EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 03

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The range (principal values) of $\sec^{-1} x$ is

A. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ **B.** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. $[0, \pi]$ **D.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. The principal value of $\cot^{-1} \sqrt{3}$ is

A. $\frac{5\pi}{6}$ **B.** $-\frac{\pi}{6}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{\pi}{3}$

3. The principal value of $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ is

A. $\frac{\pi}{6}$ **B.** $-\frac{5\pi}{6}$ **C.** $\frac{5\pi}{6}$ **D.** $-\frac{\pi}{3}$

4. $\sin^{-1} x$ means

A. the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x

B. $\frac{1}{\sin x}$

C. $-\sin x$

D. $(\sin x)^{-1}$

5. $\tan^{-1}\left(\tan \frac{2\pi}{7}\right) =$

A. $\frac{5\pi}{7}$ **B.** $\frac{2\pi}{7}$ **C.** $-\frac{2\pi}{7}$ **D.** $\frac{3\pi}{14}$

6. If $\cot^{-1} \frac{5}{4} = \alpha$, then $\cot^{-1}\left(-\frac{5}{4}\right) =$

A. $\pi + \alpha$ **B.** $\frac{\pi}{2} - \alpha$ **C.** α **D.** $\pi - \alpha$

7. If $\operatorname{cosec}^{-1} \frac{11}{3} = \alpha$, then $\operatorname{cosec}^{-1}\left(-\frac{11}{3}\right) =$

A. $-\alpha$ **B.** α **C.** $\pi + \alpha$ **D.** $\frac{\pi}{2} - \alpha$

8. $\operatorname{cosec}^{-1} \frac{9}{4}$ written as an inverse sine is

A. $\sin^{-1} \frac{4}{9}$ **B.** $\pi - \sin^{-1} \frac{4}{9}$

C. $\sin^{-1} \frac{16}{81}$ **D.** $\cos^{-1} \frac{4}{9}$

9. If $\operatorname{cosec}^{-1} x = \frac{3\pi}{10}$, then $\sec^{-1} x =$

A. $\frac{3\pi}{10}$ **B.** $\frac{\pi}{5}$ **C.** $-\frac{3\pi}{10}$ **D.** $\frac{7\pi}{10}$

10. $\sec^{-1}\frac{7}{2}+\operatorname{cosec}^{-1}\frac{7}{2}=$

A. 0 **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{2}$ **D.** π

11. $\sin^{-1}\frac{7}{25}-\sin^{-1}\frac{4}{5}=$

A. $\sin^{-1}\frac{117}{125}$ **B.** $\cos^{-1}\left(-\frac{3}{5}\right)$

C. $\pi-\sin^{-1}\left(-\frac{3}{5}\right)$ **D.** $\sin^{-1}\left(-\frac{3}{5}\right)$

12. $\tan^{-1}(-3)+\tan^{-1}(-4)=$

A. $\tan^{-1}(0.636363636363636)$ **B.** $-\pi+\tan^{-1}\frac{7}{11}$

C. $\tan^{-1}(-7)$ **D.** $\pi+\tan^{-1}(0.636363636363636)$

13. $\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}=$

A. $\frac{\pi}{4}$ **B.** $\frac{3\pi}{4}$ **C.** $\frac{\pi}{3}$ **D.** $-\frac{\pi}{4}$

14. $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{7}=$

A. $\tan^{-1}\frac{53}{114}$ **B.** $\tan^{-1}\frac{107}{210}$

C. $\tan^{-1}\frac{53}{96}$ **D.** $\tan^{-1}\frac{107}{192}$

15. $\sin\left(\cos^{-1}\frac{24}{25}\right)=$

A. $\frac{24}{7}$ **B.** $\frac{24}{25}$ **C.** $\frac{7}{24}$ **D.** $\frac{7}{25}$

16. $\cos\left(\tan^{-1}\frac{12}{5}\right)=$

A. $\frac{5}{12}$ **B.** $\frac{12}{5}$ **C.** $\frac{12}{13}$ **D.** $\frac{5}{13}$

17. $2\tan^{-1}\frac{1}{8}$ written as a single inverse sine is

A. $\cos^{-1}\frac{16}{65}$ **B.** $\sin^{-1}\frac{63}{65}$

C. $\sin^{-1}\frac{16}{65}$ **D.** $\pi-\sin^{-1}\frac{16}{65}$

18. $2\tan^{-1}7$ written as a single inverse cosine is

A. $\cos^{-1}\frac{24}{25}$ **B.** $\cos^{-1}\frac{7}{25}$

C. $\cos^{-1}\left(-\frac{24}{25}\right)$ **D.** $-\cos^{-1}\left(-\frac{24}{25}\right)$

19. For $x\geq 1$, the graph of $y=\sec^{-1}x$

A. rises from $(1,0)$ towards $y=\pi$

