EXPRESSWALA ACADEMY

Maths 1A: Inverse of a Matrix

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the cofactor of the element in row 3, column 2 of $A = \begin{bmatrix} -4 & 0 & -3 \\ 3 & -3 & 3 \\ 3 & -1 & -3 \end{bmatrix}$. [1 mark]

2. Find the adjoint of $A = \begin{bmatrix} -2 & 4 & -1 \\ -1 & 1 & 2 \\ 0 & -3 & 4 \end{bmatrix}$. [3 marks]

3. A is a 3×3 matrix with $|A| = -6$. Find $|A^{-1}|$. [1 mark]

4. If $A^{-1} = \begin{bmatrix} -1 & 5 \\ 0 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$, find $(BA)^{-1}$. [2 marks]

5. Find the values of x for which $\begin{bmatrix} x & 7 \\ 7 & x \end{bmatrix}$ is singular. [2 marks]

6. Write down the inverse of $\begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix}$. [1 mark]

7. Find the inverse of $A = \begin{bmatrix} 3 & -2 \\ 5 & -1 \end{bmatrix}$. [2 marks]

8. If $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & 3 \\ 2 & -2 & -2 \end{bmatrix}$, find A^{-1} using the adjoint. [5 marks]

9. Write the inverse of the diagonal matrix $\begin{bmatrix} -2 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 4 \end{bmatrix}$. [1 mark]

10. Show that $A = \frac{1}{3}\begin{bmatrix} 2 & -2 & -1 \\ -1 & -2 & 2 \\ 2 & 1 & 2 \end{bmatrix}$ is orthogonal and hence find A^{-1}. [3 marks]

11. If $A = \begin{bmatrix} -1 & 1 \\ -4 & 7 \end{bmatrix}$, find $\operatorname{Adj} A$ and show that $(\operatorname{Adj} A)A = |A|\, I$. [2 marks]

12. Find the adjoint of $A = \begin{bmatrix} 0 & -2 & -1 \\ -2 & 4 & 1 \\ 3 & -1 & -1 \end{bmatrix}$ and show that $A(\operatorname{Adj} A) = |A|\, I$. [5 marks]

13. A is a 3×3 matrix with $|A| = -4$. Find $|\operatorname{Adj} A|$. [1 mark]

14. For $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, find $\operatorname{Adj} A$ and verify that $|\operatorname{Adj} A| = |A|^2$. [5 marks]

15. A is a non-singular matrix of order 3 with $|A| = 2$. Express $\operatorname{Adj}(\operatorname{Adj} A)$ in terms of A. [1 mark]

16. If $A = \begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$, find $(\operatorname{Adj} A)^{-1}$. [2 marks]

17. Find S_1, S_2, S_3 for $A = \begin{bmatrix} -2 & -1 & -2 \\ -1 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ and write its characteristic equation. [3 marks]

18. If a matrix A satisfies $A^2 - 2A + 3I = O$, express A^{-1} in terms of A and I. [2 marks]

19. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 2 \end{bmatrix}$, show that $A^2 - 5A - 8I = O$. Use it to find A^{-1}. [3 marks]

20. Find the characteristic equation of $A = \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and use it to find A^{-1}. [5 marks]

