EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 04

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\tanh(-\ln 6)$.

A. $\frac{35}{6}$ **B.** $-\frac{35}{37}$ **C.** $\frac{35}{37}$ **D.** $\frac{37}{6}$

2. Find $2\sinh(\ln\frac{5}{3})\cosh(\ln\frac{5}{3})$.

A. $\frac{136}{225}$ **B.** $\frac{5}{3}$ **C.** $\frac{578}{225}$ **D.** $\frac{272}{225}$

3. Find $\tanh(\ln\frac{4}{3})$.

A. $\frac{7}{25}$ **B.** $-\frac{7}{25}$ **C.** $\frac{25}{12}$ **D.** $\frac{7}{12}$

4. Find the range of $f(x) = 3\cosh x + 3$.

A. $(6, \infty)$ **B.** $[3, \infty)$ **C.** $[0, 6]$ **D.** $[6, \infty)$

5. The function $f(x) = x^2\cosh 5x + x\cosh 5x$ is

A. both even and odd **B.** even

C. odd **D.** neither even nor odd

6. Find the range of $f(x) = -5\cosh x - 3$.

A. $(-\infty, -8]$ **B.** $(-\infty, -8)$ **C.** $(-\infty, -3]$ **D.** $[-8, \infty)$

7. If $\sinh x = \frac{12}{5}$, find $\operatorname{cosech} x$.

A. $\frac{5}{12}$ **B.** $\frac{13}{5}$ **C.** $\frac{12}{5}$ **D.** $-\frac{5}{12}$

8. If $\sinh x = -\frac{12}{5}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = \frac{13}{5},\ \tanh x = -\frac{13}{12}$

B. $\cosh x = \frac{13}{5},\ \tanh x = -\frac{12}{13}$

C. $\cosh x = \frac{\sqrt{119}}{5},\ \tanh x = -\frac{12}{13}$

D. $\cosh x = -\frac{13}{5},\ \tanh x = \frac{12}{13}$

9. Simplify $\cosh^2 x - \sinh^2 x$.

A. 1 **B.** -1 **C.** 0 **D.** $\cosh 2x$

10. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{8}{15}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{589}{300}$ **B.** $\frac{661}{300}$ **C.** $\frac{31}{12}$ **D.** $\frac{13}{12}$

11. If $\cosh x = \frac{17}{8}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x+y)$.

A. $\frac{125}{24}$ **B.** $\frac{13}{12}$ **C.** $\frac{575}{48}$ **D.** $\frac{577}{48}$

12. If $\tanh x = \frac{1}{5}$ and $\tanh y = \frac{1}{7}$, find $\tanh(x+y)$.

A. $\frac{6}{17}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{18}$ **D.** $\frac{12}{35}$

13. If $\sinh x = \frac{21}{20}$, find $\cosh 2x$.

A. $\frac{641}{200}$ **B.** $\frac{31}{10}$ **C.** $-\frac{241}{200}$ **D.** $\frac{441}{200}$

14. If $\tanh x = \frac{2}{7}$, find $\tanh 2x$.

A. $\frac{14}{53}$ **B.** $\frac{4}{7}$ **C.** $\frac{28}{53}$ **D.** $\frac{28}{45}$

15. If $\tanh x = \frac{2}{3}$, find $\cosh 2x$.

A. $\frac{12}{5}$ **B.** $\frac{13}{9}$ **C.** $\frac{13}{5}$ **D.** $\frac{5}{13}$

16. Find $(\cosh x + \sinh x)^3$ when $x = \ln 5$.

A. $\frac{1}{125}$ **B.** 15 **C.** 250 **D.** 125

17. If $\tanh x = \frac{3}{5}$, find $\cosh 2x + \sinh 2x$.

A. 4 **B.** $\frac{8}{5}$ **C.** 16 **D.** $\frac{1}{4}$

18. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{5}{4}$, find $\cosh(x+y)\cosh(x-y)$.