B. is a horizontal line

C. falls from $\left(1,\frac{\pi}{2}\right)$ towards $y=0$

D. rises from $(1,0)$ towards $y=\frac{\pi}{2}$

20. $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)=$

A. $-\frac{\pi}{4}$ **B.** $\frac{5\pi}{4}$ **C.** $\frac{3\pi}{4}$ **D.** $\frac{\pi}{4}$

21. $\tan^{-1}\left(\tan-\frac{2\pi}{3}\right)=$

A. $\frac{2\pi}{3}$ **B.** $\frac{\pi}{3}$ **C.** $-\frac{\pi}{3}$ **D.** $-\frac{2\pi}{3}$

22. If $x = 2 - \sqrt{3}$, then $\tan^{-1}\frac{3x - x^3}{1 - 3x^2} =$

A. $\frac{3\pi}{4}$ **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{4}$ **D.** $-\frac{\pi}{4}$

23. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then $xy + yz + zx =$

A. -3 **B.** 3 **C.** 0 **D.** 1

24. If $\tan^{-1}\frac{5}{x} + \tan^{-1}\frac{20}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

A. 100 **B.** $5\sqrt{17}$ **C.** 10 **D.** 25

25. The greatest value of $(\cos^{-1} x)^2 - (\sin^{-1} x)^2$ is

A. $\frac{\pi^2}{8}$ **B.** $\frac{3\pi^2}{4}$ **C.** $\frac{5\pi^2}{4}$ **D.** $\frac{\pi^3}{32}$

Key sheet

1. A	**2.** C	**3.** C	**4.** A	**5.** B
6. D	**7.** A	**8.** A	**9.** B	**10.** C
11. D	**12.** B	**13.** A	**14.** C	**15.** D
16. D	**17.** C	**18.** C	**19.** D	**20.** A
21. B	**22.** C	**23.** B	**24.** C	**25.** B

Solutions

1. The range (principal values) of $\sec^{-1} x$ is

The range of $\sec^{-1} x$ is the interval on which $\sec x$ was restricted: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Ans: (A) $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

---

2. The principal value of $\cot^{-1} \sqrt{3}$ is

The angle in $(0, \pi)$ with $\cot\theta = \sqrt{3}$ is $\frac{\pi}{6}$.

Ans: (C) $\frac{\pi}{6}$

---

3. The principal value of $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ is

The angle in $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ with $\sec\theta = -\frac{2}{\sqrt{3}}$ is $\frac{5\pi}{6}$.

Ans: (C) $\frac{5\pi}{6}$

---

4. $\sin^{-1} x$ means

$\sin^{-1} x$ is the inverse function, not the reciprocal.

Ans: (A) the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x

---

5. $\tan^{-1}\left(\tan\frac{2\pi}{7}\right) =$

$\frac{2\pi}{7}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so the value is $\frac{2\pi}{7}$ itself.

Ans: (B) $\frac{2\pi}{7}$

---

6. If $\cot^{-1}\frac{5}{4} = \alpha$, then $\cot^{-1}\left(-\frac{5}{4}\right) =$

$\cot^{-1}(-x) = \pi - \cot^{-1} x$

$\cot^{-1}\left(-\frac{5}{4}\right) = \pi - \alpha$

Ans: (D) $\pi - \alpha$

---

7. If $\operatorname{cosec}^{-1}\frac{11}{3} = \alpha$, then $\operatorname{cosec}^{-1}\left(-\frac{11}{3}\right) =$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

$\operatorname{cosec}^{-1}\left(-\frac{11}{3}\right) = -\alpha$

Ans: (A) $-\alpha$

---

8. $\operatorname{cosec}^{-1}\frac{9}{4}$ written as an inverse sine is

$\operatorname{cosec}^{-1} x = \sin^{-1}\frac{1}{x},\ |x| \geq 1$

$\operatorname{cosec}^{-1}\frac{9}{4} = \sin^{-1}\frac{4}{9}$

Ans: (A) $\sin^{-1}\frac{4}{9}$

---

9. If $\operatorname{cosec}^{-1} x = \frac{3\pi}{10}$, then $\sec^{-1} x =$

$\sec^{-1} x = \frac{\pi}{2} - \frac{3\pi}{10} = \frac{\pi}{5}$

Ans: (B) $\frac{\pi}{5}$

---

10. $\sec^{-1}\frac{7}{2}+\operatorname{cosec}^{-1}\frac{7}{2}=$

$\sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$ for every x in the common domain.

