EXPRESSWALA ACADEMY

Maths 1A: Hyperbolic Functions

MCQ Test, Set 01

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Find $\sinh(\ln 4)$.

A. $\frac{15}{8}$ **B.** $\frac{17}{4}$ **C.** $-\frac{15}{8}$ **D.** $\frac{15}{4}$

2. Find $\operatorname{sech}(\ln \frac{4}{3})$.

A. $\frac{7}{24}$ **B.** $\frac{24}{25}$ **C.** $\frac{25}{24}$ **D.** $-\frac{24}{25}$

3. Find $\cosh(\ln \frac{5}{2})$.

A. $-\frac{29}{20}$ **B.** $\frac{29}{10}$ **C.** $\frac{29}{20}$ **D.** $\frac{21}{10}$

4. Find the range of $f(x) = -4\cosh x - 5$.

A. $[-9, \infty)$ **B.** $(-\infty, -9)$ **C.** $(-\infty, -5]$ **D.** $(-\infty, -9]$

5. The function $f(x) = x\sinh 5x + x^2\cosh 5x$ is

A. neither even nor odd **B.** odd

C. both even and odd **D.** even

6. Find the range of $f(x) = 5\tanh x - 6$.

A. $(-7, -5)$ **B.** $(-11, -1)$ **C.** $[-11, -1]$ **D.** $(-5, 5)$

7. Simplify $\cosh x + \sinh x$ when $x = \ln \frac{7}{3}$.

A. $\frac{3}{7}$ **B.** $\frac{7}{3}$ **C.** $\frac{49}{9}$ **D.** $\frac{14}{3}$

8. If $\tanh x = -\frac{4}{5}$, find $\operatorname{sech} x$.

A. $\frac{3}{5}$ **B.** $-\frac{3}{5}$ **C.** $\frac{5}{4}$ **D.** $\frac{\sqrt{41}}{5}$

9. If $\cosh x = \frac{5}{4}$ and x is positive, find $\sinh x$.

A. $-\frac{3}{4}$ **B.** $\frac{3}{4}$ **C.** $\frac{4}{5}$ **D.** $\frac{\sqrt{41}}{4}$

10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{3}{4}$, with $x, y > 0$, find $\cosh(x + y)$.

A. $\frac{145}{24}$ **B.** $\frac{143}{24}$ **C.** $\frac{13}{3}$ **D.** $\frac{5}{3}$

11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{16}{63}$ **B.** $\frac{25}{7}$ **C.** $-\frac{2}{7}$ **D.** $\frac{65}{63}$

12. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{12}{5}$, with $x, y > 0$, find $\cosh(x - y)$.