A. $\frac{23}{50}$ **B.** $\frac{77}{50}$ **C.** $\frac{529}{100}$ **D.** $\frac{533}{200}$

19. Express $\sinh 6x - \sinh 4x$ as a product.

A. $2\sinh 5x \sinh x$ **B.** $2\cosh 5x \sinh x$

C. $-2\cosh 5x \sinh x$ **D.** $2\sinh 5x \cosh x$

20. If $\sinh x = \frac{5}{12}$ and $\sinh y = \frac{12}{5}$, find $\sinh(x+y)\sinh(x-y)$.

A. $\frac{14161}{3600}$ **B.** $\frac{20111}{3600}$ **C.** $-\frac{20111}{3600}$ **D.** $\frac{21361}{3600}$

21. Find $\cosh^{-1}\left(\frac{13}{12}\right)$.

A. $-\log_e \frac{3}{2}$ **B.** $\log_e \frac{3}{2}$

C. $\log_e 3$ **D.** $\frac{\log_e \frac{3}{2}}{2}$

22. Find $\cosh^{-1}\left(\frac{29}{20}\right)$.

A. $\log_e 5$ **B.** $-\log_e \frac{5}{2}$

C. $\log_e \frac{5}{2}$ **D.** $\frac{\log_e \frac{5}{2}}{2}$

23. Find $\operatorname{cosech}^{-1}\left(\frac{21}{20}\right)$.

A. $\frac{\log_e \frac{7}{3}}{2}$ **B.** $-\log_e \frac{7}{3}$

C. $\log_e \frac{7}{3}$ **D.** $\log_e \frac{14}{3}$

24. Find the domain of $f(x) = \tanh^{-1}\left(\frac{x}{2}\right)$.

A. $(-2, 2)$

B. $[-2, 2]$

C. $(-\infty, -2) \cup (2, \infty)$

D. $(-1, 1)$

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25. Express $\sinh^{-1}\left(\frac{21}{20}\right)$ in terms of $\tanh^{-1}$ and of $\cosh^{-1}$.

A. $\tanh^{-1}\left(\frac{21}{41}\right) = \cosh^{-1}\left(\frac{41}{20}\right)$

B. $\tanh^{-1}\left(\frac{29}{21}\right) = \cosh^{-1}\left(\frac{21}{20}\right)$

C. $\tanh^{-1}\left(\frac{21}{29}\right) = \cosh^{-1}\left(\frac{29}{20}\right)$

D. $\tanh^{-1}\left(\frac{441}{841}\right) = \cosh^{-1}\left(\frac{841}{400}\right)$

Key sheet

1. B	**2.** D	**3.** A	**4.** D	**5.** D
6. A	**7.** A	**8.** B	**9.** A	**10.** B
11. D	**12.** B	**13.** A	**14.** C	**15.** C
16. D	**17.** A	**18.** D	**19.** B	**20.** C
21. B	**22.** C	**23.** C	**24.** A	**25.** C

Solutions

1. Find $\tanh(-\ln 6)$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(-\ln 6) = -\frac{6 - \frac{1}{6}}{6 + \frac{1}{6}} = -\frac{35}{37}$

Ans: (B) $-\frac{35}{37}$

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2. Find $2\sinh(\ln \frac{5}{3})\cosh(\ln \frac{5}{3})$.

$\sinh(\ln \frac{5}{3}) = \frac{8}{15},\ \cosh(\ln \frac{5}{3}) = \frac{17}{15}$

$2 \times \frac{8}{15} \times \frac{17}{15} = \frac{272}{225}$

Ans: (D) $\frac{272}{225}$

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3. Find $\tanh(\ln \frac{4}{3})$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(\ln \frac{4}{3}) = \frac{\frac{4}{3} - \frac{3}{4}}{\frac{4}{3} + \frac{3}{4}} = \frac{7}{25}$

Ans: (A) $\frac{7}{25}$

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4. Find the range of $f(x) = 3\cosh x + 3$.

$\cosh x$ has range $[1, \infty)$.

$3\cosh x + 3 \geq 3(1) + 3 = 6$

Range $= [6, \infty)$

Ans: (D) $[6, \infty)$

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5. The function $f(x) = x^2 \cosh 5x + x \cosh 5x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: one term keeps its sign and the other changes, so f is neither.

Ans: (D) neither even nor odd

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6. Find the range of $f(x) = -5\cosh x - 3$.

$\cosh x \geq 1$, so $-5\cosh x \leq -5$.

$f(x) \leq -3 - 5 = -8$

Range $= (-\infty, -8]$

Ans: (A) $(-\infty, -8]$

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7. If $\sinh x = \frac{12}{5}$, find $\operatorname{cosech} x$.

$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{5}{12}$

Ans: (A) $\frac{5}{12}$

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8. If $\sinh x = -\frac{12}{5}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{144}{25} = \frac{169}{25}$

Since $\cosh x \geq 1$, $\cosh x = \frac{13}{5}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{-\frac{12}{5}}{\frac{13}{5}} = -\frac{12}{13}$

Ans: (B) $\cosh x = \frac{13}{5},\ \tanh x = -\frac{12}{13}$

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9. Simplify $\cosh^2 x - \sinh^2 x$.

From $\cosh^2 x - \sinh^2 x = 1$ and $\cosh x \pm \sinh x = e^{\pm x}$: $\cosh^2 x - \sinh^2 x = 1$

Ans: (A) 1

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10. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{8}{15}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{841}{400} - 1} = \frac{21}{20},\ \cosh y = \sqrt{1 + \frac{64}{225}} = \frac{17}{15}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{29}{20} \cdot \frac{17}{15} + \frac{21}{20} \cdot \frac{8}{15} = \frac{661}{300}$

Ans: (B) $\frac{661}{300}$

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11. If $\cosh x = \frac{17}{8}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{289}{64} - 1} = \frac{15}{8},\ \cosh y = \sqrt{1 + \frac{1225}{144}} = \frac{37}{12}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{17}{8} \cdot \frac{37}{12} + \frac{15}{8} \cdot \frac{35}{12} = \frac{577}{48}$

Ans: (D) $\frac{577}{48}$

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12. If $\tanh x = \frac{1}{5}$ and $\tanh y = \frac{1}{7}$, find $\tanh(x+y)$.

