EXPRESSWALA ACADEMY

Maths 1A: Inverse of a Matrix

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \begin{bmatrix} 4 & 3 & -1 \\ -3 & -4 & 0 \\ 0 & -2 & -1 \end{bmatrix}$, find the cofactor A_{12}. [1 mark]

2. If $A = \begin{bmatrix} 3 & 4 & -1 \\ -3 & -3 & 4 \\ -3 & -1 & 3 \end{bmatrix}$, find $\operatorname{Adj} A$. [3 marks]

3. A is a 3×3 matrix with $|A| = 8$. Find $|A^{-1}|$. [1 mark]

4. If $A^{-1} = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$, find $(AB)^{-1}$. [2 marks]

5. Find the values of x for which $\begin{bmatrix} x & -5 \\ -5 & x \end{bmatrix}$ is without an inverse. [2 marks]

6. Write down the inverse of $\begin{bmatrix} 3 & -1 \\ 7 & -2 \end{bmatrix}$. [1 mark]

7. Show that $A = \begin{bmatrix} 2 & -1 \\ 3 & -5 \end{bmatrix}$ is non-singular and find A^{-1}. [2 marks]

8. If $A = \begin{bmatrix} -2 & 3 & -1 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} using the adjoint. [5 marks]

9. Write the inverse of the diagonal matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 4 \end{bmatrix}$. [1 mark]

10. Show that $A = \frac{1}{13}\begin{bmatrix} 12 & 5 \\ -5 & 12 \end{bmatrix}$ is orthogonal and hence find A^{-1}. [3 marks]

11. If $A = \begin{bmatrix} -4 & 6 \\ -4 & 7 \end{bmatrix}$, find $\operatorname{Adj} A$ and show that $(\operatorname{Adj} A)A = |A|\, I$. [2 marks]

12. Find the adjoint of $A = \begin{bmatrix} -2 & 0 & 2 \\ 4 & 0 & -1 \\ 3 & 1 & -2 \end{bmatrix}$ and show that $A(\operatorname{Adj} A) = |A|\, I$. [5 marks]

13. If A is a square matrix of order 3 and $|A| = 5$, find $|\operatorname{Adj} A|$. [1 mark]

14. For $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $\operatorname{Adj} A$ and verify that $|\operatorname{Adj} A| = |A|^2$. [5 marks]

15. A is a non-singular matrix of order 4 with $|A| = 3$. Express $\operatorname{Adj}(\operatorname{Adj} A)$ in terms of A. [1 mark]

16. If $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$, find $\operatorname{Adj}(\operatorname{Adj} A)$ without finding $\operatorname{Adj} A$. [2 marks]

17. Find the characteristic equation of $A = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 3 & 2 \\ -1 & -2 & 4 \end{bmatrix}$. [3 marks]

18. Find the numbers p and q such that $A^2 + pA + qI = O$ for $A = \begin{bmatrix} 6 & 4 \\ 7 & 7 \end{bmatrix}$. [2 marks]

19. Show that $A = \begin{bmatrix} 6 & 2 \\ -2 & -1 \end{bmatrix}$ satisfies $A^2 - 5A - 2I = O$ and hence find A^{-1}. [3 marks]

20. Using the characteristic equation, find the inverse of $A = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 3 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. [5 marks]

