EXPRESSWALA ACADEMY

Maths 1A: Inverse Trigonometric Functions

MCQ Test, Set 01

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. The range (principal values) of $\cot^{-1} x$ is

A. $[0, \pi]$ **B.** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
C. $(0, \pi)$ **D.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. The principal value of $\cos^{-1} \frac{\sqrt{3}}{2}$ is

A. $\frac{\pi}{3}$ **B.** $\frac{\pi}{6}$ **C.** $-\frac{\pi}{6}$ **D.** $\frac{5\pi}{6}$

3. The principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is

A. $-\frac{\pi}{6}$ **B.** $\frac{2\pi}{3}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{7\pi}{6}$

4. $\sin(\sin^{-1} x) = x$ holds for

A. $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **B.** $|x| \geq 1$
C. $x \in [-1, 1]$ **D.** all real x

5. $\cos^{-1}\left(\cos \frac{5\pi}{7}\right) =$

A. $-\frac{3\pi}{14}$ **B.** $\frac{2\pi}{7}$ **C.** $\frac{5\pi}{7}$ **D.** $-\frac{5\pi}{7}$

6. If $\cos^{-1} \frac{2}{7} = \alpha$, then $\cos^{-1}\left(-\frac{2}{7}\right) =$

A. $\frac{\pi}{2} - \alpha$ **B.** $\pi + \alpha$ **C.** α **D.** $\pi - \alpha$

7. If $\tan^{-1} \frac{2}{9} = \alpha$, then $\tan^{-1}\left(-\frac{2}{9}\right) =$

A. $\frac{\pi}{2} - \alpha$ **B.** $-\alpha$ **C.** $\pi + \alpha$ **D.** α

8. $\sec^{-1} \frac{7}{4}$ written as an inverse cosine is

A. $\cos^{-1} \frac{4}{7}$ **B.** $\sin^{-1} \frac{4}{7}$
C. $\pi - \cos^{-1} \frac{4}{7}$ **D.** $\cos^{-1} \frac{3}{7}$

9. If $\sin^{-1} x = \frac{2\pi}{11}$, then $\cos^{-1} x =$

A. $\frac{2\pi}{11}$ **B.** $\frac{7\pi}{22}$ **C.** $-\frac{2\pi}{11}$ **D.** $\frac{9\pi}{11}$

10. $\tan^{-1} \frac{5}{3} + \cot^{-1} \frac{5}{3} =$

A. 0 **B.** π **C.** $\frac{\pi}{4}$ **D.** $\frac{\pi}{2}$

11. $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=$

A. $\sin^{-1}\frac{77}{85}$ **B.** $\cos^{-1}\frac{77}{85}$

C. $\sin^{-1}\frac{13}{85}$ **D.** $\pi-\sin^{-1}\frac{77}{85}$

12. $\tan^{-1}\frac{2}{3}-\tan^{-1}\frac{1}{5}=$

A. $\tan^{-1}\frac{13}{15}$ **B.** $-\pi+\tan^{-1}\frac{7}{17}$

C. $\pi+\tan^{-1}\frac{7}{17}$ **D.** $\tan^{-1}\frac{7}{17}$

13. $\tan^{-1}3+\tan^{-1}4=$

A. $-\pi+\tan^{-1}(-0.636363636363636)$ **B.** $\tan^{-1}7$

C. $\pi+\tan^{-1}\left(-\frac{7}{11}\right)$ **D.** $\tan^{-1}(-0.636363636363636)$

14. $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{7}=$

A. $\tan^{-1}\frac{30}{49}$ **B.** $\tan^{-1}\frac{6}{7}$

C. $\tan^{-1}\frac{61}{84}$ **D.** $\tan^{-1}\frac{61}{70}$

15. $\sin\left(\cos^{-1}\frac{8}{17}\right)=$

A. $\frac{15}{17}$ **B.** $\frac{15}{8}$ **C.** $\frac{8}{17}$ **D.** $\frac{8}{15}$

16. $\cos\left(\tan^{-1}\frac{3}{4}\right)=$

A. $\frac{4}{3}$ **B.** $\frac{4}{5}$ **C.** $\frac{3}{5}$ **D.** $\frac{3}{4}$

17. $2\tan^{-1}\frac{1}{7}$ written as a single inverse tangent is

A. $\tan^{-1}\frac{7}{24}$ **B.** $\tan^{-1}\frac{2}{7}$

C. $\pi+\tan^{-1}\frac{7}{24}$ **D.** $\tan^{-1}\frac{7}{25}$

18. $2\tan^{-1}3$ written as a single inverse sine is

A. $\sin^{-1}\left(-\frac{4}{5}\right)$ **B.** $\pi-\sin^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{5}$ **D.** $\sin^{-1}\frac{3}{5}$