$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

$= \frac{\frac{1}{5} + \frac{1}{7}}{1 + \frac{1}{35}} = \frac{1}{3}$

Ans: (B) $\frac{1}{3}$

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13. If $\sinh x = \frac{21}{20}$, find $\cosh 2x$.

$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2 \cdot \frac{441}{400} = \frac{641}{200}$

Ans: (A) $\frac{641}{200}$

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14. If $\tanh x = \frac{2}{7}$, find $\tanh 2x$.

$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x} = \frac{\frac{4}{7}}{1 + \frac{4}{49}} = \frac{28}{53}$

Ans: (C) $\frac{28}{53}$

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15. If $\tanh x = \frac{2}{3}$, find $\cosh 2x$.

$\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x} = \frac{1 + \frac{4}{9}}{1 - \frac{4}{9}} = \frac{13}{5}$

Ans: (C) $\frac{13}{5}$

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16. Find $(\cosh x + \sinh x)^3$ when $x = \ln 5$.

$(\cosh x + \sinh x)^3 = e^{3x}$

$= e^{3\ln 5} = 5^3 = 125$

Ans: (D) 125

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17. If $\tanh x = \frac{3}{5}$, find $\cosh 2x + \sinh 2x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 1$

$\frac{1+\frac{3}{5}}{1-\frac{3}{5}} = 4$

$\text{Answer} = (4)^1 = 4$

Ans: (A) 4

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18. If $\sinh x = \frac{21}{20}$ and $\cosh y = \frac{5}{4}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{441}{400} + \frac{25}{16} = \frac{533}{200}$

Ans: (D) $\frac{533}{200}$

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19. Express $\sinh 6x - \sinh 4x$ as a product.

$\sinh 6x - \sinh 4x = 2\cosh \frac{6+4}{2}x \ \sinh \frac{6-4}{2}x$

$= 2\cosh 5x \sinh x$

Ans: (B) $2\cosh 5x \sinh x$

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20. If $\sinh x = \frac{5}{12}$ and $\sinh y = \frac{12}{5}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{25}{144} - \frac{144}{25} = -\frac{20111}{3600}$

Ans: (C) $-\frac{20111}{3600}$

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21. Find $\cosh^{-1}\left(\frac{13}{12}\right)$.

$\cosh^{-1} x = \log_e\left(x + \sqrt{x^2-1}\right)$

$\sqrt{\frac{169}{144} - 1} = \frac{5}{12}$

$\cosh^{-1}\left(\frac{13}{12}\right) = \log_e\left(\frac{13}{12} + \frac{5}{12}\right) = \log_e \frac{3}{2}$

Ans: (B) $\log_e \frac{3}{2}$

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22. Find $\cosh^{-1}\left(\frac{29}{20}\right)$.

$\cosh^{-1} x = \log_e\left(x + \sqrt{x^2-1}\right)$

$\sqrt{\frac{841}{400} - 1} = \frac{21}{20}$

$\cosh^{-1}\left(\frac{29}{20}\right) = \log_e\left(\frac{29}{20} + \frac{21}{20}\right) = \log_e \frac{5}{2}$

Ans: (C) $\log_e \frac{5}{2}$

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23. Find $\text{cosech}^{-1}\left(\frac{21}{20}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1+\sqrt{1+x^2}}{x}$ (for $x > 0$)

$\sqrt{1 + \frac{441}{400}} = \frac{29}{20}$

$= \log_e \frac{7}{3}$

Ans: (C) $\log_e \frac{7}{3}$

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24. Find the domain of $f(x) = \tanh^{-1}\left(\frac{x}{2}\right)$.

$\tanh^{-1} u$ needs $-1 < u < 1$.

$-1 < \frac{x}{2} < 1 \Rightarrow -2 < x < 2$

Ans: (A) $(-2, 2)$

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25. Express $\sinh^{-1}\left(\frac{21}{20}\right)$ in terms of $\tanh^{-1}$ and of $\cosh^{-1}$.

$\sinh^{-1} x = \tanh^{-1} \frac{x}{\sqrt{1+x^2}} = \cosh^{-1} \sqrt{x^2+1}$ for $x > 0$.

$\sqrt{1 + \frac{441}{400}} = \frac{29}{20}$

$\tanh^{-1} \frac{\frac{21}{20}}{\frac{29}{20}} = \tanh^{-1}\left(\frac{21}{29}\right)$

$\sinh^{-1}\left(\frac{21}{20}\right) = \tanh^{-1}\left(\frac{21}{29}\right) = \cosh^{-1}\left(\frac{29}{20}\right)$

Ans: (C) $\tanh^{-1}\left(\frac{21}{29}\right) = \cosh^{-1}\left(\frac{29}{20}\right)$
