EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 10

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The domain of $\sin^{-1}(3x+4)$ is

A. $[1, \frac{5}{3}]$ **B.** $[-5, -3]$

C. $[-\frac{1}{3}, \frac{1}{3}]$ **D.** $[-\frac{5}{3}, -1]$

2. The principal value of $\operatorname{cosec}^{-1} 2$ is

A. $-\frac{\pi}{6}$ **B.** $\frac{5\pi}{6}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{\pi}{3}$

3. The principal value of $\tan^{-1} 0$ is

A. 0 **B.** π **C.** $\frac{\pi}{2}$ **D.** $\frac{\pi}{4}$

4. $\cot^{-1}\left(\cot \frac{\pi}{8}\right) =$

A. $-\frac{\pi}{8}$ **B.** $\frac{\pi}{8}$ **C.** $\frac{3\pi}{8}$ **D.** $\frac{7\pi}{8}$

5. $\tan\left(\tan^{-1} \frac{2}{9}\right) =$

A. $\frac{7}{9}$ **B.** $\frac{2}{9}$ **C.** $-\frac{2}{9}$ **D.** $\frac{9}{2}$

6. $\sec^{-1}(-x) =$

A. $\sec^{-1} x$ **B.** $2\pi - \sec^{-1} x$

C. $\pi + \sec^{-1} x$ **D.** $\pi - \sec^{-1} x$

7. $\operatorname{cosec}^{-1}(-x) =$

A. $\operatorname{cosec}^{-1} x$ **B.** $\pi + \operatorname{cosec}^{-1} x$

C. $-\operatorname{cosec}^{-1} x$ **D.** $2\pi - \operatorname{cosec}^{-1} x$

8. $\cot^{-1}\left(-\frac{2}{9}\right)$ in terms of an inverse tangent is

A. $\pi + \tan^{-1} \frac{9}{2}$ **B.** $\tan^{-1} \frac{9}{2}$

C. $\pi - \tan^{-1} \frac{9}{2}$ **D.** $\frac{\pi}{2} - \sin^{-1} \frac{2}{9}$

9. If $\tan^{-1} x = \frac{2\pi}{9}$, then $\cot^{-1} x =$

A. $-\frac{2\pi}{9}$ **B.** $\frac{5\pi}{18}$ **C.** $\frac{2\pi}{9}$ **D.** $\frac{7\pi}{9}$

10. $\cos^{-1} \frac{4}{11} + \sin^{-1} \frac{4}{11} =$

A. $\frac{\pi}{4}$ **B.** 0 **C.** $\frac{\pi}{2}$ **D.** π

11. $\cos^{-1}\frac{5}{13}+\cos^{-1}\frac{8}{17}=$

A. $\cos^{-1}\frac{220}{221}$ **B.** $-\cos^{-1}\left(-\frac{140}{221}\right)$

C. $\sin^{-1}\left(-\frac{140}{221}\right)$ **D.** $\cos^{-1}\left(-\frac{140}{221}\right)$

12. $\tan^{-1}5-\tan^{-1}\frac{1}{4}=$

A. $\pi+\tan^{-1}\frac{19}{9}$ **B.** $\tan^{-1}\frac{21}{4}$

C. $\tan^{-1}\frac{19}{9}$ **D.** $-\pi+\tan^{-1}\frac{19}{9}$

13. $\tan^{-1}\frac{2}{7}+\tan^{-1}\frac{1}{3}=$

A. $\tan^{-1}\frac{13}{19}$ **B.** $-\pi+\tan^{-1}\frac{13}{19}$

C. $\pi+\tan^{-1}\frac{13}{19}$ **D.** $\tan^{-1}\frac{13}{21}$

14. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{10}=$

A. $\tan^{-1}\frac{29}{35}$ **B.** $\tan^{-1}\frac{29}{40}$

C. $\tan^{-1}\frac{23}{28}$ **D.** $\tan^{-1}\frac{23}{36}$

15. $\sin^{-1}\left(-\frac{9}{41}\right)$ written as an inverse cosine is

A. $-\cos^{-1}\frac{9}{41}$ **B.** $\cos^{-1}\left(-\frac{40}{41}\right)$

C. $-\cos^{-1}\frac{40}{41}$ **D.** $\cos^{-1}\frac{40}{41}$

16. $\cos^{-1}\frac{15}{17}$ written as an inverse tangent is

A. $\tan^{-1}\frac{8}{15}$ **B.** $\pi-\tan^{-1}\frac{8}{15}$

C. $\tan^{-1}\frac{8}{17}$ **D.** $\tan^{-1}\frac{15}{8}$

17. $2\tan^{-1}\frac{2}{9}$ written as a single inverse tangent is

A. $\pi+\tan^{-1}\frac{36}{77}$ **B.** $\tan^{-1}\frac{4}{9}$

C. $\tan^{-1}\frac{36}{77}$ **D.** $\tan^{-1}\frac{36}{85}$

18. $2\tan^{-1}\frac{9}{4}$ written as a single inverse sine is

A. $\pi-\sin^{-1}\frac{72}{97}$ **B.** $\sin^{-1}\frac{72}{97}$

C. $\cos^{-1}\frac{72}{97}$ **D.** $\sin^{-1}\left(-\frac{65}{97}\right)$

19. The graph of $y=\operatorname{cosec}^{-1}x$ for $x\le -1$ ends at

A. $(-1,0)$ **B.** $\left(-1,-\frac{\pi}{2}\right)$

C. $\left(-1,\frac{\pi}{2}\right)$ **D.** $(-1,\pi)$

20. $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)=$

