EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 06

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(\ln 5)$.

A. $\frac{24}{5}$ **B.** $-\frac{12}{5}$ **C.** $\frac{12}{5}$ **D.** $\frac{26}{5}$

2. Find $\operatorname{cosech}(\ln 6)$.

A. $\frac{12}{35}$ **B.** $-\frac{12}{35}$ **C.** $\frac{35}{12}$ **D.** $\frac{37}{12}$

3. Find $\sinh(\ln \frac{7}{3})$.

A. $\frac{20}{21}$ **B.** $\frac{58}{21}$ **C.** $\frac{40}{21}$ **D.** $-\frac{20}{21}$

4. Find the range of $f(x) = 6\cosh x - 1$.

A. $[-7, 5]$ **B.** $(5, \infty)$ **C.** $[5, \infty)$ **D.** $[-1, \infty)$

5. The function $f(x) = \operatorname{sech} 4x + x^6 \sinh 4x$ is

A. odd **B.** neither even nor odd

C. both even and odd **D.** even

6. Find the range of $f(x) = 5 - 6\cosh x$.

A. $(-\infty, -1]$ **B.** $(-\infty, 5]$ **C.** $[-1, \infty)$ **D.** $(-\infty, -1)$

7. Simplify $\cosh x + \sinh x$ when $x = \ln 2$.

A. 4 **B.** $\frac{5}{4}$ **C.** 2 **D.** $\frac{1}{2}$

8. If $\tanh x = -\frac{3}{5}$, find $\operatorname{sech} x$.

A. $\frac{4}{5}$ **B.** $-\frac{4}{5}$ **C.** $\frac{5}{3}$ **D.** $\frac{\sqrt{34}}{5}$

9. If $\sinh x = \frac{15}{8}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = -\frac{17}{8},\ \tanh x = -\frac{15}{17}$

B. $\cosh x = \frac{17}{8},\ \tanh x = \frac{15}{17}$

C. $\cosh x = \frac{\sqrt{161}}{8},\ \tanh x = \frac{15}{17}$

D. $\cosh x = \frac{17}{8},\ \tanh x = \frac{17}{15}$

10. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{25}{24}$, with $x, y > 0$, find $\sinh(x + y)$.

A. $\frac{409}{120}$ **B.** $\frac{323}{120}$ **C.** $\frac{209}{120}$ **D.** $\frac{391}{120}$

11. If $\tanh x = \frac{3}{5}$ and $\tanh y = \frac{1}{4}$, find $\tanh(x - y)$.

A. $\frac{7}{17}$ **B.** 1 **C.** $\frac{7}{20}$ **D.** $\frac{7}{23}$

12. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{221}{60}$ **B.** $\frac{61}{60}$ **C.** $\frac{187}{60}$ **D.** $\frac{229}{60}$

13. If $\sinh x = \frac{4}{3}$, find $\cosh 2x$.

A. $\frac{11}{3}$ **B.** $\frac{32}{9}$ **C.** $-\frac{23}{9}$ **D.** $\frac{41}{9}$

14. If $\tanh x = \frac{2}{7}$, find $\cosh 2x$.

A. $\frac{53}{45}$ **B.** $\frac{28}{45}$ **C.** $\frac{53}{49}$ **D.** $\frac{45}{53}$

15. If $\tanh x = \frac{3}{4}$, find $\sinh 2x$.

A. $\frac{25}{7}$ **B.** $\frac{3}{2}$ **C.** $\frac{24}{7}$ **D.** $\frac{24}{25}$

16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 10$.