Key sheet

Q	Answer
1	-3
2	$\operatorname{Adj} A = \begin{bmatrix} -5 & -11 & 13 \\ -3 & 6 & -9 \\ -6 & -9 & 3 \end{bmatrix}$
3	$\frac{1}{8}$
4	$(AB)^{-1} = \begin{bmatrix} 1 & -5 \\ 0 & -1 \end{bmatrix}$
5	$x = \pm 5$
6	$\begin{bmatrix} -2 & 1 \\ -7 & 3 \end{bmatrix}$
7	$A^{-1} = -\frac{1}{7}\begin{bmatrix} -5 & 1 \\ -3 & 2 \end{bmatrix}$
8	$A^{-1} = \frac{1}{8}\begin{bmatrix} -2 & -2 & -4 \\ 2 & -2 & 0 \\ 2 & -2 & 8 \end{bmatrix}$
9	$\begin{bmatrix} \frac{1}{7} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$
10	$A^{-1} = A^{T} = \frac{1}{13}\begin{bmatrix} 12 & -5 \\ 5 & 12 \end{bmatrix}$
11	$\operatorname{Adj} A = \begin{bmatrix} 7 & -6 \\ 4 & -4 \end{bmatrix}$; the product is $-4I$
12	$\operatorname{Adj} A = \begin{bmatrix} 1 & 2 & 0 \\ 5 & -2 & 6 \\ 4 & 2 & 0 \end{bmatrix}$; both products equal $6I$
13	25
14	$\lvert A\rvert = -6,\ \lvert\operatorname{Adj} A\rvert = 36 = \lvert A\rvert^2$
15	$9A$
16	$\begin{bmatrix} 8 & -4 & 8 \\ -4 & 4 & 12 \\ 4 & -4 & 4 \end{bmatrix}$
17	$\lambda^3 - 10\lambda^2 + 33\lambda - 37 = 0$
18	$p = -13,\ q = 14$
19	$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ -2 & -6 \end{bmatrix}$
20	$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 2 & -2 \\ -1 & -1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

Solutions

1. If $A = \begin{bmatrix} 4 & 3 & -1 \\ -3 & -4 & 0 \\ 0 & -2 & -1 \end{bmatrix}$, find the cofactor A_{12}. [1]

$\text{Minor} = \begin{vmatrix} -3 & 0 \\ 0 & -1 \end{vmatrix} = 3$

$A_{12} = (-1)^{1+2}(3) = -3$

Ans: -3

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2. If $A = \begin{bmatrix} 3 & 4 & -1 \\ -3 & -3 & 4 \\ -3 & -1 & 3 \end{bmatrix}$, find $\text{Adj } A$. [3]

The cofactors are

$A_{11} = -5,\ A_{12} = -3,\ A_{13} = -6$

$A_{21} = -11,\ A_{22} = 6,\ A_{23} = -9$

$A_{31} = 13,\ A_{32} = -9,\ A_{33} = 3$

The cofactors of each row form a column of the adjoint:

$\text{Adj } A = \begin{bmatrix} -5 & -11 & 13 \\ -3 & 6 & -9 \\ -6 & -9 & 3 \end{bmatrix}$

Ans: $\text{Adj } A = \begin{bmatrix} -5 & -11 & 13 \\ -3 & 6 & -9 \\ -6 & -9 & 3 \end{bmatrix}$

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3. A is a 3×3 matrix with $|A| = 8$. Find $|A^{-1}|$. [1]

$|A^{-1}| = \frac{1}{|A|}$

$= \frac{1}{8}$

Ans: $\frac{1}{8}$

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4. If $A^{-1} = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$, find $(AB)^{-1}$. [2]

$(AB)^{-1} = B^{-1}A^{-1}$

$= \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -5 \\ 0 & -1 \end{bmatrix}$

Ans: $(AB)^{-1} = \begin{bmatrix} 1 & -5 \\ 0 & -1 \end{bmatrix}$

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5. Find the values of x for which $\begin{bmatrix} x & -5 \\ -5 & x \end{bmatrix}$ is without an inverse. [2]

$|A| = x^2 - 25 = 0$

$x = \pm 5$

Ans: $x = \pm 5$

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6. Write down the inverse of $\begin{bmatrix} 3 & -1 \\ 7 & -2 \end{bmatrix}$. [1]

$|A| = 1$

Swap the diagonal entries, change the signs of the other two, divide by 1.

$A^{-1} = \begin{bmatrix} -2 & 1 \\ -7 & 3 \end{bmatrix}$

Ans: $\begin{bmatrix} -2 & 1 \\ -7 & 3 \end{bmatrix}$

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7. Show that $A = \begin{bmatrix} 2 & -1 \\ 3 & -5 \end{bmatrix}$ is non-singular and find A^{-1}. [2]

$|A| = (2)(-5) - (-1)(3) = -7 \neq 0$

$\text{Adj}\, A = \begin{bmatrix} -5 & 1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = -\frac{1}{7}\begin{bmatrix} -5 & 1 \\ -3 & 2 \end{bmatrix}$

