EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 07

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(-\ln\frac{3}{2})$.

A. $\frac{5}{12}$ **B.** $-\frac{5}{12}$ **C.** $\frac{5}{6}$ **D.** $\frac{13}{6}$

2. Find $\cosh(\ln\frac{4}{3})+\sinh(\ln\frac{5}{2})$.

A. $-\frac{1}{120}$ **B.** $\frac{209}{120}$ **C.** $\frac{23}{6}$ **D.** $\frac{251}{120}$

3. Find $\sinh(\ln\frac{3}{2})$.

A. $-\frac{5}{12}$ **B.** $\frac{5}{6}$ **C.** $\frac{5}{12}$ **D.** $\frac{13}{6}$

4. Find the range of $f(x)=4\tanh x-1$.

A. $(-5,3)$ **B.** $(-4,4)$ **C.** $(-2,0)$ **D.** $[-5,3]$

5. The function $f(x)=x\sinh 5x+\tanh 5x$ is

A. even **B.** both even and odd
C. neither even nor odd **D.** odd

6. Find the range of $f(x)=-3\cosh x-1$.

A. $[-4,\infty)$ **B.** $(-\infty,-1]$ **C.** $(-\infty,-4)$ **D.** $(-\infty,-4]$

7. Simplify $\cosh x+\sinh x$ when $x=\ln 4$.

A. 4 **B.** $\frac{1}{4}$ **C.** 8 **D.** 16

8. If $\cosh x=\frac{5}{3}$ and x is positive, find $\sinh x$.

A. $\frac{3}{5}$ **B.** $-\frac{4}{3}$ **C.** $\frac{4}{3}$ **D.** $\frac{\sqrt{34}}{3}$

9. If $\tanh x=-\frac{15}{17}$, find $\operatorname{sech} x$.

A. $\frac{\sqrt{514}}{17}$ **B.** $\frac{17}{15}$ **C.** $\frac{8}{17}$ **D.** $-\frac{8}{17}$

10. If $\sinh x=\frac{3}{4}$ and $\cosh y=\frac{13}{12}$, with $x,y>0$, find $\sinh(x+y)$.

A. $\frac{7}{24}$ **B.** $\frac{5}{3}$ **C.** $\frac{7}{6}$ **D.** $\frac{4}{3}$

11. If $\tanh x=\frac{2}{7}$ and $\tanh y=\frac{1}{5}$, find $\tanh(x+y)$.

A. $\frac{3}{37}$ **B.** $\frac{17}{37}$ **C.** $\frac{17}{35}$ **D.** $\frac{17}{33}$

12. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{15}{8}$, with $x, y > 0$, find $\cosh(x+y)$.

A. $\frac{19}{6}$ **B.** $\frac{265}{96}$ **C.** $\frac{247}{96}$ **D.** $\frac{5}{3}$

13. If $\sinh x = \frac{7}{24}$ and $\cosh x = \frac{25}{24}$, find $\sinh 2x$.

A. $\frac{175}{576}$ **B.** $\frac{7}{12}$ **C.** $\frac{175}{288}$ **D.** $\frac{337}{288}$

14. If $\cosh x = \frac{5}{4}$, find $\cosh 3x$.

A. $\frac{185}{16}$ **B.** $\frac{65}{16}$ **C.** $\frac{15}{4}$ **D.** $-\frac{65}{16}$

15. If $\sinh x = \frac{2}{3}$, find $\sinh 3x$.

A. $\frac{86}{27}$ **B.** $-\frac{22}{27}$ **C.** $\frac{22}{27}$ **D.** 2

16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 4$.

A. $\frac{1}{1024}$ **B.** 20 **C.** 2048 **D.** 1024

17. Find the coefficient of x^6 in the series for $\cosh x$.

A. $\frac{1}{120}$ **B.** $\frac{1}{720}$ **C.** $\frac{1}{6}$ **D.** $-\frac{1}{720}$

18. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{5}{3}$, find $\cosh(x+y)\cosh(x-y)$.

A. $\frac{26569}{3600}$ **B.** $\frac{13969}{3600}$ **C.** $\frac{6031}{3600}$ **D.** $\frac{1169}{3600}$

19. Express $\sinh 8x - \sinh 6x$ as a product.

A. $2\cosh 7x \sinh x$ **B.** $2\sinh 7x \sinh x$

C. $2\sinh 7x \cosh x$ **D.** $-2\cosh 7x \sinh x$

20. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{5}{4}$, find $\cosh(x+y)\cosh(x-y)$.