Ans: (C) $\frac{\pi}{2}$

---

11. $\sin^{-1}\frac{7}{25}-\sin^{-1}\frac{4}{5}=$

$\sqrt{1-x^2}=\frac{24}{25},\ \sqrt{1-y^2}=\frac{3}{5}$

$x^2+y^2=\frac{449}{625}\le 1.$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)=\sin^{-1}\left(\frac{7}{25}\cdot\frac{3}{5}-\frac{4}{5}\cdot\frac{24}{25}\right)=\sin^{-1}\left(-\frac{3}{5}\right)$

Ans: (D) $\sin^{-1}\left(-\frac{3}{5}\right)$

---

12. $\tan^{-1}(-3)+\tan^{-1}(-4)=$

Here $x,y<0,\ xy=12>1.$

$\frac{x+y}{1-xy}=\frac{-7}{-11}=\frac{7}{11}$

$=-\pi+\tan^{-1}\frac{7}{11}$

Ans: (B) $-\pi+\tan^{-1}\frac{7}{11}$

---

13. $\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}=$

Here $xy=\frac{3}{20}<1.$

$\frac{x+y}{1-xy}=\frac{\frac{17}{20}}{\frac{17}{20}}=1$

$=\tan^{-1}1=\frac{\pi}{4}$

Ans: (A) $\frac{\pi}{4}$

---

14. $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{7}=$

$S_1=\frac{107}{210},\ S_2=\frac{3}{35},\ S_3=\frac{1}{210}$

$\tan^{-1}\frac{S_1-S_3}{1-S_2}=\tan^{-1}\frac{53}{96}$

Ans: (C) $\tan^{-1}\frac{53}{96}$

---

15. $\sin\left(\cos^{-1}\frac{24}{25}\right)=$

$\cos\theta=\frac{24}{25}$ with $\theta\in[0,\pi]$, so $\sin\theta=\frac{7}{25}$.

Ans: (D) $\frac{7}{25}$

---

16. $\cos\left(\tan^{-1}\frac{12}{5}\right)=$

$\tan\theta=\frac{12}{5}$: opposite 12, adjacent 5, hypotenuse 13.

$\cos\theta=\frac{5}{13}$.

Ans: (D) $\frac{5}{13}$

---

17. $2\tan^{-1}\frac{1}{8}$ written as a single inverse sine is

$\frac{2x}{1+x^2}=\frac{16}{65}$

For $x\le 1$ the formula is $\sin^{-1}\frac{2x}{1+x^2}$.

Ans: (C) $\sin^{-1}\frac{16}{65}$

---

18. $2\tan^{-1}7$ written as a single inverse cosine is

$\frac{1-x^2}{1+x^2} = -\frac{24}{25}$

For $x \geq 0$, $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$.

Ans: (C) $\cos^{-1}\left(-\frac{24}{25}\right)$

---

19. For $x \geq 1$, the graph of $y = \sec^{-1}x$

This follows from the shape of the graph described in the lesson on graphs.

Ans: (D) rises from $(1, 0)$ **towards** $y = \frac{\pi}{2}$

---

20. $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) =$

$\frac{5\pi}{4}$ lies outside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; bring it into the range using the periodic graph.

The value is $-\frac{\pi}{4}$.

Ans: (A) $-\frac{\pi}{4}$

---

21. $\tan^{-1}\left(\tan -\frac{2\pi}{3}\right) =$

$-\frac{2\pi}{3}$ lies outside $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; bring it into the range using the periodic graph.

The value is $\frac{\pi}{3}$.

Ans: (B) $\frac{\pi}{3}$

---

22. If $x = 2 - \sqrt{3}$, then $\tan^{-1}\frac{3x-x^3}{1-3x^2} =$

$x = 2 - \sqrt{3}$ lies in $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, where $\tan^{-1}\frac{3x-x^3}{1-3x^2} = 3\tan^{-1}x$.

$= \frac{\pi}{4}$

Ans: (C) $\frac{\pi}{4}$

---

23. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then $xy + yz + zx =$

Each term must take its greatest value, so $x = y = z = 1$.

Then $xy + yz + zx = 3$.

Ans: (B) 3

---

24. If $\tan^{-1}\frac{5}{x} + \tan^{-1}\frac{20}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

The product of the two values must be 1: $x^2 = 100$.

Ans: (C) 10

---

25. The greatest value of $(\cos^{-1}x)^2 - (\sin^{-1}x)^2$ is

Put $\sin^{-1}x = t$, so $\cos^{-1}x = \frac{\pi}{2} - t$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

The expression is a polynomial in t; its greatest value on this interval is $\frac{3\pi^2}{4}$.

Ans: (B) $\frac{3\pi^2}{4}$
