EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 09

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\tanh(\ln \frac{5}{2})$.

A. $-\frac{21}{29}$ **B.** $\frac{21}{10}$ **C.** $\frac{21}{29}$ **D.** $\frac{29}{10}$

2. Find $\operatorname{cosech}(\ln \frac{4}{3})$.

A. $\frac{7}{24}$ **B.** $-\frac{24}{7}$ **C.** $\frac{25}{24}$ **D.** $\frac{24}{7}$

3. Find $\cosh(\ln 5)$.

A. $-\frac{13}{5}$ **B.** $\frac{26}{5}$ **C.** $\frac{13}{5}$ **D.** $\frac{24}{5}$

4. Find the range of $f(x) = 5 - 5\cosh x$.

A. $(-\infty, 0]$ **B.** $[0, \infty)$ **C.** $(-\infty, 5]$ **D.** $(-\infty, 0)$

5. The function $f(x) = \sinh^7 x + x\cosh 4x$ is

A. even **B.** odd

C. neither even nor odd **D.** both even and odd

6. Find the range of $f(x) = 7\tanh x - 2$.

A. $(-3, -1)$ **B.** $(-7, 7)$ **C.** $[-9, 5]$ **D.** $(-9, 5)$

7. If $\tanh x = \frac{12}{13}$, find $\coth x$.

A. $\frac{13}{12}$ **B.** $\frac{1}{13}$ **C.** $\frac{12}{13}$ **D.** $-\frac{13}{12}$

8. If $\cosh x = \frac{17}{15}$ and x is positive, find $\sinh x$.

A. $\frac{15}{17}$ **B.** $\frac{8}{15}$ **C.** $\frac{\sqrt{514}}{15}$ **D.** $-\frac{8}{15}$

9. If $\cosh x = \frac{17}{8}$ and x is negative, find $\sinh x$.

A. $\frac{\sqrt{353}}{8}$ **B.** $\frac{15}{8}$ **C.** $-\frac{15}{8}$ **D.** $\frac{8}{17}$

10. If $\sinh x = \frac{5}{12}$ and $\cosh y = \frac{37}{12}$, with $x, y > 0$, find $\sinh(x + y)$.

A. $\frac{10}{3}$ **B.** $\frac{41}{9}$ **C.** $-\frac{15}{8}$ **D.** $\frac{40}{9}$

11. If $\cosh x = \frac{17}{15}$ and $\sinh y = \frac{20}{21}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{37}{35}$ **B.** $\frac{653}{315}$ **C.** $\frac{12}{35}$ **D.** $-\frac{26}{105}$

12. If $\sinh x = \frac{8}{15}$ and $\cosh y = \frac{29}{20}$, with $x, y > 0$, find $\sinh(x+y)$.

A. $\frac{589}{300}$ **B.** $\frac{19}{12}$ **C.** $\frac{661}{300}$ **D.** $-\frac{5}{12}$

13. If $\cosh x = \frac{29}{21}$, find $\cosh 2x$.

A. $\frac{2123}{441}$ **B.** $\frac{1241}{441}$ **C.** $\frac{37}{21}$ **D.** $-\frac{1241}{441}$

14. If $\tanh x = \frac{1}{2}$, find $\cosh 2x$.

A. $\frac{3}{5}$ **B.** $\frac{4}{3}$ **C.** $\frac{5}{4}$ **D.** $\frac{5}{3}$

15. If $\sinh x = \frac{1}{3}$, find $\sinh 3x$.

A. $\frac{31}{27}$ **B.** 1 **C.** $\frac{23}{27}$ **D.** $-\frac{23}{27}$

16. Find $(\cosh x + \sinh x)^2$ when $x = \ln 3$.

A. $\frac{1}{9}$ **B.** 9 **C.** 6 **D.** 18

17. Find the coefficient of x^3 in the series for $\sinh x$.

A. $-\frac{1}{6}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{2}$

18. If $\sinh x = \frac{4}{3}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

A. $\frac{16}{25}$ **B.** $-\frac{112}{75}$ **C.** $\frac{112}{75}$ **D.** $\frac{464}{225}$

19. Express $\sinh 7x - \sinh 3x$ as a product.

A. $-2\cosh 5x \sinh 2x$ **B.** $2\sinh 5x \cosh 2x$

C. $2\sinh 5x \sinh 2x$ **D.** $2\cosh 5x \sinh 2x$

20. If $\sinh x = \frac{7}{24}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

A. $\frac{319}{1600}$ **B.** $\frac{841}{14400}$ **C.** $\frac{5321}{14400}$ **D.** $-\frac{319}{1600}$

21. Find $\operatorname{sech}^{-1}\left(\frac{20}{29}\right)$.

A. $\log_e \frac{5}{2}$ **B.** $-\log_e \frac{5}{2}$

C. $\frac{\log_e \frac{5}{2}}{2}$ **D.** $\log_e 5$

22. Find $\tanh^{-1}\left(\frac{1}{4}\right)$.

A. $\frac{\log_e \frac{5}{4}}{2}$ **B.** $\log_e \frac{5}{3}$

C. $\frac{\log_e \frac{3}{5}}{2}$ **D.** $\frac{\log_e \frac{5}{3}}{2}$

23. Find $\sinh^{-1}\left(\frac{3}{4}\right)$.

A. $-\log_e 2$ **B.** $\log_e 2$ **C.** $\log_e 4$ **D.** $\frac{\log_e 2}{2}$

24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{9}\right)$.