A. $\frac{2479}{400}$ **B.** $\frac{5329}{400}$ **C.** $-\frac{1679}{400}$ **D.** $\frac{2929}{400}$

21. Find $\tanh^{-1}\left(\frac{2}{5}\right)$.

A. $\frac{\log_e \frac{7}{5}}{2}$ **B.** $\frac{\log_e \frac{3}{7}}{2}$

C. $\log_e \frac{7}{3}$ **D.** $\frac{\log_e \frac{7}{3}}{2}$

22. Find $\tanh^{-1}\left(\frac{2}{3}\right)$.

A. $\frac{\log_e 5}{2}$ **B.** $\log_e 5$

C. $-\frac{\log_e 5}{2}$ **D.** $\frac{\log_e \frac{5}{3}}{2}$

23. Find $\operatorname{cosech}^{-1}\left(\frac{24}{7}\right)$.

A. $\log_e \frac{4}{3}$ **B.** $-\log_e \frac{4}{3}$

C. $\frac{\log_e \frac{4}{3}}{2}$ **D.** $\log_e \frac{8}{3}$

24. Find the domain of $f(x) = \tanh^{-1}\left(\frac{x}{4}\right)$.

A. $(-1, 1)$ **B.** $(-4, 4)$

C. $[-4, 4]$ **D.** $(-\infty, -4) \cup (4, \infty)$

25. Express $\cosh^{-1}\left(\frac{17}{15}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

A. $\sinh^{-1}\left(\frac{8}{15}\right) = \operatorname{sech}^{-1}\left(\frac{15}{17}\right)$

B. $\sinh^{-1}\left(\frac{\sqrt{514}}{15}\right) = \operatorname{sech}^{-1}\left(\frac{15}{17}\right)$

C. $\sinh^{-1}\left(\frac{8}{15}\right) = \operatorname{sech}^{-1}\left(\frac{17}{15}\right)$

D. $\sinh^{-1}\left(\frac{64}{225}\right) = \operatorname{sech}^{-1}\left(\frac{17}{15}\right)$

Key sheet

1. B	**2.** D	**3.** C	**4.** A	**5.** C
6. D	**7.** A	**8.** C	**9.** C	**10.** D
11. B	**12.** B	**13.** C	**14.** B	**15.** A
16. A	**17.** B	**18.** B	**19.** A	**20.** D
21. D	**22.** A	**23.** A	**24.** B	**25.** A

Solutions

1. Find $\sinh(-\ln\frac{3}{2})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln\frac{3}{2}} = \frac{3}{2},\ e^{-\ln\frac{3}{2}} = \frac{2}{3}$

$\sinh(-\ln\frac{3}{2}) = -\frac{\frac{3}{2} - \frac{2}{3}}{2} = -\frac{5}{12}$

Ans: (B) $-\frac{5}{12}$

---

2. Find $\cosh(\ln\frac{4}{3}) + \sinh(\ln\frac{5}{2})$.

$\cosh(\ln\frac{4}{3}) = \frac{\frac{4}{3} + \frac{3}{4}}{2} = \frac{25}{24}$

$\sinh(\ln\frac{5}{2}) = \frac{\frac{5}{2} - \frac{2}{5}}{2} = \frac{21}{20}$

$\text{Sum} = \frac{251}{120}$

Ans: (D) $\frac{251}{120}$

---

3. Find $\sinh(\ln\frac{3}{2})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln\frac{3}{2}} = \frac{3}{2},\ e^{-\ln\frac{3}{2}} = \frac{2}{3}$

$\sinh(\ln\frac{3}{2}) = \frac{\frac{3}{2} - \frac{2}{3}}{2} = \frac{5}{12}$

Ans: (C) $\frac{5}{12}$

---

4. Find the range of $f(x) = 4\tanh x - 1$.

$-1 < \tanh x < 1.$

$-1 - 4 < f(x) < -1 + 4$

Range $= (-5, 3)$

Ans: (A) $(-5, 3)$

---

5. The function $f(x) = x\sinh 5x + \tanh 5x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: one term keeps its sign and the other changes, so f is neither.

Ans: (C) neither even nor odd

---

6. Find the range of $f(x) = -3\cosh x - 1$.

$\cosh x \geq 1$, so $-3\cosh x \leq -3$.

$f(x) \leq -1 - 3 = -4$

Range $= (-\infty, -4]$

Ans: (D) $(-\infty, -4]$

---

7. Simplify $\cosh x + \sinh x$ when $x = \ln 4$.

$\cosh x + \sinh x = e^x$

$e^{\ln 4} = 4$

Ans: (A) 4

---

8. If $\cosh x = \frac{5}{3}$ and x is positive, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{25}{9} - 1 = \frac{16}{9}$

x is positive, so $\sinh x$ is positive: $\sinh x = \frac{4}{3}$

Ans: (C) $\frac{4}{3}$

---

9. If $\tanh x = -\frac{15}{17}$, find $\operatorname{sech} x$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{225}{289} = \frac{64}{289}$

$\operatorname{sech} x > 0$, so $\operatorname{sech} x = \frac{8}{17}$

Ans: (C) $\frac{8}{17}$

---

10. If $\sinh x = \frac{3}{4}$ and $\cosh y = \frac{13}{12}$, with $x, y > 0$, find $\sinh(x+y)$.

$\cosh x = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$, $\sinh y = \sqrt{\frac{169}{144} - 1} = \frac{5}{12}$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$= \frac{3}{4} \cdot \frac{13}{12} + \frac{5}{4} \cdot \frac{5}{12} = \frac{4}{3}$