A. 100000 **B.** $\frac{1}{100000}$ **C.** 50 **D.** 200000

17. If $\tanh x = \frac{2}{3}$, find $\cosh 4x + \sinh 4x$.

A. 25 **B.** 625 **C.** $\frac{1}{25}$ **D.** $\frac{25}{9}$

18. If $\sinh x = \frac{3}{4}$ and $\cosh y = \frac{17}{15}$, find $\cosh(x + y)\cosh(x - y)$.

A. $\frac{2599}{3600}$ **B.** $\frac{12769}{3600}$ **C.** $\frac{4601}{3600}$ **D.** $\frac{6649}{3600}$

19. Express $\cosh 7x + \cosh 3x$ as a product.

A. $2\sinh 5x \cosh 2x$ **B.** $2\cosh 5x \cosh 2x$

C. $-2\cosh 5x \cosh 2x$ **D.** $2\cosh 5x \sinh 2x$

20. Express $\cosh 5x + \cosh 1x$ as a product.

A. $-2\cosh 3x \cosh 2x$ **B.** $2\sinh 3x \cosh 2x$

C. $2\cosh 3x \cosh 2x$ **D.** $2\cosh 3x \sinh 2x$

21. Find $\operatorname{sech}^{-1}\left(\frac{5}{13}\right)$.

A. $\log_e 10$ **B.** $\log_e 5$ **C.** $-\log_e 5$ **D.** $\frac{\log_e 5}{2}$

22. Find $\operatorname{cosech}^{-1}\left(\frac{12}{35}\right)$.

A. $\log_e 12$ **B.** $-\log_e 6$ **C.** $\log_e 6$ **D.** $\frac{\log_e 6}{2}$

23. Find $\sinh^{-1}\left(\frac{5}{12}\right)$.

A. $\log_e \frac{3}{2}$ **B.** $-\log_e \frac{3}{2}$

C. $\frac{\log_e \frac{3}{2}}{2}$ **D.** $\log_e 3$

24. Find the domain of $f(x) = \cosh^{-1}(x - 8)$.

A. $[8, \infty)$ **B.** $(9, \infty)$ **C.** $[7, \infty)$ **D.** $[9, \infty)$

25. Express $\sinh^{-1}\left(\frac{4}{3}\right)$ in terms of $\tanh^{-1}$ and of $\cosh^{-1}$.

A. $\tanh^{-1}\left(\frac{16}{25}\right) = \cosh^{-1}\left(\frac{25}{9}\right)$

B. $\tanh^{-1}\left(\frac{4}{5}\right) = \cosh^{-1}\left(\frac{5}{3}\right)$

C. $\tanh^{-1}\left(\frac{4}{7}\right) = \cosh^{-1}\left(\frac{7}{3}\right)$

D. $\tanh^{-1}\left(\frac{5}{4}\right) = \cosh^{-1}\left(\frac{4}{3}\right)$

Key sheet

1. C	**2.** A	**3.** A	**4.** C	**5.** B
6. A	**7.** C	**8.** A	**9.** B	**10.** D
11. A	**12.** D	**13.** D	**14.** A	**15.** C
16. B	**17.** A	**18.** D	**19.** B	**20.** C
21. B	**22.** C	**23.** A	**24.** D	**25.** B

Solutions

1. Find $\sinh(\ln 5)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln 5} = 5,\ e^{-\ln 5} = \frac{1}{5}$

$\sinh(\ln 5) = \frac{5 - \frac{1}{5}}{2} = \frac{12}{5}$

Ans: (C) $\frac{12}{5}$

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2. Find $\operatorname{cosech}(\ln 6)$.

$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$

$e^{\ln 6} = 6,\ e^{-\ln 6} = \frac{1}{6}$

$\operatorname{cosech}(\ln 6) = \frac{12}{35}$

Ans: (A) $\frac{12}{35}$

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3. Find $\sinh(\ln \frac{7}{3})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln \frac{7}{3}} = \frac{7}{3},\ e^{-\ln \frac{7}{3}} = \frac{3}{7}$

$\sinh(\ln \frac{7}{3}) = \frac{\frac{7}{3} - \frac{3}{7}}{2} = \frac{20}{21}$

Ans: (A) $\frac{20}{21}$

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4. Find the range of $f(x) = 6\cosh x - 1$.

$\cosh x$ has range $[1, \infty)$.

$6\cosh x - 1 \geq 6(1) - 1 = 5$

Range $= [5, \infty)$

Ans: (C) $[5, \infty)$

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5. The function $f(x) = \operatorname{sech} 4x + x^6 \sinh 4x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: one term keeps its sign and the other changes, so f is neither.

Ans: (B) neither even nor odd

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6. Find the range of $f(x) = 5 - 6\cosh x$.

$\cosh x \geq 1$, so $-6\cosh x \leq -6$.

$f(x) \leq 5 - 6 = -1$

Range $= (-\infty, -1]$

Ans: (A) $(-\infty, -1]$

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7. Simplify $\cosh x + \sinh x$ when $x = \ln 2$.

$\cosh x + \sinh x = e^x$

$e^{\ln 2} = 2$

Ans: (C) 2

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8. If $\tanh x = -\frac{3}{5}$, find $\operatorname{sech} x$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\operatorname{sech} x > 0$, so $\operatorname{sech} x = \frac{4}{5}$

Ans: (A) $\frac{4}{5}$

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9. If $\sinh x = \frac{15}{8}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{225}{64} = \frac{289}{64}$

Since $\cosh x \geq 1$, $\cosh x = \frac{17}{8}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{15}{8}}{\frac{17}{8}} = \frac{15}{17}$

Ans: (B) $\cosh x = \frac{17}{8},\ \tanh x = \frac{15}{17}$

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10. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{25}{24}$, with $x, y > 0$, find $\sinh(x+y)$.

$\cosh x = \sqrt{1 + \frac{144}{25}} = \frac{13}{5},\ \sinh y = \sqrt{\frac{625}{576} - 1} = \frac{7}{24}$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$= \frac{12}{5} \cdot \frac{25}{24} + \frac{13}{5} \cdot \frac{7}{24} = \frac{391}{120}$

Ans: (D) $\frac{391}{120}$

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11. If $\tanh x = \frac{3}{5}$ and $\tanh y = \frac{1}{4}$, find $\tanh(x-y)$.