A. $(0, 9)$ **B.** $[0, 9]$ **C.** $(0, 9]$ **D.** $[-9, 9]$

25. Express $\cosh^{-1}\left(\frac{29}{21}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

A. $\sinh^{-1}\left(\frac{400}{441}\right) = \operatorname{sech}^{-1}\left(\frac{29}{21}\right)$

B. $\sinh^{-1}\left(\frac{\sqrt{1282}}{21}\right) = \operatorname{sech}^{-1}\left(\frac{21}{29}\right)$

C. $\sinh^{-1}\left(\frac{20}{21}\right) = \operatorname{sech}^{-1}\left(\frac{29}{21}\right)$

D. $\sinh^{-1}\left(\frac{20}{21}\right) = \operatorname{sech}^{-1}\left(\frac{21}{29}\right)$

Key sheet

1. C	**2.** D	**3.** C	**4.** A	**5.** B
6. D	**7.** A	**8.** B	**9.** C	**10.** D
11. A	**12.** A	**13.** B	**14.** D	**15.** A
16. B	**17.** C	**18.** C	**19.** D	**20.** D
21. A	**22.** D	**23.** B	**24.** C	**25.** D

Solutions

1. Find $\tanh(\ln \frac{5}{2})$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(\ln \frac{5}{2}) = \frac{\frac{5}{2} - \frac{2}{5}}{\frac{5}{2} + \frac{2}{5}} = \frac{21}{29}$

Ans: (C) $\frac{21}{29}$

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2. Find $\operatorname{cosech}(\ln \frac{4}{3})$.

$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$

$e^{\ln \frac{4}{3}} = \frac{4}{3},\ e^{-\ln \frac{4}{3}} = \frac{3}{4}$

$\operatorname{cosech}(\ln \frac{4}{3}) = \frac{24}{7}$

Ans: (D) $\frac{24}{7}$

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3. Find $\cosh(\ln 5)$.

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\cosh(\ln 5) = \frac{5 + \frac{1}{5}}{2} = \frac{13}{5}$

Ans: (C) $\frac{13}{5}$

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4. Find the range of $f(x) = 5 - 5\cosh x$.

$\cosh x \geq 1$, so $-5\cosh x \leq -5$.

$f(x) \leq 5 - 5 = 0$

Range $= (-\infty, 0]$

Ans: (A) $(-\infty, 0]$

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5. The function $f(x) = \sinh^7 x + x\cosh 4x$ is

cosh and sech are even; sinh and tanh are odd.

Replacing x by $-x$: $f(-x) = -f(x)$, so f is odd.

Ans: (B) odd

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6. Find the range of $f(x) = 7\tanh x - 2$.

$-1 < \tanh x < 1$.

$-2 - 7 < f(x) < -2 + 7$

Range $= (-9, 5)$

Ans: (D) $(-9, 5)$

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7. If $\tanh x = \frac{12}{13}$, find $\coth x$.

$\coth x = \frac{1}{\tanh x} = \frac{13}{12}$

Ans: (A) $\frac{13}{12}$

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8. If $\cosh x = \frac{17}{15}$ and x is positive, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{289}{225} - 1 = \frac{64}{225}$

x is positive, so $\sinh x$ is positive: $\sinh x = \frac{8}{15}$

Ans: (B) $\frac{8}{15}$

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9. If $\cosh x = \frac{17}{8}$ and x is negative, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{289}{64} - 1 = \frac{225}{64}$

x is negative, so $\sinh x$ is negative: $\sinh x = -\frac{15}{8}$

Ans: (C) $-\frac{15}{8}$

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10. If $\sinh x = \frac{5}{12}$ and $\cosh y = \frac{37}{12}$, with $x, y > 0$, find $\sinh(x+y)$.

$\cosh x = \sqrt{1 + \frac{25}{144}} = \frac{13}{12}$, $\sinh y = \sqrt{\frac{1369}{144} - 1} = \frac{35}{12}$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$= \frac{5}{12} \cdot \frac{37}{12} + \frac{13}{12} \cdot \frac{35}{12} = \frac{40}{9}$

Ans: (D) $\frac{40}{9}$

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11. If $\cosh x = \frac{17}{15}$ and $\sinh y = \frac{20}{21}$, with $x, y > 0$, find $\cosh(x-y)$.