19. The graph of $y=\tan^{-1}x$ is

A. increasing, with asymptotes $y=\pm\frac{\pi}{2}$

B. increasing, with asymptotes $y=0$, $y=\pi$

C. decreasing, with asymptotes $y=\pm\frac{\pi}{2}$

D. decreasing, with asymptotes $y=0$, $y=\pi$

20. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=$

A. $\frac{\pi}{6}$ **B.** $-\frac{5\pi}{6}$ **C.** $\frac{7\pi}{6}$ **D.** $\frac{5\pi}{6}$

21. $\sin^{-1}(\sin 2)=$

A. $2-\pi$ **B.** $-2+\pi$ **C.** 2 **D.** $-2+2\pi$

22. If $x=-\frac{1}{\sqrt{3}}$, then $\sin^{-1}\frac{2x}{1+x^2}=$

A. $\frac{4\pi}{3}$ **B.** $\frac{\pi}{6}$ **C.** $-\frac{\pi}{3}$ **D.** $\frac{\pi}{3}$

23. If $\tan^{-1}\frac{2}{x}+\tan^{-1}\frac{18}{x}=\frac{\pi}{2}$, $x>0$, then $x=$

A. 6 **B.** 20 **C.** 36 **D.** $2\sqrt{82}$

24. If $\sin^{-1}\frac{7}{x}+\sin^{-1}\frac{24}{x}=\frac{\pi}{2}$, $x>0$, then $x=$

A. 31 **B.** $2\sqrt{42}$ **C.** $\sqrt{527}$ **D.** 25

25. The least value of $(\sin^{-1}x)^3+(\cos^{-1}x)^3$ is

A. $\frac{\pi^2}{8}$ **B.** $\frac{5\pi^2}{4}$ **C.** $\frac{\pi^3}{32}$ **D.** $\frac{7\pi^3}{8}$

Key sheet

1. C	**2.** B	**3.** A	**4.** C	**5.** C
6. D	**7.** B	**8.** A	**9.** B	**10.** D
11. A	**12.** D	**13.** C	**14.** B	**15.** A
16. B	**17.** A	**18.** B	**19.** A	**20.** D
21. B	**22.** C	**23.** A	**24.** D	**25.** C

Solutions

1. The range (principal values) of $\cot^{-1} x$ is

The range of $\cot^{-1} x$ is the interval on which $\cot x$ was restricted: $(0, \pi)$.

Ans: (C) $(0, \pi)$

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2. The principal value of $\cos^{-1} \frac{\sqrt{3}}{2}$ is

The angle in $[0, \pi]$ with $\cos\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$.

Ans: (B) $\frac{\pi}{6}$

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3. The principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is

The angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $\tan\theta = -\frac{1}{\sqrt{3}}$ is $-\frac{\pi}{6}$.

Ans: (A) $-\frac{\pi}{6}$

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4. $\sin(\sin^{-1} x) = x$ holds for

The identity holds on the domain of $\sin^{-1}$, which is $[-1, 1]$.

Ans: (C) $x \in [-1, 1]$

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5. $\cos^{-1}\left(\cos\frac{5\pi}{7}\right) =$

$\frac{5\pi}{7}$ is in $[0, \pi]$, so the value is $\frac{5\pi}{7}$ itself.

Ans: (C) $\frac{5\pi}{7}$

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6. If $\cos^{-1}\frac{2}{7} = \alpha$, then $\cos^{-1}\left(-\frac{2}{7}\right) =$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\cos^{-1}\left(-\frac{2}{7}\right) = \pi - \alpha$

Ans: (D) $\pi - \alpha$

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7. If $\tan^{-1}\frac{2}{9} = \alpha$, then $\tan^{-1}\left(-\frac{2}{9}\right) =$

$\tan^{-1}(-x) = -\tan^{-1} x$

$\tan^{-1}\left(-\frac{2}{9}\right) = -\alpha$

Ans: (B) $-\alpha$

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8. $\sec^{-1}\frac{7}{4}$ written as an inverse cosine is

$\sec^{-1} x = \cos^{-1}\frac{1}{x},\ |x| \geq 1$

$\sec^{-1}\frac{7}{4} = \cos^{-1}\frac{4}{7}$

Ans: (A) $\cos^{-1}\frac{4}{7}$

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9. If $\sin^{-1} x = \frac{2\pi}{11}$, then $\cos^{-1} x =$

$\cos^{-1} x = \frac{\pi}{2} - \frac{2\pi}{11} = \frac{7\pi}{22}$

Ans: (B) $\frac{7\pi}{22}$

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10. $\tan^{-1}\frac{5}{3}+\cot^{-1}\frac{5}{3}=$

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ for every x in the common domain.