Ans: (D) $\frac{4}{3}$

---

11. If $\tanh x = \frac{2}{7}$ and $\tanh y = \frac{1}{5}$, find $\tanh(x+y)$.

$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

$= \frac{\frac{2}{7} + \frac{1}{5}}{1 + \frac{2}{35}} = \frac{17}{37}$

Ans: (B) $\frac{17}{37}$

---

12. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{15}{8}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{625}{576} - 1} = \frac{7}{24}$, $\cosh y = \sqrt{1 + \frac{225}{64}} = \frac{17}{8}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{25}{24} \cdot \frac{17}{8} + \frac{7}{24} \cdot \frac{15}{8} = \frac{265}{96}$

Ans: (B) $\frac{265}{96}$

---

13. If $\sinh x = \frac{7}{24}$ and $\cosh x = \frac{25}{24}$, find $\sinh 2x$.

$\sinh 2x = 2 \sinh x \cosh x = 2 \cdot \frac{7}{24} \cdot \frac{25}{24} = \frac{175}{288}$

Ans: (C) $\frac{175}{288}$

---

14. If $\cosh x = \frac{5}{4}$, find $\cosh 3x$.

$\cosh 3x = 4\cosh^3 x - 3\cosh x$

$= 4(\frac{5}{4})^3 - 3(\frac{5}{4}) = \frac{65}{16}$

Ans: (B) $\frac{65}{16}$

---

15. If $\sinh x = \frac{2}{3}$, find $\sinh 3x$.

$\sinh 3x = 3\sinh x + 4\sinh^3 x$

$= 3(\frac{2}{3}) + 4(\frac{2}{3})^3 = \frac{86}{27}$

Ans: (A) $\frac{86}{27}$

---

16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 4$.

$(\cosh x - \sinh x)^5 = e^{-5x}$

$= e^{-5\ln 4} = 4^{-5} = \frac{1}{1024}$

Ans: (A) $\frac{1}{1024}$

---

17. Find the coefficient of x^6 in the series for $\cosh x$.

$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

The coefficient is $\frac{1}{6!} = \frac{1}{720}$.

Ans: (B) $\frac{1}{720}$

---

18. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{5}{3}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{441}{400} + \frac{25}{9} = \frac{13969}{3600}$

Ans: (B) $\frac{13969}{3600}$

---

19. Express $\sinh 8x - \sinh 6x$ as a product.

$\sinh 8x - \sinh 6x = 2\cosh \frac{8+6}{2}x \sinh \frac{8-6}{2}x$

$= 2\cosh 7x \sinh x$

Ans: (A) $2\cosh 7x \sinh x$

---

20. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{5}{4}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{144}{25} + \frac{25}{16} = \frac{2929}{400}$

Ans: (D) $\frac{2929}{400}$

---

21. Find $\tanh^{-1}\left(\frac{2}{5}\right)$.

$\tanh^{-1} x = \frac{1}{2}\log_e \frac{1+x}{1-x}$

$= \frac{1}{2}\log_e \frac{\frac{7}{5}}{\frac{3}{5}} = \frac{\log_e \frac{7}{3}}{2}$

Ans: (D) $\frac{\log_e \frac{7}{3}}{2}$

---

22. Find $\tanh^{-1}\left(\frac{2}{3}\right)$.

$\tanh^{-1} x = \frac{1}{2}\log_e \frac{1+x}{1-x}$

$= \frac{1}{2}\log_e \frac{\frac{5}{3}}{\frac{1}{3}} = \frac{\log_e 5}{2}$

Ans: (A) $\frac{\log_e 5}{2}$

---

23. Find $\operatorname{cosech}^{-1}\left(\frac{24}{7}\right)$.

$\operatorname{cosech}^{-1} x = \log_e \frac{1+\sqrt{1+x^2}}{x}$ (for $x > 0$)

$\sqrt{1 + \frac{576}{49}} = \frac{25}{7}$

$= \log_e \frac{4}{3}$

Ans: (A) $\log_e \frac{4}{3}$

---

24. Find the domain of $f(x) = \tanh^{-1}\left(\frac{x}{4}\right)$.

$\tanh^{-1} u$ needs $-1 < u < 1$.

$-1 < \frac{x}{4} < 1 \Rightarrow -4 < x < 4$

Ans: (B) $(-4, 4)$

---

25. Express $\cosh^{-1}\left(\frac{17}{15}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

$\cosh^{-1} x = \sinh^{-1}\sqrt{x^2-1} = \operatorname{sech}^{-1}\frac{1}{x}$ for $x \geq 1$.

$\sqrt{\frac{289}{225} - 1} = \frac{8}{15}$, $\frac{1}{\frac{17}{15}} = \frac{15}{17}$

$\cosh^{-1}\left(\frac{17}{15}\right) = \sinh^{-1}\left(\frac{8}{15}\right) = \operatorname{sech}^{-1}\left(\frac{15}{17}\right)$

Ans: (A) $\sinh^{-1}\left(\frac{8}{15}\right) = \operatorname{sech}^{-1}\left(\frac{15}{17}\right)$