$\sinh x = \sqrt{\frac{289}{225} - 1} = \frac{8}{15}$, $\cosh y = \sqrt{1 + \frac{400}{441}} = \frac{29}{21}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{17}{15} \cdot \frac{29}{21} - \frac{8}{15} \cdot \frac{20}{21} = \frac{37}{35}$

Ans: (A) $\frac{37}{35}$

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12. If $\sinh x = \frac{8}{15}$ and $\cosh y = \frac{29}{20}$, with $x, y > 0$, find $\sinh(x+y)$.

$\cosh x = \sqrt{1 + \frac{64}{225}} = \frac{17}{15}$, $\sinh y = \sqrt{\frac{841}{400} - 1} = \frac{21}{20}$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$= \frac{8}{15} \cdot \frac{29}{20} + \frac{17}{15} \cdot \frac{21}{20} = \frac{589}{300}$

Ans: (A) $\frac{589}{300}$

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13. If $\cosh x = \frac{29}{21}$, find $\cosh 2x$.

$\cosh 2x = 2\cosh^2 x - 1 = 2 \cdot \frac{841}{441} - 1 = \frac{1241}{441}$

Ans: (B) $\frac{1241}{441}$

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14. If $\tanh x = \frac{1}{2}$, find $\cosh 2x$.

$\cosh 2x = \frac{1+\tanh^2 x}{1-\tanh^2 x} = \frac{1+\frac{1}{4}}{1-\frac{1}{4}} = \frac{5}{3}$

Ans: (D) $\frac{5}{3}$

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15. If $\sinh x = \frac{1}{3}$, find $\sinh 3x$.

$\sinh 3x = 3\sinh x + 4\sinh^3 x$

$= 3(\frac{1}{3}) + 4(\frac{1}{3})^3 = \frac{31}{27}$

Ans: (A) $\frac{31}{27}$

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16. Find $(\cosh x + \sinh x)^2$ when $x = \ln 3$.

$(\cosh x + \sinh x)^2 = e^{2x}$

$= e^{2\ln 3} = 3^2 = 9$

Ans: (B) 9

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17. Find the coefficient of x^3 in the series for $\sinh x$.

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

The coefficient is $\frac{1}{3!} = \frac{1}{6}$.

Ans: (C) $\frac{1}{6}$

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18. If $\sinh x = \frac{4}{3}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{16}{9} - \frac{64}{225} = \frac{112}{75}$

Ans: (C) $\frac{112}{75}$

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19. Express $\sinh 7x - \sinh 3x$ as a product.

$\sinh 7x - \sinh 3x = 2\cosh \frac{7+3}{2}x \ \sinh \frac{7-3}{2}x$

$= 2\cosh 5x \sinh 2x$

Ans: (D) $2\cosh 5x \sinh 2x$

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20. If $\sinh x = \frac{7}{24}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{49}{576} - \frac{64}{225} = -\frac{319}{1600}$

Ans: (D) $-\frac{319}{1600}$

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21. Find $\operatorname{sech}^{-1}\left(\frac{20}{29}\right)$.

$\operatorname{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1 - \frac{400}{841}} = \frac{21}{29}$

$= \log_e \frac{1+\frac{21}{29}}{\frac{20}{29}} = \log_e \frac{5}{2}$

Ans: (A) $\log_e \frac{5}{2}$

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22. Find $\tanh^{-1}\left(\frac{1}{4}\right)$.

$\tanh^{-1} x = \frac{1}{2}\log_e \frac{1+x}{1-x}$

$= \frac{1}{2}\log_e \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{\log_e \frac{5}{3}}{2}$

Ans: (D) $\frac{\log_e \frac{5}{3}}{2}$

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23. Find $\sinh^{-1}\left(\frac{3}{4}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{9}{16} + 1} = \frac{5}{4}$

$\sinh^{-1}\left(\frac{3}{4}\right) = \log_e\left(\frac{3}{4} + \frac{5}{4}\right) = \log_e 2$

Ans: (B) $\log_e 2$

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24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{9}\right)$.

$\operatorname{sech}^{-1} u$ needs $0 < u \leq 1$.

$0 < \frac{x}{9} \leq 1 \Rightarrow 0 < x \leq 9$

Ans: (C) $(0, 9]$

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25. Express $\cosh^{-1}\left(\frac{29}{21}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

$\cosh^{-1} x = \sinh^{-1}\sqrt{x^2-1} = \operatorname{sech}^{-1}\frac{1}{x}$ for $x \geq 1$.

$\sqrt{\frac{841}{441}-1} = \frac{20}{21},\ \frac{1}{\frac{29}{21}} = \frac{21}{29}$

$\cosh^{-1}\left(\frac{29}{21}\right) = \sinh^{-1}\left(\frac{20}{21}\right) = \operatorname{sech}^{-1}\left(\frac{21}{29}\right)$

Ans: (D) $\sinh^{-1}\left(\frac{20}{21}\right) = \operatorname{sech}^{-1}\left(\frac{21}{29}\right)$