Key sheet

Q	Answer
1	3
2	$\operatorname{Adj} A = \begin{bmatrix} 10 & -13 & 9 \\ 4 & -8 & 5 \\ 3 & -6 & 2 \end{bmatrix}$
3	$-\frac{1}{6}$
4	$(BA)^{-1} = \begin{bmatrix} 11 & -6 \\ 2 & -1 \end{bmatrix}$
5	$x = \pm 7$
6	$\begin{bmatrix} 1 & 2 \\ -3 & -7 \end{bmatrix}$
7	$A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & 2 \\ -5 & 3 \end{bmatrix}$
8	$A^{-1} = \frac{1}{8}\begin{bmatrix} 8 & 0 & 8 \\ 8 & -2 & 5 \\ 0 & 2 & -1 \end{bmatrix}$
9	$\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$
10	$A^{-1} = A^{T} = \frac{1}{3}\begin{bmatrix} 2 & -1 & 2 \\ -2 & -2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$
11	$\operatorname{Adj} A = \begin{bmatrix} 7 & -1 \\ 4 & -1 \end{bmatrix}$; the product is $-3I$
12	$\operatorname{Adj} A = \begin{bmatrix} -3 & -1 & 2 \\ 1 & 3 & 2 \\ -10 & -6 & -4 \end{bmatrix}$; both products equal $8I$
13	16
14	$\lvert A\rvert = -5,\ \lvert\operatorname{Adj} A\rvert = 25 = \lvert A\rvert^2$
15	$2A$
16	$\frac{1}{2}\begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$
17	$\lambda^3 + \lambda^2 + \lambda - 11 = 0$
18	$A^{-1} = \frac{1}{3}(2I - A)$
19	$A^{-1} = \frac{1}{8}\begin{bmatrix} -2 & 7 \\ 2 & -3 \end{bmatrix}$
20	$A^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ -8 & -4 & 0 \end{bmatrix}$

Solutions

1. Find the cofactor of the element in row 3, column 2 of $A = \begin{bmatrix} -4 & 0 & -3 \\ 3 & -3 & 3 \\ 3 & -1 & -3 \end{bmatrix}$. [1]

$\text{Minor} = \begin{vmatrix} -4 & -3 \\ 3 & 3 \end{vmatrix} = -3$

$A_{32} = (-1)^{3+2}(-3) = 3$

Ans: 3

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2. Find the adjoint of $A = \begin{bmatrix} -2 & 4 & -1 \\ -1 & 1 & 2 \\ 0 & -3 & 4 \end{bmatrix}$. [3]

The cofactors are

$A_{11} = 10,\ A_{12} = 4,\ A_{13} = 3$

$A_{21} = -13,\ A_{22} = -8,\ A_{23} = -6$

$A_{31} = 9,\ A_{32} = 5,\ A_{33} = 2$

The cofactors of each row form a column of the adjoint:

$\text{Adj}\, A = \begin{bmatrix} 10 & -13 & 9 \\ 4 & -8 & 5 \\ 3 & -6 & 2 \end{bmatrix}$

Ans: $\text{Adj}\, A = \begin{bmatrix} 10 & -13 & 9 \\ 4 & -8 & 5 \\ 3 & -6 & 2 \end{bmatrix}$

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3. A is a 3×3 matrix with $|A| = -6$. Find $|A^{-1}|$. [1]

$|A^{-1}| = \frac{1}{|A|}$

$= -\frac{1}{6}$

Ans: $-\frac{1}{6}$

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4. If $A^{-1} = \begin{bmatrix} -1 & 5 \\ 0 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$, find $(BA)^{-1}$. [2]

$(BA)^{-1} = A^{-1}B^{-1}$

$= \begin{bmatrix} -1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 11 & -6 \\ 2 & -1 \end{bmatrix}$

Ans: $(BA)^{-1} = \begin{bmatrix} 11 & -6 \\ 2 & -1 \end{bmatrix}$

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5. Find the values of x for which $\begin{bmatrix} x & 7 \\ 7 & x \end{bmatrix}$ is singular. [2]

$|A| = x^2 - 49 = 0$

$x = \pm 7$

Ans: $x = \pm 7$

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6. Write down the inverse of $\begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix}$. [1]

$|A| = -1$

Swap the diagonal entries, change the signs of the other two, divide by -1.