Ans: (D) $\frac{\pi}{2}$

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11. $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=$

$\sqrt{1-x^2}=\frac{4}{5},\ \sqrt{1-y^2}=\frac{15}{17}$

$x^2+y^2=\frac{4201}{7225}\le 1.$

$\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)=\sin^{-1}\left(\frac{3}{5}\cdot\frac{15}{17}+\frac{8}{17}\cdot\frac{4}{5}\right)=\sin^{-1}\frac{77}{85}$

Ans: (A) $\sin^{-1}\frac{77}{85}$

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12. $\tan^{-1}\frac{2}{3}-\tan^{-1}\frac{1}{5}=$

Here $xy=\frac{2}{15}>-1.$

$\frac{x-y}{1+xy}=\frac{\frac{7}{15}}{\frac{17}{15}}=\frac{7}{17}$

$=\tan^{-1}\frac{7}{17}$

Ans: (D) $\tan^{-1}\frac{7}{17}$

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13. $\tan^{-1}3+\tan^{-1}4=$

Here $x,y>0,\ xy=12>1.$

$\frac{x+y}{1-xy}=\frac{7}{-11}=-\frac{7}{11}$

$=\pi+\tan^{-1}\left(-\frac{7}{11}\right)$

Ans: (C) $\pi+\tan^{-1}\left(-\frac{7}{11}\right)$

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14. $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{7}=$

$S_1=\frac{61}{84},\ S_2=\frac{1}{6},\ S_3=\frac{1}{84}$

$\tan^{-1}\frac{S_1-S_3}{1-S_2}=\tan^{-1}\frac{6}{7}$

Ans: (B) $\tan^{-1}\frac{6}{7}$

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15. $\sin\left(\cos^{-1}\frac{8}{17}\right)=$

$\cos\theta=\frac{8}{17}$ with $\theta\in[0,\pi]$, so $\sin\theta=\frac{15}{17}$.

Ans: (A) $\frac{15}{17}$

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16. $\cos\left(\tan^{-1}\frac{3}{4}\right)=$

$\tan\theta=\frac{3}{4}$: opposite 3, adjacent 4, hypotenuse 5.

$\cos\theta=\frac{4}{5}$.

Ans: (B) $\frac{4}{5}$

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17. $2\tan^{-1}\frac{1}{7}$ written as a single inverse tangent is

Use $2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}$ for $x<1$.

$\frac{2x}{1-x^2}=\frac{7}{24}$

Ans: (A) $\tan^{-1}\frac{7}{24}$

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18. $2\tan^{-1}3$ written as a single inverse sine is

$\frac{2x}{1+x^2} = \frac{3}{5}$

For $x > 1$ the formula is $\pi - \sin^{-1}\frac{2x}{1+x^2}$.

Ans: (B) $\pi - \sin^{-1}\frac{3}{5}$

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19. The graph of $y = \tan^{-1}x$ is

This follows from the shape of the graph described in the lesson on graphs.

Ans: (A) increasing, with asymptotes $y = \pm\frac{\pi}{2}$

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20. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) =$

$\frac{7\pi}{6}$ lies outside $[0, \pi]$; bring it into the range using the periodic graph.

The value is $\frac{5\pi}{6}$.

Ans: (D) $\frac{5\pi}{6}$

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21. $\sin^{-1}(\sin 2) =$

2 lies outside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; bring it into the range using the periodic graph.

The value is $-2 + \pi$.

Ans: (B) $-2 + \pi$

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22. If $x = -\frac{1}{\sqrt{3}}$, then $\sin^{-1}\frac{2x}{1+x^2} =$

$x = -\frac{1}{\sqrt{3}}$ lies in $-1 < x < 1$, where $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$.

$= -\frac{\pi}{3}$

Ans: (C) $-\frac{\pi}{3}$

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23. If $\tan^{-1}\frac{2}{x} + \tan^{-1}\frac{18}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

The product of the two values must be 1: $x^2 = 36$.

Ans: (A) 6

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24. If $\sin^{-1}\frac{7}{x} + \sin^{-1}\frac{24}{x} = \frac{\pi}{2}$, $x > 0$, then $x =$

$x = \sqrt{7^2 + 24^2} = 25$.

Ans: (D) 25

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25. The least value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is

Put $\sin^{-1}x = t$, so $\cos^{-1}x = \frac{\pi}{2} - t$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

The expression is a polynomial in t; its least value on this interval is $\frac{\pi^3}{32}$.

Ans: (C) $\frac{\pi^3}{32}$