A. $\frac{\pi}{8}$ **B.** $\frac{7\pi}{8}$ **C.** $-\frac{\pi}{8}$ **D.** $\frac{9\pi}{8}$

21. $\sec^{-1}\left(\sec\frac{5\pi}{4}\right)=$

A. $\frac{\pi}{4}$ **B.** $\frac{5\pi}{4}$ **C.** $-\frac{3\pi}{4}$ **D.** $\frac{3\pi}{4}$

22. For $x < -\frac{1}{\sqrt{3}}$, $\tan^{-1}\frac{3x-x^3}{1-3x^2} =$

A. $-\pi + 3\tan^{-1}x$ **B.** $\pi + 3\tan^{-1}x$

C. $-(\pi + 3\tan^{-1}x)$ **D.** $3\tan^{-1}x$

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23. If $\tan^{-1}\frac{1}{x} + \tan^{-1}\frac{49}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

A. 49 **B.** 7 **C.** $\sqrt{2402}$ **D.** 50

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24. If $\sin^{-1}\frac{28}{x} + \sin^{-1}\frac{45}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

A. 53 **B.** $\sqrt{1241}$ **C.** $6\sqrt{35}$ **D.** 73

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25. To remove the root in $\sqrt{x^2 - 36}$, use

A. the cosine substitution $x = 6\cos\theta$ **B.** the tangent substitution $x = 6\tan\theta$

C. the sine substitution $x = 6\sin\theta$ **D.** the secant substitution $x = 6\sec\theta$

Key sheet

1. D	**2.** C	**3.** A	**4.** B	**5.** B
6. D	**7.** C	**8.** C	**9.** B	**10.** C
11. D	**12.** C	**13.** A	**14.** C	**15.** C
16. A	**17.** C	**18.** A	**19.** B	**20.** A
21. D	**22.** B	**23.** B	**24.** A	**25.** D

Solutions

1. The domain of $\sin^{-1}(3x+4)$ is

$-1 \le 3x+4 \le 1 \Rightarrow -\frac{5}{3} \le x \le -1$

Ans: (D) $\left[-\frac{5}{3}, -1\right]$

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2. The principal value of $\operatorname{cosec}^{-1} 2$ is

The angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ with $\operatorname{cosec}\theta = 2$ is $\frac{\pi}{6}$.

Ans: (C) $\frac{\pi}{6}$

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3. The principal value of $\tan^{-1} 0$ is

The angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $\tan\theta = 0$ is 0.

Ans: (A) 0

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4. $\cot^{-1}\left(\cot\frac{\pi}{8}\right) =$

$\frac{\pi}{8}$ is in $(0, \pi)$, so the value is $\frac{\pi}{8}$ itself.

Ans: (B) $\frac{\pi}{8}$

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5. $\tan\left(\tan^{-1}\frac{2}{9}\right) =$

$\tan(\tan^{-1} x) = x$ on the domain of $\tan^{-1}$.

Ans: (B) $\frac{2}{9}$

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6. $\sec^{-1}(-x) =$

The rule for negative arguments: $\sec^{-1}(-x) = \pi - \sec^{-1} x$.

Ans: (D) $\pi - \sec^{-1} x$

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7. $\operatorname{cosec}^{-1}(-x) =$

The rule for negative arguments: $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$.

Ans: (C) $-\operatorname{cosec}^{-1} x$

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8. $\cot^{-1}\left(-\frac{2}{9}\right)$ in terms of an inverse tangent is

$\cot^{-1} x = \pi + \tan^{-1}\frac{1}{x},\ x < 0$

$\cot^{-1}\left(-\frac{2}{9}\right) = \pi + \tan^{-1}\left(-\frac{9}{2}\right) = \pi - \tan^{-1}\frac{9}{2}$

Ans: (C) $\pi - \tan^{-1}\frac{9}{2}$

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9. If $\tan^{-1} x = \frac{2\pi}{9}$, then $\cot^{-1} x =$

$\cot^{-1} x = \frac{\pi}{2} - \frac{2\pi}{9} = \frac{5\pi}{18}$

Ans: (B) $\frac{5\pi}{18}$

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10. $\cos^{-1}\frac{4}{11} + \sin^{-1}\frac{4}{11} =$

$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$ for every x in the common domain.