$A^{-1} = \begin{bmatrix} 1 & 2 \\ -3 & -7 \end{bmatrix}$

Ans: $\begin{bmatrix} 1 & 2 \\ -3 & -7 \end{bmatrix}$

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7. Find the inverse of $A = \begin{bmatrix} 3 & -2 \\ 5 & -1 \end{bmatrix}$. [2]

$|A| = (3)(-1) - (-2)(5) = 7 \neq 0$

$\text{Adj } A = \begin{bmatrix} -1 & 2 \\ -5 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & 2 \\ -5 & 3 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & 2 \\ -5 & 3 \end{bmatrix}$

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8. If $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & 3 \\ 2 & -2 & -2 \end{bmatrix}$, find A^{-1} using the adjoint. [5]

Cofactors:

$A_{11} = 8,\ A_{12} = 8,\ A_{13} = 0$

$A_{21} = 0,\ A_{22} = -2,\ A_{23} = 2$

$A_{31} = 8,\ A_{32} = 5,\ A_{33} = -1$

$|A| = -1(8) + 2(8) + 2(0) = 8 \neq 0$

$\text{Adj } A = \begin{bmatrix} 8 & 0 & 8 \\ 8 & -2 & 5 \\ 0 & 2 & -1 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{8}\begin{bmatrix} 8 & 0 & 8 \\ 8 & -2 & 5 \\ 0 & 2 & -1 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{8}\begin{bmatrix} 8 & 0 & 8 \\ 8 & -2 & 5 \\ 0 & 2 & -1 \end{bmatrix}$

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9. Write the inverse of the diagonal matrix $\begin{bmatrix} -2 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 4 \end{bmatrix}$. [1]

$\text{diag}(a, b, c)^{-1} = \text{diag}\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

$= \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

Ans: $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

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10. Show that $A = \frac{1}{3}\begin{bmatrix} 2 & -2 & -1 \\ -1 & -2 & 2 \\ 2 & 1 & 2 \end{bmatrix}$ is orthogonal and hence find A^{-1}. [3]

$A^T = \frac{1}{3}\begin{bmatrix} 2 & -1 & 2 \\ -2 & -2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

So A is orthogonal, and for an orthogonal matrix $A^{-1} = A^T$.

$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 & 2 \\ -2 & -2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Ans: $A^{-1} = A^T = \frac{1}{3}\begin{bmatrix} 2 & -1 & 2 \\ -2 & -2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

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11. If $A = \begin{bmatrix} -1 & 1 \\ -4 & 7 \end{bmatrix}$, find $\text{Adj}\, A$ and show that $(\text{Adj}\, A)A = |A|\, I$. [2]

$\text{Adj}\, A = \begin{bmatrix} 7 & -1 \\ 4 & -1 \end{bmatrix}$

$|A| = -3$

$A(\text{Adj}\, A) = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} = -3I$

Ans: $\text{Adj}\, A = \begin{bmatrix} 7 & -1 \\ 4 & -1 \end{bmatrix}$**; the product is** $-3I$

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12. Find the adjoint of $A = \begin{bmatrix} 0 & -2 & -1 \\ -2 & 4 & 1 \\ 3 & -1 & -1 \end{bmatrix}$ and show that $A(\text{Adj}\, A) = |A|\, I$. [5]

Cofactors:

$A_{11} = -3,\ A_{12} = 1,\ A_{13} = -10$

$A_{21} = -1,\ A_{22} = 3,\ A_{23} = -6$

$A_{31} = 2,\ A_{32} = 2,\ A_{33} = -4$

$\text{Adj}\, A = \begin{bmatrix} -3 & -1 & 2 \\ 1 & 3 & 2 \\ -10 & -6 & -4 \end{bmatrix}$

$|A| = 0(-3) + (-2)(1) + (-1)(-10) = 8$

$A(\text{Adj}\, A) = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$

$(\text{Adj}\, A)A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$

Ans: $\text{Adj}\, A = \begin{bmatrix} -3 & -1 & 2 \\ 1 & 3 & 2 \\ -10 & -6 & -4 \end{bmatrix}$**; both products equal** $8I$

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13. A is a 3×3 matrix with $|A| = -4$. Find $|\operatorname{Adj} A|$. [1]

$|\operatorname{Adj} A| = |A|^{n-1}$

$= (-4)^2 = 16$

Ans: 16

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14. For $A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, find $\operatorname{Adj} A$ and verify that $|\operatorname{Adj} A| = |A|^2$. [5]

