EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 05

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(-\ln\frac{4}{3})$.

A. $\frac{7}{12}$ **B.** $\frac{7}{24}$ **C.** $-\frac{7}{24}$ **D.** $\frac{25}{12}$

2. Find $\operatorname{sech}(\ln\frac{5}{3})$.

A. $\frac{8}{15}$ **B.** $\frac{15}{17}$ **C.** $\frac{17}{15}$ **D.** $-\frac{15}{17}$

3. Find $\cosh(\ln\frac{7}{3})$.

A. $-\frac{29}{21}$ **B.** $\frac{58}{21}$ **C.** $\frac{40}{21}$ **D.** $\frac{29}{21}$

4. Which of these functions is even?

A. $\operatorname{sech} x$ **B.** $\tanh x$ **C.** $\coth x$ **D.** $\operatorname{cosech} x$

5. The function $f(x) = \operatorname{sech} 4x + \cosh^4 x$ is

A. neither even nor odd **B.** both even and odd
C. odd **D.** even

6. Find the range of $f(x) = 5\operatorname{sech} x + 1$.

A. $(1, 6]$ **B.** $[1, 6]$ **C.** $(0, 5]$ **D.** $(1, 6)$

7. If $\tanh x = \frac{21}{29}$, find $\coth x$.

A. $\frac{8}{29}$ **B.** $-\frac{29}{21}$ **C.** $\frac{21}{29}$ **D.** $\frac{29}{21}$

8. If $\cosh x = \frac{29}{21}$ and x is negative, find $\sinh x$.

A. $\frac{\sqrt{1282}}{21}$ **B.** $\frac{20}{21}$ **C.** $\frac{21}{29}$ **D.** $-\frac{20}{21}$

9. Simplify $\cosh x - \sinh x$.

A. 1 **B.** e^{-x} **C.** $\cosh 2x$ **D.** e^{x}

10. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{17}{15}$, with $x, y > 0$, find $\sinh(x - y)$.

A. $\frac{5}{12}$ **B.** $\frac{31}{60}$ **C.** $-\frac{13}{12}$ **D.** $\frac{589}{300}$

11. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{13}{12}$, with $x, y > 0$, find $\sinh(x - y)$.

A. $-\frac{109}{60}$ **B.** $\frac{91}{60}$ **C.** $\frac{119}{60}$ **D.** $\frac{221}{60}$

12. If $\sinh x = \frac{35}{12}$ and $\cosh y = \frac{29}{21}$, with $x, y > 0$, find $\sinh(x - y)$.

A. $-\frac{373}{252}$ **B.** $\frac{275}{252}$ **C.** $\frac{55}{28}$ **D.** $\frac{195}{28}$

13. If $\cosh x = \frac{29}{20}$, find $\cosh 2x$.

A. $\frac{1041}{200}$ **B.** $\frac{19}{10}$ **C.** $-\frac{641}{200}$ **D.** $\frac{641}{200}$

14. If $\tanh x = \frac{1}{3}$, find $\cosh 2x$.

A. $\frac{3}{4}$ **B.** $\frac{10}{9}$ **C.** $\frac{5}{4}$ **D.** $\frac{4}{5}$

15. If $\tanh x = \frac{3}{5}$, find $\cosh 2x$.

A. $\frac{17}{8}$ **B.** $\frac{15}{8}$ **C.** $\frac{8}{17}$ **D.** $\frac{34}{25}$

16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 3$.