$\tanh(x-y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$

$= \frac{\frac{3}{5} - \frac{1}{4}}{1 - \frac{3}{20}} = \frac{7}{17}$

Ans: (A) $\frac{7}{17}$

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12. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{841}{400} - 1} = \frac{21}{20},\ \cosh y = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{29}{20} \cdot \frac{5}{3} + \frac{21}{20} \cdot \frac{4}{3} = \frac{229}{60}$

Ans: (D) $\frac{229}{60}$

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13. If $\sinh x = \frac{4}{3}$, find $\cosh 2x$.

$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2 \cdot \frac{16}{9} = \frac{41}{9}$

Ans: (D) $\frac{41}{9}$

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14. If $\tanh x = \frac{2}{7}$, find $\cosh 2x$.

$\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x} = \frac{1 + \frac{4}{49}}{1 - \frac{4}{49}} = \frac{53}{45}$

Ans: (A) $\frac{53}{45}$

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15. If $\tanh x = \frac{3}{4}$, find $\sinh 2x$.

$\sinh 2x = \frac{2\tanh x}{1 - \tanh^2 x} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{24}{7}$

Ans: (C) $\frac{24}{7}$

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16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 10$.

$(\cosh x - \sinh x)^5 = e^{-5x}$

$= e^{-5\ln 10} = 10^{-5} = \frac{1}{100000}$

Ans: (B) $\frac{1}{100000}$

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17. If $\tanh x = \frac{2}{3}$, find $\cosh 4x + \sinh 4x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 2$

$\frac{1+\frac{2}{3}}{1-\frac{2}{3}} = 5$

$\text{Answer} = (5)^2 = 25$

Ans: (A) 25

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18. If $\sinh x = \frac{3}{4}$ and $\cosh y = \frac{17}{15}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{9}{16} + \frac{289}{225} = \frac{6649}{3600}$

Ans: (D) $\frac{6649}{3600}$

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19. Express $\cosh 7x + \cosh 3x$ as a product.

$\cosh 7x + \cosh 3x = 2\cosh\frac{7+3}{2}x\ \cosh\frac{7-3}{2}x$

$= 2\cosh 5x \cosh 2x$

Ans: (B) $2\cosh 5x \cosh 2x$

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20. Express $\cosh 5x + \cosh 1x$ as a product.

$\cosh 5x + \cosh 1x = 2\cosh\frac{5+1}{2}x\ \cosh\frac{5-1}{2}x$

$= 2\cosh 3x \cosh 2x$

Ans: (C) $2\cosh 3x \cosh 2x$

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21. Find $\text{sech}^{-1}\left(\frac{5}{13}\right)$.

$\text{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1-\frac{25}{169}} = \frac{12}{13}$

$= \log_e \frac{1+\frac{12}{13}}{\frac{5}{13}} = \log_e 5$

Ans: (B) $\log_e 5$

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22. Find $\text{cosech}^{-1}\left(\frac{12}{35}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1+\sqrt{1+x^2}}{x}$ (for $x > 0$)

$\sqrt{1+\frac{144}{1225}} = \frac{37}{35}$

$= \log_e 6$

Ans: (C) $\log_e 6$

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23. Find $\sinh^{-1}\left(\frac{5}{12}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{25}{144}+1} = \frac{13}{12}$

$\sinh^{-1}\left(\frac{5}{12}\right) = \log_e\left(\frac{5}{12}+\frac{13}{12}\right) = \log_e \frac{3}{2}$

Ans: (A) $\log_e \frac{3}{2}$

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24. Find the domain of $f(x) = \cosh^{-1}(x-8)$.

$\cosh^{-1} u$ needs $u \geq 1$.

$x - 8 \geq 1 \Rightarrow x \geq 9$

Ans: (D) $[9, \infty)$

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25. Express $\sinh^{-1}\left(\frac{4}{3}\right)$ in terms of $\tanh^{-1}$ and of $\cosh^{-1}$.

$\sinh^{-1} x = \tanh^{-1}\frac{x}{\sqrt{1+x^2}} = \cosh^{-1}\sqrt{x^2+1}$ for $x > 0$.

$\sqrt{1+\frac{16}{9}} = \frac{5}{3}$

$\tanh^{-1}\frac{\frac{4}{3}}{\frac{5}{3}} = \tanh^{-1}\left(\frac{4}{5}\right)$

$\sinh^{-1}\left(\frac{4}{3}\right) = \tanh^{-1}\left(\frac{4}{5}\right) = \cosh^{-1}\left(\frac{5}{3}\right)$

Ans: (B) $\tanh^{-1}\left(\frac{4}{5}\right) = \cosh^{-1}\left(\frac{5}{3}\right)$
