EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 02

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(\ln 2)$.

A. $\frac{5}{2}$ **B.** $-\frac{3}{4}$ **C.** $\frac{3}{4}$ **D.** $\frac{3}{2}$

2. Find $\cosh(\ln 3) + \sinh(\ln \frac{5}{2})$.

A. $\frac{163}{60}$ **B.** $\frac{167}{60}$ **C.** $\frac{11}{2}$ **D.** $\frac{37}{60}$

3. Find $\tanh(\ln \frac{5}{3})$.

A. $\frac{8}{17}$ **B.** $\frac{16}{15}$ **C.** $\frac{34}{15}$ **D.** $-\frac{8}{17}$

4. Find the range of $f(x) = 4 - 2\cosh x$.

A. $(-\infty, 2)$ **B.** $(-\infty, 2]$ **C.** $(-\infty, 4]$ **D.** $[2, \infty)$

5. The function $f(x) = \tanh 5x + x^2 \sinh 5x$ is

A. neither even nor odd **B.** even

C. both even and odd **D.** odd

6. Find the range of $f(x) = 6\,\mathrm{sech}\, x - 1$.

A. $(-1, 5)$ **B.** $[-1, 5]$ **C.** $(-1, 5]$ **D.** $(0, 6]$

7. If $\tanh x = \frac{3}{5}$, find $\coth x$.

A. $\frac{2}{5}$ **B.** $-\frac{5}{3}$ **C.** $\frac{3}{5}$ **D.** $\frac{5}{3}$

8. If $\tanh x = \frac{7}{25}$, find $\mathrm{sech}\, x$.

A. $\frac{\sqrt{674}}{25}$ **B.** $-\frac{24}{25}$ **C.** $\frac{24}{25}$ **D.** $\frac{25}{7}$

9. If $\cosh x = \frac{25}{24}$ and x is negative, find $\sinh x$.

A. $-\frac{7}{24}$ **B.** $\frac{\sqrt{1201}}{24}$ **C.** $\frac{7}{24}$ **D.** $\frac{24}{25}$

10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{19}{4}$ **B.** $\frac{323}{36}$ **C.** $\frac{325}{36}$ **D.** $\frac{5}{4}$

11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{125}{28}$ **B.** $\frac{197}{28}$ **C.** $\frac{373}{252}$ **D.** $\frac{195}{28}$

12. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{77}{36}$ **B.** $\frac{65}{16}$ **C.** $-\frac{49}{24}$ **D.** $\frac{85}{36}$

