EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 03

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\cosh(\ln 2)$.

A. $-\frac{5}{4}$ **B.** $\frac{3}{2}$ **C.** $\frac{5}{2}$ **D.** $\frac{5}{4}$

2. Find $2\sinh(\ln\frac{4}{3})\cosh(\ln\frac{4}{3})$.

A. $\frac{625}{288}$ **B.** $\frac{175}{576}$ **C.** $\frac{4}{3}$ **D.** $\frac{175}{288}$

3. Find $\sinh(-\ln 5)$.

A. $\frac{26}{5}$ **B.** $-\frac{12}{5}$ **C.** $\frac{12}{5}$ **D.** $\frac{24}{5}$

4. Find the range of $f(x) = 3\cosh x + 1$.

A. $[-2, 4]$ **B.** $[1, \infty)$ **C.** $[4, \infty)$ **D.** $(4, \infty)$

5. The function $f(x) = x^4\cosh 2x + \tanh 2x$ is

A. both even and odd **B.** odd
C. neither even nor odd **D.** even

6. Find the range of $f(x) = 5\,\mathrm{sech}\,x + 3$.

A. $[3, 8]$ **B.** $(3, 8)$ **C.** $(0, 5]$ **D.** $(3, 8]$

7. If $\tanh x = \frac{4}{5}$, find $\coth x$.

A. $\frac{4}{5}$ **B.** $\frac{1}{5}$ **C.** $\frac{5}{4}$ **D.** $-\frac{5}{4}$

8. If $\sinh x = -\frac{3}{4}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = \frac{\sqrt{7}}{4},\ \tanh x = -\frac{3}{5}$
B. $\cosh x = \frac{5}{4},\ \tanh x = -\frac{5}{3}$
C. $\cosh x = \frac{5}{4},\ \tanh x = -\frac{3}{5}$
D. $\cosh x = -\frac{5}{4},\ \tanh x = \frac{3}{5}$

9. If $\cosh x = \frac{37}{12}$ and x is positive, find $\sinh x$.

A. $\frac{35}{12}$ **B.** $\frac{\sqrt{1513}}{12}$ **C.** $\frac{12}{37}$ **D.** $-\frac{35}{12}$

10. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{5}{12}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{145}{144}$ **B.** $\frac{17}{144}$ **C.** $-\frac{1}{24}$ **D.** $\frac{5}{4}$