Ans: (C) $\frac{\pi}{2}$

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11. $\cos^{-1}\frac{5}{13}+\cos^{-1}\frac{8}{17}=$

$\sqrt{1-x^2}=\frac{12}{13},\ \sqrt{1-y^2}=\frac{15}{17}$

Here $x+y\geq 0$.

$xy-\sqrt{1-x^2}\sqrt{1-y^2}=\frac{5}{13}\cdot\frac{8}{17}-\frac{12}{13}\cdot\frac{15}{17}=-\frac{140}{221}$

So the formula gives $\cos^{-1}\left(-\frac{140}{221}\right)$.

Ans: (D) $\cos^{-1}\left(-\frac{140}{221}\right)$

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12. $\tan^{-1}5-\tan^{-1}\frac{1}{4}=$

Here $xy=\frac{5}{4}>-1$.

$\frac{x-y}{1+xy}=\frac{\frac{19}{4}}{\frac{9}{4}}=\frac{19}{9}$

$=\tan^{-1}\frac{19}{9}$

Ans: (C) $\tan^{-1}\frac{19}{9}$

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13. $\tan^{-1}\frac{2}{7}+\tan^{-1}\frac{1}{3}=$

Here $xy=\frac{2}{21}<1$.

$\frac{x+y}{1-xy}=\frac{\frac{13}{21}}{\frac{19}{21}}=\frac{13}{19}$

$=\tan^{-1}\frac{13}{19}$

Ans: (A) $\tan^{-1}\frac{13}{19}$

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14. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{10}=$

$S_1=\frac{29}{40},\ S_2=\frac{1}{8},\ S_3=\frac{1}{160}$

$\tan^{-1}\frac{S_1-S_3}{1-S_2}=\tan^{-1}\frac{23}{28}$

Ans: (C) $\tan^{-1}\frac{23}{28}$

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15. $\sin^{-1}\left(-\frac{9}{41}\right)$ written as an inverse cosine is

$x<0$, so $\sin^{-1}x=-\cos^{-1}\sqrt{1-x^2}=-\cos^{-1}\frac{40}{41}$.

Ans: (C) $-\cos^{-1}\frac{40}{41}$

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16. $\cos^{-1}\frac{15}{17}$ written as an inverse tangent is

$\cos\theta=\frac{15}{17}$, $\sin\theta=\frac{8}{17}$, so $\tan\theta=\frac{8}{15}$.

Ans: (A) $\tan^{-1}\frac{8}{15}$

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17. $2\tan^{-1}\frac{2}{9}$ written as a single inverse tangent is

Use $2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}$ for $x<1$.

$\frac{2x}{1-x^2}=\frac{36}{77}$

Ans: (C) $\tan^{-1}\frac{36}{77}$

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18. $2\tan^{-1}\frac{9}{4}$ written as a single inverse sine is

$\frac{2x}{1+x^2}=\frac{72}{97}$

For $x>1$ the formula is $\pi-\sin^{-1}\frac{2x}{1+x^2}$.

Ans: (A) $\pi-\sin^{-1}\frac{72}{97}$

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19. The graph of $y = \operatorname{cosec}^{-1} x$ for $x \le -1$ ends at

This follows from the shape of the graph described in the lesson on graphs.

Ans: (B) $\left(-1, -\frac{\pi}{2}\right)$

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20. $\tan^{-1}\left(\tan \frac{9\pi}{8}\right) =$

$\frac{9\pi}{8}$ lies outside $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; bring it into the range using the periodic graph.

The value is $\frac{\pi}{8}$.

Ans: (A) $\frac{\pi}{8}$

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21. $\sec^{-1}\left(\sec \frac{5\pi}{4}\right) =$

$\frac{5\pi}{4}$ lies outside $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$; bring it into the range using the periodic graph.

The value is $\frac{3\pi}{4}$.

Ans: (D) $\frac{3\pi}{4}$

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22. For $x < -\frac{1}{\sqrt{3}}$, $\tan^{-1} \frac{3x - x^3}{1 - 3x^2} =$

Put $x = \tan\theta$, so that the expression inside is a ratio of 3θ.

For $x < -\frac{1}{\sqrt{3}}$ the angle has to be brought back into the principal range.

$\tan^{-1} \frac{3x - x^3}{1 - 3x^2} = \pi + 3\tan^{-1} x$

Ans: (B) $\pi + 3\tan^{-1} x$

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23. If $\tan^{-1} \frac{1}{x} + \tan^{-1} \frac{49}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

The product of the two values must be 1: $x^2 = 49$.

Ans: (B) 7

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24. If $\sin^{-1} \frac{28}{x} + \sin^{-1} \frac{45}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

$x = \sqrt{28^2 + 45^2} = 53$.

Ans: (A) 53

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25. To remove the root in $\sqrt{x^2 - 36}$, use

$36\sec^2\theta - 36 = 36\tan^2\theta$.

Ans: (D) the secant substitution $x = 6\sec\theta$