13. If $\sinh x = \frac{12}{5}$, find $\cosh 2x$.

A. $\frac{29}{5}$ **B.** $\frac{288}{25}$ **C.** $\frac{313}{25}$ **D.** $-\frac{263}{25}$

14. If $\tanh x = \frac{3}{5}$, find $\sinh 2x$.

A. $\frac{6}{5}$ **B.** $\frac{15}{8}$ **C.** $\frac{15}{17}$ **D.** $\frac{17}{8}$

15. If $\tanh x = \frac{1}{7}$, find $\tanh 2x$.

A. $\frac{7}{24}$ **B.** $\frac{2}{7}$ **C.** $\frac{7}{50}$ **D.** $\frac{7}{25}$

16. Find $(\cosh x - \sinh x)^2$ when $x = \ln 2$.

A. 8 **B.** 4 **C.** $\frac{1}{4}$ **D.** $\frac{1}{8}$

17. If $\tanh x = \frac{1}{5}$, find $\cosh 4x + \sinh 4x$.

A. $\frac{81}{16}$ **B.** $\frac{4}{9}$ **C.** $\frac{36}{25}$ **D.** $\frac{9}{4}$

18. Express $\cosh 4x + \cosh 2x$ as a product.

A. $2\cosh 3x \cosh x$ **B.** $-2\cosh 3x \cosh x$

C. $2\sinh 3x \cosh x$ **D.** $2\cosh 3x \sinh x$

19. Express $\cosh 6x - \cosh 4x$ as a product.

A. $2\cosh 5x \sinh x$ **B.** $-2\sinh 5x \sinh x$

C. $2\sinh 5x \sinh x$ **D.** $2\sinh 5x \cosh x$

20. If $\sinh x = \frac{35}{12}$ and $\sinh y = \frac{3}{4}$, find $\sinh(x + y)\sinh(x - y)$.

A. $\frac{143}{18}$ **B.** $\frac{653}{72}$ **C.** $-\frac{143}{18}$ **D.** $\frac{169}{36}$

21. Find $\operatorname{cosech}^{-1}\left(\frac{15}{8}\right)$.

A. $\log_e \frac{10}{3}$ **B.** $\log_e \frac{5}{3}$

C. $-\log_e \frac{5}{3}$ **D.** $\frac{\log_e \frac{5}{3}}{2}$

22. Find $\operatorname{sech}^{-1}\left(\frac{15}{17}\right)$.

A. $\log_e \frac{5}{3}$ **B.** $-\log_e \frac{5}{3}$

C. $\frac{\log_e \frac{5}{3}}{2}$ **D.** $\log_e \frac{10}{3}$

23. Find $\operatorname{sech}^{-1}\left(\frac{8}{17}\right)$.

A. $\log_e 4$ **B.** $-\log_e 4$ **C.** $\log_e 8$ **D.** $\frac{\log_e 4}{2}$

24. Find the domain of $f(x) = \coth^{-1}(x - 9)$.

A. $[8, 10]$ **B.** $(8, 10)$

C. $(-\infty, 8) \cup (10, \infty)$ **D.** $(-\infty, 8] \cup [10, \infty)$

25. Express $\cosh^{-1}\left(\frac{29}{20}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

A. $\sinh^{-1}\left(\frac{441}{400}\right) = \operatorname{sech}^{-1}\left(\frac{29}{20}\right)$

B. $\sinh^{-1}\left(\frac{21}{20}\right) = \operatorname{sech}^{-1}\left(\frac{20}{29}\right)$

C. $\sinh^{-1}\left(\frac{21}{20}\right) = \operatorname{sech}^{-1}\left(\frac{29}{20}\right)$

D. $\sinh^{-1}\left(\frac{\sqrt{1241}}{20}\right) = \operatorname{sech}^{-1}\left(\frac{20}{29}\right)$

Key sheet

1. C	**2.** A	**3.** A	**4.** B	**5.** D
6. C	**7.** D	**8.** C	**9.** A	**10.** C
11. B	**12.** D	**13.** C	**14.** B	**15.** D
16. C	**17.** D	**18.** A	**19.** C	**20.** A
21. B	**22.** A	**23.** A	**24.** C	**25.** B

Solutions

1. Find $\sinh(\ln 2)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln 2} = 2,\ e^{-\ln 2} = \frac{1}{2}$

$\sinh(\ln 2) = \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$

Ans: (C) $\frac{3}{4}$

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2. Find $\cosh(\ln 3) + \sinh(\ln \frac{5}{2})$.

$\cosh(\ln 3) = \frac{3 + \frac{1}{3}}{2} = \frac{5}{3}$

$\sinh(\ln \frac{5}{2}) = \frac{\frac{5}{2} - \frac{2}{5}}{2} = \frac{21}{20}$

$\text{Sum} = \frac{163}{60}$

Ans: (A) $\frac{163}{60}$

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3. Find $\tanh(\ln \frac{5}{3})$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(\ln \frac{5}{3}) = \frac{\frac{5}{3} - \frac{3}{5}}{\frac{5}{3} + \frac{3}{5}} = \frac{8}{17}$

Ans: (A) $\frac{8}{17}$

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4. Find the range of $f(x) = 4 - 2\cosh x$.

$\cosh x \geq 1$, so $-2\cosh x \leq -2$.

$f(x) \leq 4 - 2 = 2$

Range $= (-\infty, 2]$

Ans: (B) $(-\infty, 2]$

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5. The function $f(x) = \tanh 5x + x^2 \sinh 5x$ is

cosh and sech are even; sinh and tanh are odd.

Replacing x by $-x$: $f(-x) = -f(x)$, so f is odd.

Ans: (D) odd

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6. Find the range of $f(x) = 6\,\text{sech}\, x - 1$.

$0 < \text{sech}\, x \leq 1$.

$-1 < f(x) \leq 5$

Range $= (-1, 5]$

Ans: (C) $(-1, 5]$

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7. If $\tanh x = \frac{3}{5}$, find $\coth x$.

$\coth x = \frac{1}{\tanh x} = \frac{5}{3}$

Ans: (D) $\frac{5}{3}$

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8. If $\tanh x = \frac{7}{25}$, find $\operatorname{sech} x$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{49}{625} = \frac{576}{625}$

$\operatorname{sech} x > 0$, so $\operatorname{sech} x = \frac{24}{25}$

Ans: (C) $\frac{24}{25}$

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9. If $\cosh x = \frac{25}{24}$ and x is negative, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{625}{576} - 1 = \frac{49}{576}$

x is negative, so $\sinh x$ is negative: $\sinh x = -\frac{7}{24}$

Ans: (A) $-\frac{7}{24}$

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10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{1369}{144} - 1} = \frac{35}{12}$, $\cosh y = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{37}{12} \cdot \frac{5}{3} + \frac{35}{12} \cdot \frac{4}{3} = \frac{325}{36}$

Ans: (C) $\frac{325}{36}$

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11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{841}{441} - 1} = \frac{20}{21}$, $\cosh y = \sqrt{1 + \frac{1225}{144}} = \frac{37}{12}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{29}{21} \cdot \frac{37}{12} + \frac{20}{21} \cdot \frac{35}{12} = \frac{197}{28}$

Ans: (B) $\frac{197}{28}$

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12. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{35}{12}$, with $x, y > 0$, find $\cosh(x-y)$.