11. If $\sinh x = \frac{35}{12}$ and $\cosh y = \frac{25}{24}$, with $x, y > 0$, find $\sinh(x - y)$.

A. $-\frac{85}{36}$ **B.** $\frac{77}{36}$ **C.** $\frac{63}{16}$ **D.** $\frac{21}{8}$

12. If $\sinh x = \frac{20}{21}$ and $\cosh y = \frac{29}{20}$, with $x, y > 0$, find $\sinh(x + y)$.

A. $-\frac{29}{420}$ **B.** $\frac{841}{420}$ **C.** $\frac{1189}{420}$ **D.** $\frac{1261}{420}$

13. If $\sinh x = \frac{20}{21}$ and $\cosh x = \frac{29}{21}$, find $\sinh 2x$.

A. $\frac{1160}{441}$ **B.** $\frac{40}{21}$ **C.** $\frac{1241}{441}$ **D.** $\frac{580}{441}$

14. If $\cosh x = 3$, find $\cosh 3x$.

A. 9 **B.** 99 **C.** 117 **D.** -99

15. If $\tanh x = \frac{3}{7}$, find $\cosh 2x$.

A. $\frac{20}{29}$ **B.** $\frac{21}{20}$ **C.** $\frac{29}{20}$ **D.** $\frac{58}{49}$

16. Find $(\cosh x - \sinh x)^3$ when $x = \ln 3$.

A. 9 **B.** $\frac{1}{27}$ **C.** 54 **D.** 27

17. If $\tanh x = \frac{1}{4}$, find $\cosh 4x + \sinh 4x$.

A. $\frac{625}{81}$ **B.** $\frac{25}{16}$ **C.** $\frac{9}{25}$ **D.** $\frac{25}{9}$

18. Express $\sinh 9x - \sinh 1x$ as a product.

A. $2\cosh 5x \sinh 4x$ **B.** $-2\cosh 5x \sinh 4x$

C. $2\sinh 5x \cosh 4x$ **D.** $2\sinh 5x \sinh 4x$

19. Express $\sinh 9x - \sinh 7x$ as a product.

A. $2\sinh 8x \sinh x$ **B.** $2\cosh 8x \sinh x$

C. $-2\cosh 8x \sinh x$ **D.** $2\sinh 8x \cosh x$

20. Express $\sinh 7x + \sinh 3x$ as a product.

A. $2\sinh 5x \cosh 2x$ **B.** $-2\sinh 5x \cosh 2x$

C. $2\cosh 5x \sinh 2x$ **D.** $2\cosh 5x \cosh 2x$

21. Find $\operatorname{cosech}^{-1}\left(-\frac{8}{15}\right)$.

A. $\frac{\log_e 4}{2}$ **B.** $\log_e 8$ **C.** $-\log_e 4$ **D.** $\log_e 4$

22. Find $\operatorname{sech}^{-1}\left(\frac{12}{37}\right)$.

A. $\frac{\log_e 6}{2}$ **B.** $-\log_e 6$ **C.** $\log_e 12$ **D.** $\log_e 6$

23. Find $\cosh^{-1}\left(\frac{37}{12}\right)$.

A. $\log_e 12$ **B.** $\log_e 6$ **C.** $-\log_e 6$ **D.** $\frac{\log_e 6}{2}$

24. Find the domain of $f(x) = \cosh^{-1}(x - 7)$.

A. $[6, \infty)$ **B.** $[7, \infty)$ **C.** $[8, \infty)$ **D.** $(8, \infty)$

25. Express $\cosh^{-1}\left(\frac{37}{12}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

A. $\sinh^{-1}\left(\frac{\sqrt{1513}}{12}\right) = \operatorname{sech}^{-1}\left(\frac{12}{37}\right)$

B. $\sinh^{-1}\left(\frac{1225}{144}\right) = \operatorname{sech}^{-1}\left(\frac{37}{12}\right)$

C. $\sinh^{-1}\left(\frac{35}{12}\right) = \operatorname{sech}^{-1}\left(\frac{37}{12}\right)$

D. $\sinh^{-1}\left(\frac{35}{12}\right) = \operatorname{sech}^{-1}\left(\frac{12}{37}\right)$

Key sheet

1. D	**2.** D	**3.** B	**4.** C	**5.** C
6. D	**7.** C	**8.** C	**9.** A	**10.** A
11. B	**12.** C	**13.** A	**14.** B	**15.** C
16. B	**17.** D	**18.** A	**19.** B	**20.** A
21. C	**22.** D	**23.** B	**24.** C	**25.** D

Solutions

1. Find $\cosh(\ln 2)$.

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\cosh(\ln 2) = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$

Ans: (D) $\frac{5}{4}$

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2. Find $2\sinh(\ln \frac{4}{3})\cosh(\ln \frac{4}{3})$.

$\sinh(\ln \frac{4}{3}) = \frac{7}{24},\ \cosh(\ln \frac{4}{3}) = \frac{25}{24}$

$2 \times \frac{7}{24} \times \frac{25}{24} = \frac{175}{288}$

Ans: (D) $\frac{175}{288}$

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3. Find $\sinh(-\ln 5)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln 5} = 5,\ e^{-\ln 5} = \frac{1}{5}$

$\sinh(-\ln 5) = -\frac{5 - \frac{1}{5}}{2} = -\frac{12}{5}$

Ans: (B) $-\frac{12}{5}$

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4. Find the range of $f(x) = 3\cosh x + 1$.

$\cosh x$ has range $[1, \infty)$.

$3\cosh x + 1 \geq 3(1) + 1 = 4$

Range $= [4, \infty)$

Ans: (C) $[4, \infty)$

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5. The function $f(x) = x^4 \cosh 2x + \tanh 2x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: one term keeps its sign and the other changes, so f is neither.

Ans: (C) neither even nor odd

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6. Find the range of $f(x) = 5\operatorname{sech} x + 3$.

$0 < \operatorname{sech} x \leq 1$.

$3 < f(x) \leq 8$

Range $= (3, 8]$

Ans: (D) $(3, 8]$

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7. If $\tanh x = \frac{4}{5}$, find $\coth x$.

$\coth x = \frac{1}{\tanh x} = \frac{5}{4}$

Ans: (C) $\frac{5}{4}$

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8. If $\sinh x = -\frac{3}{4}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

Since $\cosh x \geq 1$, $\cosh x = \frac{5}{4}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{-\frac{3}{4}}{\frac{5}{4}} = -\frac{3}{5}$

Ans: (C) $\cosh x = \frac{5}{4},\ \tanh x = -\frac{3}{5}$

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9. If $\cosh x = \frac{37}{12}$ and x is positive, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{1369}{144} - 1 = \frac{1225}{144}$

x is positive, so $\sinh x$ is positive: $\sinh x = \frac{35}{12}$

Ans: (A) $\frac{35}{12}$

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10. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{5}{12}$, with $x, y > 0$, find $\cosh(x - y)$.

$\sinh x = \sqrt{\frac{625}{576} - 1} = \frac{7}{24},\ \cosh y = \sqrt{1 + \frac{25}{144}} = \frac{13}{12}$

$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{25}{24} \cdot \frac{13}{12} - \frac{7}{24} \cdot \frac{5}{12} = \frac{145}{144}$

Ans: (A) $\frac{145}{144}$

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11. If $\sinh x = \frac{35}{12}$ and $\cosh y = \frac{25}{24}$, with $x, y > 0$, find $\sinh(x - y)$.

$\cosh x = \sqrt{1 + \frac{1225}{144}} = \frac{37}{12},\ \sinh y = \sqrt{\frac{625}{576} - 1} = \frac{7}{24}$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$= \frac{35}{12} \cdot \frac{25}{24} - \frac{37}{12} \cdot \frac{7}{24} = \frac{77}{36}$

Ans: (B) $\frac{77}{36}$

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12. If $\sinh x = \frac{20}{21}$ and $\cosh y = \frac{29}{20}$, with $x, y > 0$, find $\sinh(x + y)$.

