EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 08

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(-\ln\frac{5}{3})$.

A. $-\frac{8}{15}$ **B.** $\frac{34}{15}$ **C.** $\frac{16}{15}$ **D.** $\frac{8}{15}$

2. Find $\coth(\ln\frac{3}{2})$.

A. $\frac{5}{12}$ **B.** $-\frac{13}{5}$ **C.** $\frac{5}{13}$ **D.** $\frac{13}{5}$

3. Find $\tanh(\ln 4)$.

A. $-\frac{15}{17}$ **B.** $\frac{17}{4}$ **C.** $\frac{15}{17}$ **D.** $\frac{15}{4}$

4. Find the range of $f(x) = 2\tanh x + 4$.

A. $(-2, 2)$ **B.** $(3, 5)$ **C.** $[2, 6]$ **D.** $(2, 6)$

5. The function $f(x) = x^6\cosh 5x + x\sinh 5x$ is

A. odd **B.** even

C. neither even nor odd **D.** both even and odd

6. Find the range of $f(x) = 4\cosh x + 1$.

A. $(5, \infty)$ **B.** $[-3, 5]$ **C.** $[5, \infty)$ **D.** $[1, \infty)$

7. If $\cosh x = \frac{29}{21}$, find $\operatorname{sech} x$.

A. $\frac{21}{29}$ **B.** $\frac{29}{21}$ **C.** $-\frac{21}{29}$ **D.** $\frac{8}{29}$

8. If $\sinh x = -\frac{20}{21}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = -\frac{29}{21},\ \tanh x = \frac{20}{29}$

B. $\cosh x = \frac{29}{21},\ \tanh x = -\frac{20}{29}$

C. $\cosh x = \frac{\sqrt{41}}{21},\ \tanh x = -\frac{20}{29}$

D. $\cosh x = \frac{29}{21},\ \tanh x = -\frac{29}{20}$

9. If $\tanh x = -\frac{12}{13}$, find $\operatorname{sech} x$.

A. $\frac{13}{12}$ **B.** $\frac{\sqrt{313}}{13}$ **C.** $\frac{5}{13}$ **D.** $-\frac{5}{13}$

10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{7}{24}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{63}{16}$ **B.** $\frac{65}{16}$ **C.** $\frac{33}{8}$ **D.** $\frac{85}{36}$

11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{21}{20}$, with $x, y > 0$, find $\cosh(x+y)$.