Cofactors:

$A_{11} = 3,\ A_{12} = 1,\ A_{13} = -3$

$A_{21} = -1,\ A_{22} = -2,\ A_{23} = 1$

$A_{31} = 1,\ A_{32} = 2,\ A_{33} = -6$

$\operatorname{Adj} A = \begin{bmatrix} 3 & -1 & 1 \\ 1 & -2 & 2 \\ -3 & 1 & -6 \end{bmatrix}$

$|A| = -2(3) + 1(1) + 0(-3) = -5$

$|\operatorname{Adj} A| = \begin{vmatrix} 3 & -1 & 1 \\ 1 & -2 & 2 \\ -3 & 1 & -6 \end{vmatrix} = 25$

$|A|^2 = (-5)^2 = 25$

So $|\operatorname{Adj} A| = |A|^2$.

Ans: $|A| = -5,\ |\operatorname{Adj} A| = 25 = |A|^2$

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15. A is a non-singular matrix of order 3 with $|A| = 2$. Express $\operatorname{Adj}(\operatorname{Adj} A)$ in terms of A. [1]

$\operatorname{Adj}(\operatorname{Adj} A) = |A|^{n-2} A$

$= (2)^1 A = 2A$

Ans: $2A$

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16. If $A = \begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$, find $(\operatorname{Adj} A)^{-1}$. [2]

$(\operatorname{Adj} A)^{-1} = \frac{1}{|A|} A$

$|A| = 2$

$(\operatorname{Adj} A)^{-1} = \frac{1}{2} \begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$

Ans: $\frac{1}{2} \begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$

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17. Find S_1, S_2, S_3 for $A = \begin{bmatrix} -2 & -1 & -2 \\ -1 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ and write its characteristic equation. [3]

$S_1 = -2 + 1 + 0 = -1$

$S_2 = \begin{vmatrix} -2 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -2 \\ 3 & 0 \end{vmatrix} = 1$

$S_3 = |A| = 11$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 + \lambda^2 + \lambda - 11 = 0$

Ans: $\lambda^3 + \lambda^2 + \lambda - 11 = 0$

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18. If a matrix A satisfies $A^2 - 2A + 3I = O$, express A^{-1} in terms of A and I. [2]

Multiply by A^{-1}: $A - 2I + 3A^{-1} = O$

$A^{-1} = \frac{1}{3}(2I - A)$

Ans: $A^{-1} = \frac{1}{3}(2I - A)$

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19. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 2 \end{bmatrix}$, show that $A^2 - 5A - 8I = O$. Use it to find A^{-1}. [3]

$A^2 = \begin{bmatrix} 23 & 35 \\ 10 & 18 \end{bmatrix}$

$A^2 - 5A - 8I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Multiply by A^{-1}: $A - 5I - 8A^{-1} = O$

$A^{-1} = -\frac{1}{8}(5I - A) = \frac{1}{8}\begin{bmatrix} -2 & 7 \\ 2 & -3 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{8}\begin{bmatrix} -2 & 7 \\ 2 & -3 \end{bmatrix}$

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20. Find the characteristic equation of $A = \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and use it to find A^{-1}. [5]

$S_1 = 1,\ S_2 = \begin{vmatrix} -1 & -1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ 3 & 0 \end{vmatrix} = 4,\ S_3 = |A| = 4$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0:\ \lambda^3 - \lambda^2 + 4\lambda - 4 = 0$

$A^3 - A^2 + 4A - 4I = O$

$A^{-1} = \frac{1}{S_3}(A^2 - S_1A + S_2I) = \frac{1}{4}(A^2 - A + 4I)$

$A^2 = \begin{bmatrix} -4 & 0 & 0 \\ 5 & 1 & 0 \\ -5 & -5 & -4 \end{bmatrix}$

$A^2 - A + 4I = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ -8 & -4 & 0 \end{bmatrix}$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ -8 & -4 & 0 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ -8 & -4 & 0 \end{bmatrix}$