EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 05

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The range (principal values) of $\cos^{-1} x$ is

A. $[0, \pi]$ **B.** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **D.** $(0, \pi)$

2. The principal value of $\tan^{-1} \sqrt{3}$ is

A. $\frac{2\pi}{3}$ **B.** $\frac{\pi}{6}$ **C.** $-\frac{\pi}{3}$ **D.** $\frac{\pi}{3}$

3. The principal value of $\operatorname{cosec}^{-1}(-1)$ is

A. $\frac{3\pi}{2}$ **B.** π **C.** $-\frac{\pi}{2}$ **D.** $\frac{\pi}{2}$

4. $\operatorname{cosec}\left(\operatorname{cosec}^{-1} \frac{7}{3}\right) =$

A. $-\frac{7}{3}$ **B.** $-\frac{4}{3}$ **C.** $\frac{7}{3}$ **D.** $\frac{3}{7}$

5. $\operatorname{cosec}^{-1}\left(\operatorname{cosec} \frac{\pi}{5}\right) =$

A. $\frac{\pi}{5}$ **B.** $\frac{3\pi}{10}$ **C.** $-\frac{\pi}{5}$ **D.** $\frac{4\pi}{5}$

6. If $\sec^{-1} \frac{9}{4} = \alpha$, then $\sec^{-1}\left(-\frac{9}{4}\right) =$

A. $\pi + \alpha$ **B.** α **C.** $\pi - \alpha$ **D.** $\frac{\pi}{2} - \alpha$

7. If $\sin^{-1} \frac{2}{13} = \alpha$, then $\sin^{-1}\left(-\frac{2}{13}\right) =$

A. $-\alpha$ **B.** $\pi + \alpha$ **C.** $\frac{\pi}{2} - \alpha$ **D.** α

8. $\sec^{-1} \frac{6}{5}$ written as an inverse cosine is

A. $\pi - \cos^{-1} \frac{5}{6}$ **B.** $\cos^{-1} \frac{5}{6}$

C. $\cos^{-1} \frac{1}{6}$ **D.** $\sin^{-1} \frac{5}{6}$

9. If $\tan^{-1} x = \frac{\pi}{12}$, then $\cot^{-1} x =$

A. $\frac{\pi}{12}$ **B.** $\frac{11\pi}{12}$ **C.** $-\frac{\pi}{12}$ **D.** $\frac{5\pi}{12}$

10. $\cos^{-1}\left(-\frac{3}{7}\right) + \sin^{-1}\left(-\frac{3}{7}\right) =$

A. $\frac{\pi}{2}$ **B.** π **C.** $\frac{\pi}{4}$ **D.** 0

11. $\sin^{-1}\frac{15}{17}+\sin^{-1}\frac{4}{5}=$

A. $\sin^{-1}\frac{77}{85}$ **B.** $\sin^{-1}\frac{13}{85}$

C. $\cos^{-1}\frac{77}{85}$ **D.** $\pi-\sin^{-1}\frac{77}{85}$

12. $\tan^{-1}2+\tan^{-1}5=$

A. $\tan^{-1}(-0.777777777777778)$ **B.** $\pi+\tan^{-1}\left(-\frac{7}{9}\right)$

C. $-\pi+\tan^{-1}(-0.777777777777778)$ **D.** $\tan^{-1}7$

13. $\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{5}{7}=$

A. $\frac{\pi}{4}$ **B.** $\frac{3\pi}{4}$ **C.** $\frac{\pi}{3}$ **D.** $-\frac{\pi}{4}$

14. $\tan^{-1}\frac{2}{5}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{6}=$

