EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 10

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\tanh(-\ln 4)$.

A. $\frac{15}{17}$ **B.** $\frac{17}{4}$ **C.** $\frac{15}{4}$ **D.** $-\frac{15}{17}$

2. Find $\coth(\ln \frac{5}{2})$.

A. $-\frac{29}{21}$ **B.** $\frac{21}{29}$ **C.** $\frac{29}{21}$ **D.** $\frac{21}{20}$

3. Find $\sinh(\ln \frac{5}{2})$.

A. $\frac{29}{10}$ **B.** $\frac{21}{10}$ **C.** $\frac{21}{20}$ **D.** $-\frac{21}{20}$

4. The range of $\tanh x$ is

A. $(-1, 1)$ **B.** $\mathbb{R}$ **C.** $[-1, 1]$ **D.** $(0, 1)$

5. The function $f(x) = x \sinh 2x + \operatorname{sech} 2x$ is

A. neither even nor odd **B.** even

C. both even and odd **D.** odd

6. Find the range of $f(x) = 6 - 6\cosh x$.

A. $(-\infty, 0]$ **B.** $(-\infty, 0)$ **C.** $(-\infty, 6]$ **D.** $[0, \infty)$

7. If $\sinh x = \frac{7}{24}$, find $\operatorname{cosech} x$.

A. $\frac{7}{24}$ **B.** $\frac{25}{24}$ **C.** $\frac{24}{7}$ **D.** $-\frac{24}{7}$

8. If $\sinh x = \frac{5}{12}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = \frac{13}{12},\ \tanh x = \frac{5}{13}$

B. $\cosh x = -\frac{13}{12},\ \tanh x = -\frac{5}{13}$

C. $\cosh x = \frac{\sqrt{119}}{12},\ \tanh x = \frac{5}{13}$

D. $\cosh x = \frac{13}{12},\ \tanh x = \frac{13}{5}$

9. If $\sinh x = -\frac{35}{12}$, find $\cosh x$ and $\tanh x$.

A. $\cosh x = \frac{\sqrt{1081}}{12},\ \tanh x = -\frac{35}{37}$

B. $\cosh x = \frac{37}{12},\ \tanh x = -\frac{37}{35}$

C. $\cosh x = -\frac{37}{12},\ \tanh x = \frac{35}{37}$

D. $\cosh x = \frac{37}{12},\ \tanh x = -\frac{35}{37}$

10. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{21}{20}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{109}{60}$ **B.** $-\frac{49}{120}$ **C.** $\frac{161}{240}$ **D.** $\frac{289}{240}$

11. If $\cosh x = \frac{13}{12}$ and $\sinh y = \frac{12}{5}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{221}{60}$ **B.** $\frac{169}{30}$ **C.** $\frac{229}{60}$ **D.** $\frac{109}{60}$

12. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{15}{8}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{23}{24}$ **B.** $-\frac{5}{12}$ **C.** $\frac{577}{48}$ **D.** $\frac{13}{12}$