A. 243 **B.** $\frac{1}{243}$ **C.** 486 **D.** 15

17. If $\tanh x = \frac{3}{5}$, find $\cosh 6x + \sinh 6x$.

A. 64 **B.** $\frac{512}{125}$ **C.** $\frac{1}{64}$ **D.** 4096

18. If $\sinh x = \frac{15}{8}$ and $\cosh y = \frac{13}{5}$, find $\cosh(x + y)\cosh(x - y)$.

A. $\frac{16441}{1600}$ **B.** $\frac{32041}{1600}$ **C.** $\frac{5191}{1600}$ **D.** $-\frac{1991}{1600}$

19. Express $\cosh 8x - \cosh 6x$ as a product.

A. $2\sinh 7x \cosh x$ **B.** $2\cosh 7x \sinh x$

C. $2\sinh 7x \sinh x$ **D.** $-2\sinh 7x \sinh x$

20. If $\sinh x = \frac{7}{24}$ and $\cosh y = \frac{29}{20}$, find $\cosh(x + y)\cosh(x - y)$.

A. $\frac{31501}{14400}$ **B.** $-\frac{251}{14400}$ **C.** $\frac{29051}{14400}$ **D.** $\frac{43681}{14400}$

21. Find $\operatorname{sech}^{-1}\left(\frac{4}{5}\right)$.

A. $\frac{\log_e 2}{2}$ **B.** $\log_e 4$ **C.** $\log_e 2$ **D.** $-\log_e 2$

22. Find $\operatorname{cosech}^{-1}\left(-\frac{5}{12}\right)$.

A. $\frac{\log_e 5}{2}$ **B.** $-\log_e 5$ **C.** $\log_e 5$ **D.** $\log_e 10$

23. Find $\operatorname{cosech}^{-1}\left(-\frac{3}{4}\right)$.

A. $\log_e 6$ **B.** $\log_e 3$ **C.** $-\log_e 3$ **D.** $\frac{\log_e 3}{2}$

24. Find the domain of $f(x) = \cosh^{-1}(x - 2)$.

A. $[1, \infty)$ **B.** $(3, \infty)$ **C.** $[3, \infty)$ **D.** $[2, \infty)$

25. $\sinh^{-1}(-x) =$

A. $\frac{1}{\sinh^{-1} x}$ **B.** $\sinh^{-1} x$

C. $\cosh^{-1} x$ **D.** $-\sinh^{-1} x$

Key sheet

1. C	**2.** B	**3.** D	**4.** A	**5.** D
6. A	**7.** D	**8.** D	**9.** B	**10.** A
11. B	**12.** B	**13.** D	**14.** C	**15.** A
16. B	**17.** A	**18.** A	**19.** C	**20.** A
21. C	**22.** B	**23.** C	**24.** C	**25.** D

Solutions

1. Find $\sinh(-\ln\frac{4}{3})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln\frac{4}{3}} = \frac{4}{3},\ e^{-\ln\frac{4}{3}} = \frac{3}{4}$

$\sinh(-\ln\frac{4}{3}) = -\frac{\frac{4}{3} - \frac{3}{4}}{2} = -\frac{7}{24}$

Ans: (C) $-\frac{7}{24}$

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2. Find $\operatorname{sech}(\ln\frac{5}{3})$.

$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

$e^{\ln\frac{5}{3}} = \frac{5}{3},\ e^{-\ln\frac{5}{3}} = \frac{3}{5}$

$\operatorname{sech}(\ln\frac{5}{3}) = \frac{15}{17}$

Ans: (B) $\frac{15}{17}$

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3. Find $\cosh(\ln\frac{7}{3})$.

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\cosh(\ln\frac{7}{3}) = \frac{\frac{7}{3} + \frac{3}{7}}{2} = \frac{29}{21}$

Ans: (D) $\frac{29}{21}$

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4. Which of these functions is even?

cosh and sech are the even ones.

Ans: (A) $\operatorname{sech} x$

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5. The function $f(x) = \operatorname{sech} 4x + \cosh^4 x$ is

cosh and sech are even; sinh and tanh are odd.

Replacing x by $-x$: $f(-x) = f(x)$, so f is even.

Ans: (D) even

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6. Find the range of $f(x) = 5\operatorname{sech} x + 1$.

$0 < \operatorname{sech} x \leq 1$.

$1 < f(x) \leq 6$

Range $= (1, 6]$

Ans: (A) $(1, 6]$

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7. If $\tanh x = \frac{21}{29}$, find $\coth x$.

$\coth x = \frac{1}{\tanh x} = \frac{29}{21}$

Ans: (D) $\frac{29}{21}$

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8. If $\cosh x = \frac{29}{21}$ and x is negative, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{841}{441} - 1 = \frac{400}{441}$

x is negative, so $\sinh x$ is negative: $\sinh x = -\frac{20}{21}$

Ans: (D) $-\frac{20}{21}$

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9. Simplify $\cosh x - \sinh x$.

From $\cosh^2 x - \sinh^2 x = 1$ and $\cosh x \pm \sinh x = e^{\pm x}$: $\cosh x - \sinh x = e^{-x}$

Ans: (B) e^{-x}

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10. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{17}{15}$, with $x, y > 0$, find $\sinh(x - y)$.

$\cosh x = \sqrt{1 + \frac{441}{400}} = \frac{29}{20}$, $\sinh y = \sqrt{\frac{289}{225} - 1} = \frac{8}{15}$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$= \frac{21}{20} \cdot \frac{17}{15} - \frac{29}{20} \cdot \frac{8}{15} = \frac{5}{12}$

Ans: (A) $\frac{5}{12}$

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11. If $\sinh x = \frac{12}{5}$ and $\cosh y = \frac{13}{12}$, with $x, y > 0$, find $\sinh(x - y)$.