$\cosh x = \sqrt{1 + \frac{400}{441}} = \frac{29}{21},\ \sinh y = \sqrt{\frac{841}{400} - 1} = \frac{21}{20}$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$= \frac{20}{21} \cdot \frac{29}{20} + \frac{29}{21} \cdot \frac{21}{20} = \frac{1189}{420}$

Ans: (C) $\frac{1189}{420}$

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13. If $\sinh x = \frac{20}{21}$ and $\cosh x = \frac{29}{21}$, find $\sinh 2x$.

$\sinh 2x = 2 \sinh x \cosh x = 2 \cdot \frac{20}{21} \cdot \frac{29}{21} = \frac{1160}{441}$

Ans: (A) $\frac{1160}{441}$

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14. If $\cosh x = 3$, find $\cosh 3x$.

$\cosh 3x = 4 \cosh^3 x - 3 \cosh x$

$= 4(3)^3 - 3(3) = 99$

Ans: (B) 99

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15. If $\tanh x = \frac{3}{7}$, find $\cosh 2x$.

$\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x} = \frac{1 + \frac{9}{49}}{1 - \frac{9}{49}} = \frac{29}{20}$

Ans: (C) $\frac{29}{20}$

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16. Find $(\cosh x - \sinh x)^3$ when $x = \ln 3$.

$(\cosh x - \sinh x)^3 = e^{-3x}$

$= e^{-3\ln 3} = 3^{-3} = \frac{1}{27}$

Ans: (B) $\frac{1}{27}$

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17. If $\tanh x = \frac{1}{4}$, find $\cosh 4x + \sinh 4x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 2$

$\frac{1+\frac{1}{4}}{1-\frac{1}{4}} = \frac{5}{3}$

$\text{Answer} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

Ans: (D) $\frac{25}{9}$

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18. Express $\sinh 9x - \sinh 1x$ as a product.

$\sinh 9x - \sinh 1x = 2\cosh \frac{9+1}{2}x \ \sinh \frac{9-1}{2}x$

$= 2\cosh 5x \sinh 4x$

Ans: (A) $2\cosh 5x \sinh 4x$

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19. Express $\sinh 9x - \sinh 7x$ as a product.

$\sinh 9x - \sinh 7x = 2\cosh \frac{9+7}{2}x \ \sinh \frac{9-7}{2}x$

$= 2\cosh 8x \sinh x$

Ans: (B) $2\cosh 8x \sinh x$

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20. Express $\sinh 7x + \sinh 3x$ as a product.

$\sinh 7x + \sinh 3x = 2\sinh \frac{7+3}{2}x \ \cosh \frac{7-3}{2}x$

$= 2\sinh 5x \cosh 2x$

Ans: (A) $2\sinh 5x \cosh 2x$

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21. Find $\text{cosech}^{-1}\left(-\frac{8}{15}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1-\sqrt{1+x^2}}{x}$ (for $x < 0$)

$\sqrt{1+\frac{64}{225}} = \frac{17}{15}$

$= -\log_e 4$

Ans: (C) $-\log_e 4$

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22. Find $\text{sech}^{-1}\left(\frac{12}{37}\right)$.

$\text{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1-\frac{144}{1369}} = \frac{35}{37}$

$= \log_e \frac{1+\frac{35}{37}}{\frac{12}{37}} = \log_e 6$

Ans: (D) $\log_e 6$

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23. Find $\cosh^{-1}\left(\frac{37}{12}\right)$.

$\cosh^{-1} x = \log_e\left(x + \sqrt{x^2-1}\right)$

$\sqrt{\frac{1369}{144} - 1} = \frac{35}{12}$

$\cosh^{-1}\left(\frac{37}{12}\right) = \log_e\left(\frac{37}{12} + \frac{35}{12}\right) = \log_e 6$

Ans: (B) $\log_e 6$

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24. Find the domain of $f(x) = \cosh^{-1}(x-7)$.

$\cosh^{-1} u$ needs $u \geq 1$.

$x - 7 \geq 1 \Rightarrow x \geq 8$

Ans: (C) $[8, \infty)$

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25. Express $\cosh^{-1}\left(\frac{37}{12}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

$\cosh^{-1} x = \sinh^{-1}\sqrt{x^2-1} = \operatorname{sech}^{-1}\frac{1}{x}$ for $x \geq 1$.

$\sqrt{\frac{1369}{144} - 1} = \frac{35}{12},\ \frac{1}{\frac{37}{12}} = \frac{12}{37}$

$\cosh^{-1}\left(\frac{37}{12}\right) = \sinh^{-1}\left(\frac{35}{12}\right) = \operatorname{sech}^{-1}\left(\frac{12}{37}\right)$

Ans: (D) $\sinh^{-1}\left(\frac{35}{12}\right) = \operatorname{sech}^{-1}\left(\frac{12}{37}\right)$