13. If $\sinh x = \frac{20}{21}$, find $\cosh 2x$.

A. $\frac{61}{21}$ **B.** $\frac{800}{441}$ **C.** $-\frac{359}{441}$ **D.** $\frac{1241}{441}$

14. If $\cosh x = 2$, find $\cosh 3x$.

A. 38 **B.** 26 **C.** -26 **D.** 6

15. If $\tanh x = \frac{1}{3}$, find $\tanh 2x$.

A. $\frac{2}{3}$ **B.** $\frac{3}{4}$ **C.** $\frac{3}{10}$ **D.** $\frac{3}{5}$

16. Find $(\cosh x + \sinh x)^3$ when $x = \ln 4$.

A. 128 **B.** $\frac{1}{64}$ **C.** 12 **D.** 64

17. If $\tanh x = \frac{1}{4}$, find $\cosh 6x + \sinh 6x$.

A. $\frac{125}{27}$ **B.** $\frac{27}{125}$ **C.** $\frac{15625}{729}$ **D.** $\frac{125}{64}$

18. If $\sinh x = \frac{5}{12}$ and $\sinh y = \frac{7}{24}$, find $\sinh(x + y)\sinh(x - y)$.

A. $\frac{1}{64}$ **B.** $\frac{17}{192}$ **C.** $-\frac{17}{192}$ **D.** $\frac{149}{576}$

19. Express $\sinh 5x + \sinh 1x$ as a product.

A. $-2\sinh 3x \cosh 2x$ **B.** $2\sinh 3x \cosh 2x$

C. $2\cosh 3x \sinh 2x$ **D.** $2\cosh 3x \cosh 2x$

20. If $\sinh x = \frac{4}{3}$ and $\cosh y = \frac{29}{21}$, find $\cosh(x + y)\cosh(x - y)$.

A. $\frac{1625}{441}$ **B.** $\frac{275}{147}$ **C.** $\frac{361}{49}$ **D.** $\frac{19}{147}$

21. Find $\sinh^{-1}\left(\frac{12}{5}\right)$.

A. $\log_e 5$ **B.** $-\log_e 5$ **C.** $\log_e 10$ **D.** $\frac{\log_e 5}{2}$

22. Find $\tanh^{-1}\left(\frac{1}{3}\right)$.

A. $\log_e 2$ **B.** $\frac{\log_e 2}{2}$

C. $\frac{\log_e \frac{4}{3}}{2}$ **D.** $-\frac{\log_e 2}{2}$

23. Find $\operatorname{cosech}^{-1}\left(-\frac{20}{21}\right)$.

A. $\log_e 5$ **B.** $\frac{\log_e \frac{5}{2}}{2}$

C. $-\log_e \frac{5}{2}$ **D.** $\log_e \frac{5}{2}$

24. Find the domain of $f(x) = \coth^{-1}(x - 7)$.

A. $(6, 8)$

B. $(-\infty, 6] \cup [8, \infty)$

C. $(-\infty, 6) \cup (8, \infty)$

D. $[6, 8]$

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25. $\tanh^{-1}(-x) =$

A. $\coth^{-1} x$

B. $\frac{1}{\tanh^{-1} x}$

C. $\tanh^{-1} x$

D. $-\tanh^{-1} x$

Key sheet

1. D	**2.** C	**3.** C	**4.** A	**5.** B
6. A	**7.** C	**8.** A	**9.** D	**10.** D
11. C	**12.** D	**13.** D	**14.** B	**15.** D
16. D	**17.** A	**18.** B	**19.** B	**20.** A
21. A	**22.** B	**23.** C	**24.** C	**25.** D

Solutions

1. Find $\tanh(-\ln 4)$.

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(-\ln 4) = -\frac{4 - \frac{1}{4}}{4 + \frac{1}{4}} = -\frac{15}{17}$

Ans: (D) $-\frac{15}{17}$

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2. Find $\coth(\ln \frac{5}{2})$.

$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$e^{\ln \frac{5}{2}} = \frac{5}{2},\ e^{-\ln \frac{5}{2}} = \frac{2}{5}$

$\coth(\ln \frac{5}{2}) = \frac{29}{21}$

Ans: (C) $\frac{29}{21}$

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3. Find $\sinh(\ln \frac{5}{2})$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln \frac{5}{2}} = \frac{5}{2},\ e^{-\ln \frac{5}{2}} = \frac{2}{5}$

$\sinh(\ln \frac{5}{2}) = \frac{\frac{5}{2} - \frac{2}{5}}{2} = \frac{21}{20}$

Ans: (C) $\frac{21}{20}$

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4. The range of $\tanh x$ is

$|\tanh x| < 1$, and it takes every value in $(-1, 1)$.

Ans: (A) $(-1, 1)$

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5. The function $f(x) = x \sinh 2x + \operatorname{sech} 2x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: $f(-x) = f(x)$, so f is even.

Ans: (B) even

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6. Find the range of $f(x) = 6 - 6\cosh x$.

$\cosh x \geq 1$, so $-6\cosh x \leq -6$.

$f(x) \leq 6 - 6 = 0$

Range $= (-\infty, 0]$

Ans: (A) $(-\infty, 0]$

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7. If $\sinh x = \frac{7}{24}$, find $\operatorname{cosech} x$.

$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{24}{7}$

Ans: (C) $\frac{24}{7}$

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8. If $\sinh x = \frac{5}{12}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{25}{144} = \frac{169}{144}$

Since $\cosh x \geq 1$, $\cosh x = \frac{13}{12}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{5}{12}}{\frac{13}{12}} = \frac{5}{13}$

Ans: (A) $\cosh x = \frac{13}{12},\ \tanh x = \frac{5}{13}$

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9. If $\sinh x = -\frac{35}{12}$, find $\cosh x$ and $\tanh x$.

$\cosh^2 x = 1 + \sinh^2 x = 1 + \frac{1225}{144} = \frac{1369}{144}$

Since $\cosh x \geq 1$, $\cosh x = \frac{37}{12}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{-\frac{35}{12}}{\frac{37}{12}} = -\frac{35}{37}$

Ans: (D) $\cosh x = \frac{37}{12},\ \tanh x = -\frac{35}{37}$

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10. If $\cosh x = \frac{25}{24}$ and $\sinh y = \frac{21}{20}$, with $x, y > 0$, find $\cosh(x - y)$.