A. $\tan^{-1}\frac{9}{10}$ **B.** $\tan^{-1}\frac{81}{67}$

C. $\tan^{-1}\frac{79}{67}$ **D.** $\tan^{-1}\frac{79}{113}$

15. $\sin\left(\cos^{-1}\frac{21}{29}\right)=$

A. $\frac{21}{29}$ **B.** $\frac{21}{20}$ **C.** $\frac{20}{21}$ **D.** $\frac{20}{29}$

16. $\cos\left(\tan^{-1}\frac{15}{8}\right)=$

A. $\frac{15}{8}$ **B.** $\frac{8}{17}$ **C.** $\frac{15}{17}$ **D.** $\frac{8}{15}$

17. $2\tan^{-1}\frac{1}{9}$ written as a single inverse cosine is

A. $\cos^{-1}\left(-\frac{40}{41}\right)$ **B.** $-\cos^{-1}\frac{40}{41}$

C. $\cos^{-1}\frac{9}{41}$ **D.** $\cos^{-1}\frac{40}{41}$

18. $2\tan^{-1}\frac{7}{3}$ written as a single inverse tangent is

A. $\pi+\tan^{-1}\frac{21}{20}$ **B.** $\tan^{-1}\frac{21}{20}$

C. $\tan^{-1}\frac{14}{3}$ **D.** $\pi-\tan^{-1}\frac{21}{20}$

19. The graph of $y=\cos^{-1}x$ passes through

A. $\left(0,\frac{\pi}{2}\right)$ **B.** $(-1,0)$

C. $(0,0)$ **D.** $(1,\pi)$

20. $\sin^{-1}(\sin 10)=$

A. $10-3\pi$ **B.** $10-2\pi$ **C.** $-10+3\pi$ **D.** 10

21. $\tan^{-1}\left(\tan\frac{7\pi}{5}\right)=$

A. $\frac{3\pi}{5}$ **B.** $\frac{2\pi}{5}$ **C.** $-\frac{2\pi}{5}$ **D.** $\frac{7\pi}{5}$

22. If $x=-\frac{1}{2}$, then $\cos^{-1}(4x^3-3x)=$

A. 0 **B.** $\frac{\pi}{3}$ **C.** π **D.** $\frac{\pi}{2}$

23. $\tan^{-1}\frac{3}{8}+\tan^{-1}\frac{5}{11}=$

A. $\frac{\pi}{6}$ **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{3}$ **D.** $\frac{\pi}{4}$

24. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx =$

A. 1 **B.** -3 **C.** 0 **D.** 3

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25. To remove the root in $\sqrt{25 - x^2}$, use

A. the tangent substitution $x = 5\tan\theta$

B. the sine substitution $x = 5\sin\theta$

C. the cosine substitution $x = 5\cos\theta$

D. the secant substitution $x = 5\sec\theta$

Key sheet

1. A	**2.** D	**3.** C	**4.** C	**5.** A
6. C	**7.** A	**8.** B	**9.** D	**10.** A
11. D	**12.** B	**13.** A	**14.** C	**15.** D
16. B	**17.** D	**18.** D	**19.** A	**20.** C
21. B	**22.** A	**23.** D	**24.** D	**25.** B

Solutions

1. The range (principal values) of $\cos^{-1} x$ is

The range of $\cos^{-1} x$ is the interval on which $\cos x$ was restricted: $[0, \pi]$.

Ans: (A) $[0, \pi]$

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2. The principal value of $\tan^{-1} \sqrt{3}$ is

The angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $\tan\theta = \sqrt{3}$ is $\frac{\pi}{3}$.

Ans: (D) $\frac{\pi}{3}$

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3. The principal value of $\text{cosec}^{-1}(-1)$ is

The angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ with $\text{cosec}\,\theta = -1$ is $-\frac{\pi}{2}$.

Ans: (C) $-\frac{\pi}{2}$

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4. $\text{cosec}\left(\text{cosec}^{-1} \frac{7}{3}\right) =$

$\text{cosec}(\text{cosec}^{-1} x) = x$ on the domain of cosec^{-1}.

Ans: (C) $\frac{7}{3}$

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5. $\text{cosec}^{-1}\left(\text{cosec}\,\frac{\pi}{5}\right) =$

$\frac{\pi}{5}$ is in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$, so the value is $\frac{\pi}{5}$ itself.

Ans: (A) $\frac{\pi}{5}$

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6. If $\sec^{-1}\frac{9}{4} = \alpha$, then $\sec^{-1}\left(-\frac{9}{4}\right) =$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\sec^{-1}\left(-\frac{9}{4}\right) = \pi - \alpha$

Ans: (C) $\pi - \alpha$

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7. If $\sin^{-1}\frac{2}{13} = \alpha$, then $\sin^{-1}\left(-\frac{2}{13}\right) =$

$\sin^{-1}(-x) = -\sin^{-1} x$

$\sin^{-1}\left(-\frac{2}{13}\right) = -\alpha$

Ans: (A) $-\alpha$

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8. $\sec^{-1}\frac{6}{5}$ written as an inverse cosine is

$\sec^{-1} x = \cos^{-1}\frac{1}{x},\ |x| \geq 1$

$\sec^{-1}\frac{6}{5} = \cos^{-1}\frac{5}{6}$

Ans: (B) $\cos^{-1}\frac{5}{6}$

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9. If $\tan^{-1} x = \frac{\pi}{12}$, then $\cot^{-1} x =$

$\cot^{-1} x = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Ans: (D) $\frac{5\pi}{12}$

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10. $\cos^{-1}\left(-\frac{3}{7}\right) + \sin^{-1}\left(-\frac{3}{7}\right) =$

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$ for every x in the common domain.