Ans: $A^{-1} = -\frac{1}{7}\begin{bmatrix} -5 & 1 \\ -3 & 2 \end{bmatrix}$

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8. If $A = \begin{bmatrix} -2 & 3 & -1 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1} using the adjoint. [5]

Cofactors:

$A_{11} = -2,\ A_{12} = 2,\ A_{13} = 2$

$A_{21} = -2,\ A_{22} = -2,\ A_{23} = -2$

$A_{31} = -4,\ A_{32} = 0,\ A_{33} = 8$

$|A| = -2(-2) + 3(2) + (-1)(2) = 8 \neq 0$

$\text{Adj}\, A = \begin{bmatrix} -2 & -2 & -4 \\ 2 & -2 & 0 \\ 2 & -2 & 8 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj}\, A}{|A|} = \frac{1}{8}\begin{bmatrix} -2 & -2 & -4 \\ 2 & -2 & 0 \\ 2 & -2 & 8 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{8}\begin{bmatrix} -2 & -2 & -4 \\ 2 & -2 & 0 \\ 2 & -2 & 8 \end{bmatrix}$

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9. Write the inverse of the diagonal matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 4 \end{bmatrix}$. [1]

$\text{diag}(a, b, c)^{-1} = \text{diag}\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$

$= \begin{bmatrix} \frac{1}{7} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

Ans: $\begin{bmatrix} \frac{1}{7} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

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10. Show that $A = \frac{1}{13}\begin{bmatrix} 12 & 5 \\ -5 & 12 \end{bmatrix}$ is orthogonal and hence find A^{-1}. [3]

$A^T = \frac{1}{13}\begin{bmatrix} 12 & -5 \\ 5 & 12 \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

So A is orthogonal, and for an orthogonal matrix $A^{-1} = A^T$.

$A^{-1} = \frac{1}{13}\begin{bmatrix} 12 & -5 \\ 5 & 12 \end{bmatrix}$

Ans: $A^{-1} = A^T = \frac{1}{13}\begin{bmatrix} 12 & -5 \\ 5 & 12 \end{bmatrix}$

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11. If $A = \begin{bmatrix} -4 & 6 \\ -4 & 7 \end{bmatrix}$, find $\operatorname{Adj} A$ and show that $(\operatorname{Adj} A)A = |A|\, I$. [2]

$\operatorname{Adj} A = \begin{bmatrix} 7 & -6 \\ 4 & -4 \end{bmatrix}$

$|A| = -4$

$A(\operatorname{Adj} A) = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

Ans: $\operatorname{Adj} A = \begin{bmatrix} 7 & -6 \\ 4 & -4 \end{bmatrix}$; **the product is** $-4I$

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12. Find the adjoint of $A = \begin{bmatrix} -2 & 0 & 2 \\ 4 & 0 & -1 \\ 3 & 1 & -2 \end{bmatrix}$ and show that $A(\operatorname{Adj} A) = |A|\, I$. [5]

Cofactors:

$A_{11} = 1,\ A_{12} = 5,\ A_{13} = 4$

$A_{21} = 2,\ A_{22} = -2,\ A_{23} = 2$

$A_{31} = 0,\ A_{32} = 6,\ A_{33} = 0$

$\operatorname{Adj} A = \begin{bmatrix} 1 & 2 & 0 \\ 5 & -2 & 6 \\ 4 & 2 & 0 \end{bmatrix}$

$|A| = -2(1) + 0(5) + 2(4) = 6$

$A(\operatorname{Adj} A) = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$

$(\operatorname{Adj} A)A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$

Ans: $\operatorname{Adj} A = \begin{bmatrix} 1 & 2 & 0 \\ 5 & -2 & 6 \\ 4 & 2 & 0 \end{bmatrix}$; **both products equal** $6I$

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13. If A is a square matrix of order 3 and $|A| = 5$, find $|\operatorname{Adj} A|$. [1]

$|\operatorname{Adj} A| = |A|^{n-1}$

$= (5)^2 = 25$

Ans: 25

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14. For $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $\text{Adj}\, A$ and verify that $|\text{Adj}\, A| = |A|^2$. [5]

Cofactors:

$A_{11} = -1,\ A_{12} = -1,\ A_{13} = -2$

$A_{21} = -4,\ A_{22} = 2,\ A_{23} = 4$

$A_{31} = 3,\ A_{32} = 3,\ A_{33} = 0$

$\text{Adj}\, A = \begin{bmatrix} -1 & -4 & 3 \\ -1 & 2 & 3 \\ -2 & 4 & 0 \end{bmatrix}$

$|A| = 2(-1) + (-2)(-1) + 3(-2) = -6$

$|\text{Adj}\, A| = \begin{vmatrix} -1 & -4 & 3 \\ -1 & 2 & 3 \\ -2 & 4 & 0 \end{vmatrix} = 36$

$|A|^2 = (-6)^2 = 36$

So $|\text{Adj}\, A| = |A|^2$.

Ans: $|A| = -6,\ |\text{Adj}\, A| = 36 = |A|^2$

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15. A is a non-singular matrix of order 4 with $|A| = 3$. Express $\text{Adj}(\text{Adj}\, A)$ in terms of A. [1]

$\text{Adj}(\text{Adj}\, A) = |A|^{n-2} A$

$= (3)^2 A = 9A$

Ans: $9A$

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16. If $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$, find $\text{Adj}(\text{Adj}\, A)$ without finding $\text{Adj}\, A$. [2]

$\text{Adj}(\text{Adj}\, A) = |A|^{n-2} A = |A|\, A$ for $n = 3$

$|A| = 2(4) + (-1)(4) + 2(0) = 4$

$\text{Adj}(\text{Adj}\, A) = \begin{bmatrix} 8 & -4 & 8 \\ -4 & 4 & 12 \\ 4 & -4 & 4 \end{bmatrix}$

Ans: $\begin{bmatrix} 8 & -4 & 8 \\ -4 & 4 & 12 \\ 4 & -4 & 4 \end{bmatrix}$

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17. Find the characteristic equation of $A = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 3 & 2 \\ -1 & -2 & 4 \end{bmatrix}$. [3]

$S_1 = 3 + 3 + 4 = 10$

$S_2 = \begin{vmatrix} 3 & 1 \\ 3 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ -2 & 4 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 4 \end{vmatrix} = 33$

$S_3 = |A| = 37$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 10\lambda^2 + 33\lambda - 37 = 0$

Ans: $\lambda^3 - 10\lambda^2 + 33\lambda - 37 = 0$

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18. Find the numbers p and q such that $A^2 + pA + qI = O$ for $A = \begin{bmatrix} 6 & 4 \\ 7 & 7 \end{bmatrix}$. [2]

$A^2 - (a+d)A + (ad - bc)I = O$

$a + d = 13,\ ad - bc = 14$

$p = -13,\ q = 14$

Ans: $p = -13,\ q = 14$

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19. Show that $A = \begin{bmatrix} 6 & 2 \\ -2 & -1 \end{bmatrix}$ satisfies $A^2 - 5A - 2I = O$ and hence find A^{-1}. [3]

$A^2 = \begin{bmatrix} 32 & 10 \\ -10 & -3 \end{bmatrix}$

$A^2 - 5A - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Multiply by A^{-1}: $A - 5I - 2A^{-1} = O$

$A^{-1} = -\frac{1}{2}(5I - A) = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ -2 & -6 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ -2 & -6 \end{bmatrix}$

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20. Using the characteristic equation, find the inverse of $A = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 3 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. [5]

$S_1 = 3,\ S_2 = \begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 3,\ S_3 = |A| = 2$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0 :\ \lambda^3 - 3\lambda^2 + 3\lambda - 2 = 0$

$A^3 - 3A^2 + 3A - 2I = O$

$A^{-1} = \frac{1}{S_3}(A^2 - S_1 A + S_2 I) = \frac{1}{2}(A^2 - 3A + 3I)$

$A^2 = \begin{bmatrix} -1 & -1 & 1 \\ 5 & 5 & -1 \\ 4 & 5 & -1 \end{bmatrix}$

$A^2 - 3A + 3I = \begin{bmatrix} 2 & 2 & -2 \\ -1 & -1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 2 & -2 \\ -1 & -1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

Ans: $A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 2 & -2 \\ -1 & -1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$