$\sinh x = \sqrt{\frac{625}{576} - 1} = \frac{7}{24},\ \cosh y = \sqrt{1 + \frac{441}{400}} = \frac{29}{20}$

$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{25}{24} \cdot \frac{29}{20} - \frac{7}{24} \cdot \frac{21}{20} = \frac{289}{240}$

Ans: (D) $\frac{289}{240}$

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11. If $\cosh x = \frac{13}{12}$ and $\sinh y = \frac{12}{5}$, with $x, y > 0$, find $\cosh(x + y)$.

$\sinh x = \sqrt{\frac{169}{144} - 1} = \frac{5}{12},\ \cosh y = \sqrt{1 + \frac{144}{25}} = \frac{13}{5}$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{13}{12} \cdot \frac{13}{5} + \frac{5}{12} \cdot \frac{12}{5} = \frac{229}{60}$

Ans: (C) $\frac{229}{60}$

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12. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{15}{8}$, with $x, y > 0$, find $\cosh(x - y)$.

$\sinh x = \sqrt{\frac{1369}{144} - 1} = \frac{35}{12},\ \cosh y = \sqrt{1 + \frac{225}{64}} = \frac{17}{8}$

$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{37}{12} \cdot \frac{17}{8} - \frac{35}{12} \cdot \frac{15}{8} = \frac{13}{12}$

Ans: (D) $\frac{13}{12}$

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13. If $\sinh x = \frac{20}{21}$, find $\cosh 2x$.

$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2 \cdot \frac{400}{441} = \frac{1241}{441}$

Ans: (D) $\frac{1241}{441}$

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14. If $\cosh x = 2$, find $\cosh 3x$.

$\cosh 3x = 4\cosh^3 x - 3\cosh x$

$= 4(2)^3 - 3(2) = 26$

Ans: (B) 26

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15. If $\tanh x = \frac{1}{3}$, find $\tanh 2x$.

$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x} = \frac{\frac{2}{3}}{1 + \frac{1}{9}} = \frac{3}{5}$

Ans: (D) $\frac{3}{5}$

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16. Find $(\cosh x + \sinh x)^3$ when $x = \ln 4$.

$(\cosh x + \sinh x)^3 = e^{3x}$

$= e^{3\ln 4} = 4^3 = 64$

Ans: (D) 64

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17. If $\tanh x = \frac{1}{4}$, find $\cosh 6x + \sinh 6x$.

$\cosh 2nx + \sinh 2nx = \left(\frac{1+\tanh x}{1-\tanh x}\right)^n$ with $n = 3$

$\frac{1+\frac{1}{4}}{1-\frac{1}{4}} = \frac{5}{3}$

$\text{Answer} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

Ans: (A) $\frac{125}{27}$

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18. If $\sinh x = \frac{5}{12}$ and $\sinh y = \frac{7}{24}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{25}{144} - \frac{49}{576} = \frac{17}{192}$

Ans: (B) $\frac{17}{192}$

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19. Express $\sinh 5x + \sinh 1x$ as a product.

$\sinh 5x + \sinh 1x = 2\sinh \frac{5+1}{2}x \cosh \frac{5-1}{2}x$

$= 2\sinh 3x \cosh 2x$

Ans: (B) $2\sinh 3x \cosh 2x$

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20. If $\sinh x = \frac{4}{3}$ and $\cosh y = \frac{29}{21}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{16}{9} + \frac{841}{441} = \frac{1625}{441}$

Ans: (A) $\frac{1625}{441}$

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21. Find $\sinh^{-1}\left(\frac{12}{5}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{144}{25}+1} = \frac{13}{5}$

$\sinh^{-1}\left(\frac{12}{5}\right) = \log_e\left(\frac{12}{5} + \frac{13}{5}\right) = \log_e 5$

Ans: (A) $\log_e 5$

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22. Find $\tanh^{-1}\left(\frac{1}{3}\right)$.

$\tanh^{-1} x = \frac{1}{2}\log_e \frac{1+x}{1-x}$

$= \frac{1}{2}\log_e \frac{\frac{4}{3}}{\frac{2}{3}} = \frac{\log_e 2}{2}$

Ans: (B) $\frac{\log_e 2}{2}$

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23. Find $\text{cosech}^{-1}\left(-\frac{20}{21}\right)$.

$\text{cosech}^{-1} x = \log_e \frac{1-\sqrt{1+x^2}}{x}$ (for $x < 0$)

$\sqrt{1+\frac{400}{441}} = \frac{29}{21}$

$= -\log_e \frac{5}{2}$

Ans: (C) $-\log_e \frac{5}{2}$

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24. Find the domain of $f(x) = \coth^{-1}(x-7)$.

$\coth^{-1} u$ needs $|u| > 1$.

$|x-7| > 1 \Rightarrow x < 6$ or $x > 8$

Ans: (C) $(-\infty, 6) \cup (8, \infty)$

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25. $\tanh^{-1}(-x) =$

From the table of inverse hyperbolic functions: $-\tanh^{-1} x$.

Ans: (D) $-\tanh^{-1} x$