Ans: (A) $\frac{\pi}{2}$

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11. $\sin^{-1}\frac{15}{17}+\sin^{-1}\frac{4}{5}=$

$\sqrt{1-x^2}=\frac{8}{17},\ \sqrt{1-y^2}=\frac{3}{5}$

$x^2+y^2=\frac{10249}{7225}>1$ and both are positive.

$\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)=\pi-\sin^{-1}\left(\frac{15}{17}\cdot\frac{3}{5}+\frac{4}{5}\cdot\frac{8}{17}\right)=\pi-\sin^{-1}\frac{77}{85}$

Ans: (D) $\pi-\sin^{-1}\frac{77}{85}$

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12. $\tan^{-1}2+\tan^{-1}5=$

Here $x,y>0,\ xy=10>1$.

$\frac{x+y}{1-xy}=\frac{7}{-9}=-\frac{7}{9}$

$=\pi+\tan^{-1}\left(-\frac{7}{9}\right)$

Ans: (B) $\pi+\tan^{-1}\left(-\frac{7}{9}\right)$

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13. $\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{5}{7}=$

Here $xy=\frac{5}{42}<1$.

$\frac{x+y}{1-xy}=\frac{\frac{37}{42}}{\frac{37}{42}}=1$

$=\tan^{-1}1=\frac{\pi}{4}$

Ans: (A) $\frac{\pi}{4}$

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14. $\tan^{-1}\frac{2}{5}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{6}=$

$S_1=\frac{9}{10},\ S_2=\frac{23}{90},\ S_3=\frac{1}{45}$

$\tan^{-1}\frac{S_1-S_3}{1-S_2}=\tan^{-1}\frac{79}{67}$

Ans: (C) $\tan^{-1}\frac{79}{67}$

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15. $\sin\left(\cos^{-1}\frac{21}{29}\right)=$

$\cos\theta=\frac{21}{29}$ with $\theta\in[0,\pi]$, so $\sin\theta=\frac{20}{29}$.

Ans: (D) $\frac{20}{29}$

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16. $\cos\left(\tan^{-1}\frac{15}{8}\right)=$

$\tan\theta=\frac{15}{8}$: opposite 15, adjacent 8, hypotenuse 17.

$\cos\theta=\frac{8}{17}$.

Ans: (B) $\frac{8}{17}$

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17. $2\tan^{-1}\frac{1}{9}$ written as a single inverse cosine is

$\frac{1-x^2}{1+x^2}=\frac{40}{41}$

For $x\geq 0$, $2\tan^{-1}x=\cos^{-1}\frac{1-x^2}{1+x^2}$.

Ans: (D) $\cos^{-1}\frac{40}{41}$

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18. $2\tan^{-1}\frac{7}{3}$ written as a single inverse tangent is

Use $2\tan^{-1}x=\pi+\tan^{-1}\frac{2x}{1-x^2}$ for $x>1$.

$\frac{2x}{1-x^2}=-\frac{21}{20}$

Ans: (D) $\pi-\tan^{-1}\frac{21}{20}$

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19. The graph of $y = \cos^{-1} x$ passes through

This follows from the shape of the graph described in the lesson on graphs.

Ans: (A) $\left(0, \frac{\pi}{2}\right)$

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20. $\sin^{-1}(\sin 10) =$

10 lies outside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; bring it into the range using the periodic graph.
The value is $-10 + 3\pi$.

Ans: (C) $-10 + 3\pi$

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21. $\tan^{-1}\left(\tan \frac{7\pi}{5}\right) =$

$\frac{7\pi}{5}$ lies outside $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; bring it into the range using the periodic graph.
The value is $\frac{2\pi}{5}$.

Ans: (B) $\frac{2\pi}{5}$

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22. If $x = -\frac{1}{2}$, then $\cos^{-1}(4x^3 - 3x) =$

$x = -\frac{1}{2}$ lies in $-1 \le x \le -\frac{1}{2}$, where $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1} x - 2\pi$.
$= 0$

Ans: (A) 0

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23. $\tan^{-1}\frac{3}{8} + \tan^{-1}\frac{5}{11} =$

Standard result: $\tan^{-1}\frac{p}{q} + \tan^{-1}\frac{q-p}{q+p} = \frac{\pi}{4}$.

Ans: (D) $\frac{\pi}{4}$

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24. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx =$

Each term must take its greatest value, so $x = y = z = -1$.
Then $xy + yz + zx = 3$.

Ans: (D) 3

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25. To remove the root in $\sqrt{25 - x^2}$, use

$25 - 25\sin^2\theta = 25\cos^2\theta$.

Ans: (B) the sine substitution $x = 5\sin\theta$
