\left(-\frac{4}{9}\right) = \pi - \alpha$

Ans: (B) $\pi - \alpha$

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7. If $\sec^{-1}\frac{5}{3} = \alpha$, then $\sec^{-1}\left(-\frac{5}{3}\right) =$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\sec^{-1}\left(-\frac{5}{3}\right) = \pi - \alpha$

Ans: (D) $\pi - \alpha$

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8. $\operatorname{cosec}^{-1}\frac{7}{6}$ written as an inverse sine is

$\operatorname{cosec}^{-1} x = \sin^{-1}\frac{1}{x},\ |x| \geq 1$

$\operatorname{cosec}^{-1}\frac{7}{6} = \sin^{-1}\frac{6}{7}$

Ans: (A) $\sin^{-1}\frac{6}{7}$

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9. If $\sin^{-1} x = -\frac{\pi}{8}$, then $\cos^{-1} x =$

$\cos^{-1} x = \frac{\pi}{2} - -\frac{\pi}{8} = \frac{5\pi}{8}$

Ans: (A) $\frac{5\pi}{8}$

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10. $\tan^{-1}(-6) + \cot^{-1}(-6) =$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ for every x in the common domain.

Ans: (D) $\frac{\pi}{2}$

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11. $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} =$

$\sqrt{1-x^2} = \frac{12}{13},\ \sqrt{1-y^2} = \frac{24}{25}$

$x^2 + y^2 = \frac{23906}{105625} \le 1.$

$\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \sin^{-1}\left(\frac{5}{13}\cdot\frac{24}{25} + \frac{7}{25}\cdot\frac{12}{13}\right) = \sin^{-1}\frac{204}{325}$

Ans: (D) $\sin^{-1}\frac{204}{325}$

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12. $\tan^{-1}\frac{2}{5} + \tan^{-1}\frac{3}{7} =$

Here $xy = \frac{6}{35} < 1.$

$\frac{x+y}{1-xy} = \frac{\frac{29}{35}}{\frac{29}{35}} = 1$

$= \tan^{-1} 1 = \frac{\pi}{4}$

Ans: (A) $\frac{\pi}{4}$

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13. $\tan^{-1} 4 - \tan^{-1}(-3) =$

Here $x > 0,\ y < 0,\ xy < -1.$

$\frac{x-y}{1+xy} = \frac{7}{-11} = -\frac{7}{11}$

$= \pi + \tan^{-1}\left(-\frac{7}{11}\right)$

Ans: (D) $\pi + \tan^{-1}\left(-\frac{7}{11}\right)$

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14. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{9} =$

$S_1 = \frac{31}{36},\ S_2 = \frac{5}{24},\ S_3 = \frac{1}{72}$

$\tan^{-1}\frac{S_1 - S_3}{1 - S_2} = \tan^{-1}\frac{61}{57}$

Ans: (A) $\tan^{-1}\frac{61}{57}$

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15. $\sin\left(\cos^{-1}\frac{40}{41}\right) =$

$\cos\theta = \frac{40}{41}$ with $\theta \in [0, \pi]$, so $\sin\theta = \frac{9}{41}$.

Ans: (A) $\frac{9}{41}$

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16. $\cos\left(\tan^{-1}\frac{7}{24}\right) =$

$\tan\theta = \frac{7}{24}$: opposite 7, adjacent 24, hypotenuse 25.

$\cos\theta = \frac{24}{25}$.

Ans: (B) $\frac{24}{25}$

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17. $2\tan^{-1}\frac{1}{9}$ written as a single inverse tangent is

Use $2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}$ for $x < 1$.

$\frac{2x}{1-x^2} = \frac{9}{40}$

Ans: (B) $\tan^{-1}\frac{9}{40}$

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18. $2\tan^{-1}6$ written as a single inverse sine is

$\frac{2x}{1+x^2} = \frac{12}{37}$

For $x > 1$ the formula is $\pi - \sin^{-1}\frac{2x}{1+x^2}$.

Ans: (D) $\pi - \sin^{-1}\frac{12}{37}$

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19. Which graph is decreasing over its whole domain?

This follows from the shape of the graph described in the lesson on graphs.

Ans: (B) the inverse cotangent, $y = \cot^{-1}x$

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20. $\cos^{-1}(\cos 8) =$

8 lies outside $[0, \pi]$; bring it into the range using the periodic graph.

The value is $8 - 2\pi$.

Ans: (C) $8 - 2\pi$

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21. $\sin^{-1}(\sin -4) =$

-4 lies outside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; bring it into the range using the periodic graph.

The value is $4 - \pi$.

Ans: (D) $4 - \pi$

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22. If $x = \sqrt{3} + 2$, then $\cos^{-1}\frac{1-x^2}{1+x^2} =$

$x = \sqrt{3} + 2$ lies in $x \geq 0$, where $\cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$.

$= \frac{5\pi}{6}$

Ans: (D) $\frac{5\pi}{6}$

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23. If $\cos^{-1}x + \cos^{-1}y = \frac{2\pi}{3}$, then $\sin^{-1}x + \sin^{-1}y =$

$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

Ans: (A) $\frac{\pi}{3}$

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24. $\tan^{-1}\frac{4}{9} + \tan^{-1}\frac{5}{13} =$

Standard result: $\tan^{-1}\frac{p}{q} + \tan^{-1}\frac{q-p}{q+p} = \frac{\pi}{4}$.

Ans: (C) $\frac{\pi}{4}$

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25. To remove the root in $\sqrt{64 - x^2}$, use

$64 - 64\sin^2\theta = 64\cos^2\theta$.

Ans: (B) the sine substitution $x = 8\sin\theta$