$\cosh x = \sqrt{1 + \frac{144}{25}} = \frac{13}{5}$, $\sinh y = \sqrt{\frac{169}{144} - 1} = \frac{5}{12}$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$= \frac{12}{5} \cdot \frac{13}{12} - \frac{13}{5} \cdot \frac{5}{12} = \frac{91}{60}$

Ans: (B) $\frac{91}{60}$

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12. If $\sinh x = \frac{35}{12}$ and $\cosh y = \frac{29}{21}$, with $x, y > 0$, find $\sinh(x - y)$.

$\cosh x = \sqrt{1 + \frac{1225}{144}} = \frac{37}{12}$, $\sinh y = \sqrt{\frac{841}{441} - 1} = \frac{20}{21}$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$= \frac{35}{12} \cdot \frac{29}{21} - \frac{37}{12} \cdot \frac{20}{21} = \frac{275}{252}$

Ans: (B) $\frac{275}{252}$

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13. If $\cosh x = \frac{29}{20}$, find $\cosh 2x$.

$\cosh 2x = 2\cosh^2 x - 1 = 2 \cdot \frac{841}{400} - 1 = \frac{641}{200}$

Ans: (D) $\frac{641}{200}$

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14. If $\tanh x = \frac{1}{3}$, find $\cosh 2x$.

$\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x} = \frac{1 + \frac{1}{9}}{1 - \frac{1}{9}} = \frac{5}{4}$

Ans: (C) $\frac{5}{4}$

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15. If $\tanh x = \frac{3}{5}$, find $\cosh 2x$.

$\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x} = \frac{1 + \frac{9}{25}}{1 - \frac{9}{25}} = \frac{17}{8}$

Ans: (A) $\frac{17}{8}$

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16. Find $(\cosh x - \sinh x)^5$ when $x = \ln 3$.

$(\cosh x - \sinh x)^5 = e^{-5x}$

$= e^{-5 \ln 3} = 3^{-5} = \frac{1}{243}$

Ans: (B) $\frac{1}{243}$

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17. If $\tanh x = \frac{3}{5}$, find $\cosh 6x + \sinh 6x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 3$

$\frac{1+\frac{3}{5}}{1-\frac{3}{5}} = 4$

$\text{Answer} = (4)^3 = 64$

Ans: (A) 64

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18. If $\sinh x = \frac{15}{8}$ and $\cosh y = \frac{13}{5}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{225}{64} + \frac{169}{25} = \frac{16441}{1600}$

Ans: (A) $\frac{16441}{1600}$

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19. Express $\cosh 8x - \cosh 6x$ as a product.

$\cosh 8x - \cosh 6x = 2\sinh \frac{8+6}{2}x \ \sinh \frac{8-6}{2}x$

$= 2\sinh 7x \sinh x$

Ans: (C) $2\sinh 7x \sinh x$

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20. If $\sinh x = \frac{7}{24}$ and $\cosh y = \frac{29}{20}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{49}{576} + \frac{841}{400} = \frac{31501}{14400}$

Ans: (A) $\frac{31501}{14400}$

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21. Find $\text{sech}^{-1}\left(\frac{4}{5}\right)$.

$\text{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$= \log_e \frac{1+\frac{3}{5}}{\frac{4}{5}} = \log_e 2$

Ans: (C) $\log_e 2$

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22. Find $\text{cosech}^{-1}\left(-\frac{5}{12}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1-\sqrt{1+x^2}}{x}$ (for $x < 0$)

$\sqrt{1+\frac{25}{144}} = \frac{13}{12}$

$= -\log_e 5$

Ans: (B) $-\log_e 5$

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23. Find $\text{cosech}^{-1}\left(-\frac{3}{4}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1-\sqrt{1+x^2}}{x}$ (for $x < 0$)

$\sqrt{1+\frac{9}{16}} = \frac{5}{4}$

$= -\log_e 3$

Ans: (C) $-\log_e 3$

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24. Find the domain of $f(x) = \cosh^{-1}(x-2)$.

$\cosh^{-1} u$ needs $u \geq 1$.

$x - 2 \geq 1 \Rightarrow x \geq 3$

Ans: (C) $[3, \infty)$

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25. $\sinh^{-1}(-x) =$

From the table of inverse hyperbolic functions: $-\sinh^{-1} x$.

Ans: (D) $-\sinh^{-1} x$