A. $\frac{421}{420}$ **B.** $\frac{841}{210}$ **C.** $\frac{1261}{420}$ **D.** $\frac{1189}{420}$

12. If $\cosh x = \frac{17}{8}$ and $\sinh y = \frac{5}{12}$, with $x, y > 0$, find $\cosh(x-y)$.

A. $\frac{73}{48}$ **B.** $\frac{37}{12}$ **C.** $-\frac{55}{48}$ **D.** $\frac{25}{24}$

13. If $\sinh x = \frac{21}{20}$ and $\cosh x = \frac{29}{20}$, find $\cosh 2x$.

A. $\frac{609}{200}$ **B.** $\frac{641}{200}$ **C.** $\frac{29}{10}$ **D.** 1

14. If $\tanh x = \frac{1}{7}$, find $\sinh 2x$.

A. $\frac{7}{24}$ **B.** $\frac{7}{25}$ **C.** $\frac{25}{24}$ **D.** $\frac{2}{7}$

15. If $\tanh x = \frac{3}{4}$, find $\tanh 2x$.

A. $\frac{12}{25}$ **B.** $\frac{24}{25}$ **C.** $\frac{3}{2}$ **D.** $\frac{24}{7}$

16. Find $(\cosh x + \sinh x)^4$ when $x = \ln \frac{3}{2}$.

A. $\frac{81}{16}$ **B.** $\frac{16}{81}$ **C.** 6 **D.** $\frac{81}{8}$

17. If $\tanh x = \frac{1}{7}$, find $\cosh 6x + \sinh 6x$.

A. $\frac{27}{64}$ **B.** $\frac{4096}{729}$ **C.** $\frac{64}{27}$ **D.** $\frac{512}{343}$

18. If $\sinh x = \frac{3}{4}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

A. $\frac{1001}{3600}$ **B.** $\frac{3049}{3600}$ **C.** $\frac{169}{3600}$ **D.** $-\frac{1001}{3600}$

19. Express $\cosh 8x + \cosh 2x$ as a product.

A. $2\cosh 5x \sinh 3x$ **B.** $-2\cosh 5x \cosh 3x$

C. $2\cosh 5x \cosh 3x$ **D.** $2\sinh 5x \cosh 3x$

20. If $\sinh x = \frac{21}{20}$ and $\sinh y = \frac{7}{24}$, find $\sinh(x+y)\sinh(x-y)$.

A. $\frac{17101}{14400}$ **B.** $\frac{8281}{14400}$ **C.** $\frac{14651}{14400}$ **D.** $-\frac{14651}{14400}$

21. Find $\sinh^{-1}\left(-\frac{12}{5}\right)$.

A. $\log_e 5$ **B.** $\log_e 10$ **C.** $\frac{\log_e 5}{2}$ **D.** $-\log_e 5$

22. Find $\sinh^{-1}\left(-\frac{8}{15}\right)$.

A. $\log_e \frac{5}{3}$ **B.** $\log_e \frac{10}{3}$

C. $\frac{\log_e \frac{5}{3}}{2}$ **D.** $-\log_e \frac{5}{3}$

23. Find $\operatorname{cosech}^{-1}\left(-\frac{21}{20}\right)$.

A. $\log_e \frac{14}{3}$ **B.** $\frac{\log_e \frac{7}{3}}{2}$

C. $-\log_e \frac{7}{3}$ **D.** $\log_e \frac{7}{3}$

24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{5}\right)$.

A. $[0, 5]$ **B.** $[-5, 5]$ **C.** $(0, 5)$ **D.** $(0, 5]$

25. The domain of $\operatorname{sech}^{-1} x$ is

A. $(0, 1]$ **B.** $(-1, 1)$ **C.** $[1, \infty)$ **D.** $[0, 1]$

Key sheet

1. A	**2.** D	**3.** C	**4.** D	**5.** B
6. C	**7.** A	**8.** B	**9.** C	**10.** B
11. C	**12.** A	**13.** B	**14.** A	**15.** B
16. A	**17.** C	**18.** A	**19.** C	**20.** C
21. D	**22.** D	**23.** C	**24.** D	**25.** A

Solutions

1. Find $\sinh(-\ln\frac{5}{3})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln\frac{5}{3}} = \frac{5}{3},\ e^{-\ln\frac{5}{3}} = \frac{3}{5}$

$\sinh(-\ln\frac{5}{3}) = -\frac{\frac{5}{3} - \frac{3}{5}}{2} = -\frac{8}{15}$

Ans: (A) $-\frac{8}{15}$

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2. Find $\coth(\ln\frac{3}{2})$.

$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$e^{\ln\frac{3}{2}} = \frac{3}{2},\ e^{-\ln\frac{3}{2}} = \frac{2}{3}$

$\coth(\ln\frac{3}{2}) = \frac{13}{5}$

Ans: (D) $\frac{13}{5}$

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3. Find $\tanh(\ln 4)$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(\ln 4) = \frac{4 - \frac{1}{4}}{4 + \frac{1}{4}} = \frac{15}{17}$

Ans: (C) $\frac{15}{17}$

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4. Find the range of $f(x) = 2\tanh x + 4$.

$-1 < \tanh x < 1.$

$4 - 2 < f(x) < 4 + 2$

Range $= (2, 6)$

Ans: (D) $(2, 6)$

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5. The function $f(x) = x^6\cosh 5x + x\sinh 5x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: $f(-x) = f(x)$, so f is even.

Ans: (B) even

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6. Find the range of $f(x) = 4\cosh x + 1$.

$\cosh x$ has range $[1, \infty)$.

$4\cosh x + 1 \geq 4(1) + 1 = 5$

Range $= [5, \infty)$

Ans: (C) $[5, \infty)$

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7. If $\cosh x = \frac{29}{21}$, find $\operatorname{sech} x$.

$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{21}{29}$

Ans: (A) $\frac{21}{29}$

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8. If $\sinh x = -\frac{20}{21}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{400}{441} = \frac{841}{441}$

Since $\cosh x \geq 1$, $\cosh x = \frac{29}{21}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{-\frac{20}{21}}{\frac{29}{21}} = -\frac{20}{29}$

Ans: (B) $\cosh x = \frac{29}{21},\ \tanh x = -\frac{20}{29}$

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9. If $\tanh x = -\frac{12}{13}$, find $\operatorname{sech} x$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{144}{169} = \frac{25}{169}$

$\operatorname{sech} x > 0$, so $\operatorname{sech} x = \frac{5}{13}$

Ans: (C) $\frac{5}{13}$

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10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{7}{24}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{1369}{144} - 1} = \frac{35}{12},\ \cosh y = \sqrt{1 + \frac{49}{576}} = \frac{25}{24}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{37}{12} \cdot \frac{25}{24} + \frac{35}{12} \cdot \frac{7}{24} = \frac{65}{16}$

Ans: (B) $\frac{65}{16}$

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11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{21}{20}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{841}{441} - 1} = \frac{20}{21},\ \cosh y = \sqrt{1 + \frac{441}{400}} = \frac{29}{20}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{29}{21} \cdot \frac{29}{20} + \frac{20}{21} \cdot \frac{21}{20} = \frac{1261}{420}$

Ans: (C) $\frac{1261}{420}$

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12. If $\cosh x = \frac{17}{8}$ and $\sinh y = \frac{5}{12}$, with $x, y > 0$, find $\cosh(x-y)$.