$\sinh x = \sqrt{\frac{625}{576} - 1} = \frac{7}{24}$, $\cosh y = \sqrt{1 + \frac{1225}{144}} = \frac{37}{12}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{25}{24} \cdot \frac{37}{12} - \frac{7}{24} \cdot \frac{35}{12} = \frac{85}{36}$

Ans: (D) $\frac{85}{36}$

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13. If $\sinh x = \frac{12}{5}$, find $\cosh 2x$.

$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2 \cdot \frac{144}{25} = \frac{313}{25}$

Ans: (C) $\frac{313}{25}$

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14. If $\tanh x = \frac{3}{5}$, find $\sinh 2x$.

$\sinh 2x = \frac{2\tanh x}{1 - \tanh^2 x} = \frac{\frac{6}{5}}{1 - \frac{9}{25}} = \frac{15}{8}$

Ans: (B) $\frac{15}{8}$

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15. If $\tanh x = \frac{1}{7}$, find $\tanh 2x$.

$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x} = \frac{\frac{2}{7}}{1 + \frac{1}{49}} = \frac{7}{25}$

Ans: (D) $\frac{7}{25}$

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16. Find $(\cosh x - \sinh x)^2$ when $x = \ln 2$.

$(\cosh x - \sinh x)^2 = e^{-2x}$

$= e^{-2\ln 2} = 2^{-2} = \frac{1}{4}$

Ans: (C) $\frac{1}{4}$

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17. If $\tanh x = \frac{1}{5}$, find $\cosh 4x + \sinh 4x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 2$

$\frac{1+\frac{1}{5}}{1-\frac{1}{5}} = \frac{3}{2}$

$\text{Answer} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Ans: (D) $\frac{9}{4}$

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18. Express $\cosh 4x + \cosh 2x$ as a product.

$\cosh 4x + \cosh 2x = 2\cosh \frac{4+2}{2}x \cosh \frac{4-2}{2}x$

$= 2\cosh 3x \cosh x$

Ans: (A) $2\cosh 3x \cosh x$

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19. Express $\cosh 6x - \cosh 4x$ as a product.

$\cosh 6x - \cosh 4x = 2\sinh \frac{6+4}{2}x \sinh \frac{6-4}{2}x$

$= 2\sinh 5x \sinh x$

Ans: (C) $2\sinh 5x \sinh x$

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20. If $\sinh x = \frac{35}{12}$ and $\sinh y = \frac{3}{4}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{1225}{144} - \frac{9}{16} = \frac{143}{18}$

Ans: (A) $\frac{143}{18}$

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21. Find $\text{cosech}^{-1}\left(\frac{15}{8}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1+\sqrt{1+x^2}}{x}$ (for $x > 0$)

$\sqrt{1 + \frac{225}{64}} = \frac{17}{8}$

$= \log_e \frac{5}{3}$

Ans: (B) $\log_e \frac{5}{3}$

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22. Find $\text{sech}^{-1}\left(\frac{15}{17}\right)$.

$\text{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1 - \frac{225}{289}} = \frac{8}{17}$

$= \log_e \frac{1+\frac{8}{17}}{\frac{15}{17}} = \log_e \frac{5}{3}$

Ans: (A) $\log_e \frac{5}{3}$

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23. Find $\operatorname{sech}^{-1}\left(\frac{8}{17}\right)$.

$\operatorname{sech}^{-1} x = \log_e \frac{1+\sqrt{1-x^2}}{x}$

$\sqrt{1-\frac{64}{289}} = \frac{15}{17}$

$= \log_e \frac{1+\frac{15}{17}}{\frac{8}{17}} = \log_e 4$

Ans: (A) $\log_e 4$

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24. Find the domain of $f(x) = \coth^{-1}(x-9)$.

$\coth^{-1} u$ needs $|u| > 1$.

$|x-9| > 1 \Rightarrow x < 8$ or $x > 10$

Ans: (C) $(-\infty, 8) \cup (10, \infty)$

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25. Express $\cosh^{-1}\left(\frac{29}{20}\right)$ in terms of $\sinh^{-1}$ and of sech^{-1}.

$\cosh^{-1} x = \sinh^{-1}\sqrt{x^2-1} = \operatorname{sech}^{-1}\frac{1}{x}$ for $x \geq 1$.

$\sqrt{\frac{841}{400}-1} = \frac{21}{20}$, $\frac{1}{\frac{29}{20}} = \frac{20}{29}$

$\cosh^{-1}\left(\frac{29}{20}\right) = \sinh^{-1}\left(\frac{21}{20}\right) = \operatorname{sech}^{-1}\left(\frac{20}{29}\right)$

Ans: (B) $\sinh^{-1}\left(\frac{21}{20}\right) = \operatorname{sech}^{-1}\left(\frac{20}{29}\right)$