$\sinh x = \sqrt{\frac{289}{64} - 1} = \frac{15}{8},\ \cosh y = \sqrt{1 + \frac{25}{144}} = \frac{13}{12}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{17}{8} \cdot \frac{13}{12} - \frac{15}{8} \cdot \frac{5}{12} = \frac{73}{48}$

Ans: (A) $\frac{73}{48}$

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13. If $\sinh x = \frac{21}{20}$ and $\cosh x = \frac{29}{20}$, find $\cosh 2x$.

$\cosh 2x = \cosh^2 x + \sinh^2 x = \frac{841}{400} + \frac{441}{400} = \frac{641}{200}$

Ans: (B) $\frac{641}{200}$

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14. If $\tanh x = \frac{1}{7}$, find $\sinh 2x$.

$\sinh 2x = \frac{2\tanh x}{1 - \tanh^2 x} = \frac{\frac{2}{7}}{1 - \frac{1}{49}} = \frac{7}{24}$

Ans: (A) $\frac{7}{24}$

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15. If $\tanh x = \frac{3}{4}$, find $\tanh 2x$.

$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x} = \frac{\frac{3}{2}}{1 + \frac{9}{16}} = \frac{24}{25}$

Ans: (B) $\frac{24}{25}$

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16. Find $(\cosh x + \sinh x)^4$ when $x = \ln \frac{3}{2}$.

$(\cosh x + \sinh x)^4 = e^{4x}$

$= e^{4 \ln \frac{3}{2}} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

Ans: (A) $\frac{81}{16}$

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17. If $\tanh x = \frac{1}{7}$, find $\cosh 6x + \sinh 6x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 3$

$\frac{1+\frac{1}{7}}{1-\frac{1}{7}} = \frac{4}{3}$

$\text{Answer} = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

Ans: (C) $\frac{64}{27}$

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18. If $\sinh x = \frac{3}{4}$ and $\sinh y = \frac{8}{15}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{9}{16} - \frac{64}{225} = \frac{1001}{3600}$

Ans: (A) $\frac{1001}{3600}$

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19. Express $\cosh 8x + \cosh 2x$ as a product.

$\cosh 8x + \cosh 2x = 2\cosh \frac{8+2}{2}x \cosh \frac{8-2}{2}x$

$= 2\cosh 5x \cosh 3x$

Ans: (C) $2\cosh 5x \cosh 3x$

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20. If $\sinh x = \frac{21}{20}$ and $\sinh y = \frac{7}{24}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{441}{400} - \frac{49}{576} = \frac{14651}{14400}$

Ans: (C) $\frac{14651}{14400}$

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21. Find $\sinh^{-1}\left(-\frac{12}{5}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{144}{25}+1} = \frac{13}{5}$

$\sinh^{-1}\left(-\frac{12}{5}\right) = \log_e\left(-\frac{12}{5} + \frac{13}{5}\right) = -\log_e 5$

Ans: (D) $-\log_e 5$

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22. Find $\sinh^{-1}\left(-\frac{8}{15}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{64}{225}+1} = \frac{17}{15}$

$\sinh^{-1}\left(-\frac{8}{15}\right) = \log_e\left(-\frac{8}{15} + \frac{17}{15}\right) = -\log_e \frac{5}{3}$

Ans: (D) $-\log_e \frac{5}{3}$

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23. Find $\operatorname{cosech}^{-1}\left(-\frac{21}{20}\right)$.

$\operatorname{cosech}^{-1} x = \log_e \frac{1-\sqrt{1+x^2}}{x}$ (for $x < 0$)

$\sqrt{1+\frac{441}{400}} = \frac{29}{20}$

$= -\log_e \frac{7}{3}$

Ans: (C) $-\log_e \frac{7}{3}$

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24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{5}\right)$.

$\operatorname{sech}^{-1} u$ needs $0 < u \leq 1$.

$0 < \frac{x}{5} \leq 1 \Rightarrow 0 < x \leq 5$

Ans: (D) $(0, 5]$

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25. The domain of $\operatorname{sech}^{-1} x$ is

From the table of inverse hyperbolic functions: $(0, 1]$.

Ans: (A) $(0, 1]$