A. $\frac{5}{4}$ **B.** $\frac{629}{100}$ **C.** $\frac{3}{4}$ **D.** $-\frac{23}{20}$

13. If $\sinh x = \frac{35}{12}$, find $\cosh 2x$.

A. $\frac{1297}{72}$ **B.** $\frac{41}{6}$ **C.** $\frac{1225}{72}$ **D.** $-\frac{1153}{72}$

14. If $\sinh x = 2$, find $\sinh 3x$.

A. 6 **B.** 26 **C.** 38 **D.** -26

15. If $\tanh x = \frac{2}{3}$, find $\sinh 2x$.

A. $\frac{4}{3}$ **B.** $\frac{13}{5}$ **C.** $\frac{12}{5}$ **D.** $\frac{12}{13}$

16. Find $(\cosh x + \sinh x)^4$ when $x = \ln 10$.

A. $\frac{1}{10000}$ **B.** 40 **C.** 20000 **D.** 10000

17. Find the coefficient of x^4 in the series for $\cosh x$.

A. $\frac{1}{6}$ **B.** $\frac{1}{24}$ **C.** $\frac{1}{4}$ **D.** $-\frac{1}{24}$

18. If $\sinh x = \frac{8}{15}$ and $\sinh y = \frac{15}{8}$, find $\sinh(x + y)\sinh(x - y)$.

A. $-\frac{46529}{14400}$ **B.** $\frac{25921}{14400}$ **C.** $\frac{46529}{14400}$ **D.** $\frac{54721}{14400}$

19. Express $\sinh 5x - \sinh 3x$ as a product.

A. $2\sinh 4x \sinh x$ **B.** $-2\cosh 4x \sinh x$

C. $2\sinh 4x \cosh x$ **D.** $2\cosh 4x \sinh x$

20. If $\sinh x = \frac{4}{3}$ and $\cosh y = \frac{25}{24}$, find $\cosh(x + y)\cosh(x - y)$.

A. $\frac{361}{64}$ **B.** $\frac{517}{192}$ **C.** $-\frac{133}{192}$ **D.** $\frac{1649}{576}$

21. Find $\coth^{-1}(2)$.

A. $\frac{\log_e 2}{2}$ **B.** $-\frac{\log_e 3}{2}$ **C.** $\frac{\log_e 3}{2}$ **D.** $\log_e 3$

22. Find $\coth^{-1}(3)$.

A. $\log_e 2$ **B.** $-\frac{\log_e 2}{2}$ **C.** $\frac{\log_e 4}{2}$ **D.** $\frac{\log_e 2}{2}$

23. Find $\sinh^{-1}\left(-\frac{5}{12}\right)$.

A. $\log_e 3$ **B.** $-\log_e \frac{3}{2}$

C. $\frac{\log_e \frac{3}{2}}{2}$ **D.** $\log_e \frac{3}{2}$

24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{2}\right)$.

A. $(0, 2)$ **B.** $[0, 2]$ **C.** $(0, 2]$ **D.** $[-2, 2]$

25. The range of $\tanh^{-1} x$ is

A. $\mathbb{R} - \{0\}$ **B.** $(-1, 1)$ **C.** $\mathbb{R}$ **D.** $[0, \infty)$

Key sheet

1. A	**2.** B	**3.** C	**4.** D	**5.** D
6. B	**7.** B	**8.** A	**9.** B	**10.** A
11. D	**12.** A	**13.** A	**14.** C	**15.** C
16. D	**17.** B	**18.** A	**19.** D	**20.** D
21. C	**22.** D	**23.** B	**24.** C	**25.** C

Solutions

1. Find $\sinh(\ln 4)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\ln 4} = 4,\ e^{-\ln 4} = \frac{1}{4}$

$\sinh(\ln 4) = \frac{4 - \frac{1}{4}}{2} = \frac{15}{8}$

Ans: (A) $\frac{15}{8}$

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2. Find $\operatorname{sech}(\ln \frac{4}{3})$.

$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

$e^{\ln \frac{4}{3}} = \frac{4}{3},\ e^{-\ln \frac{4}{3}} = \frac{3}{4}$

$\operatorname{sech}(\ln \frac{4}{3}) = \frac{24}{25}$

Ans: (B) $\frac{24}{25}$

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3. Find $\cosh(\ln \frac{5}{2})$.

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\cosh(\ln \frac{5}{2}) = \frac{\frac{5}{2} + \frac{2}{5}}{2} = \frac{29}{20}$

Ans: (C) $\frac{29}{20}$

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4. Find the range of $f(x) = -4\cosh x - 5$.

$\cosh x \geq 1$, so $-4\cosh x \leq -4$.

$f(x) \leq -5 - 4 = -9$

Range $= (-\infty, -9]$

Ans: (D) $(-\infty, -9]$

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5. The function $f(x) = x\sinh 5x + x^2\cosh 5x$ is

$\cosh$ and sech are even; $\sinh$ and $\tanh$ are odd.

Replacing x by $-x$: $f(-x) = f(x)$, so f is even.

Ans: (D) even

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6. Find the range of $f(x) = 5\tanh x - 6$.

$-1 < \tanh x < 1$.

$-6 - 5 < f(x) < -6 + 5$

Range $= (-11, -1)$

Ans: (B) $(-11, -1)$

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7. Simplify $\cosh x + \sinh x$ when $x = \ln \frac{7}{3}$.

$\cosh x + \sinh x = e^x$

$e^{\ln \frac{7}{3}} = \frac{7}{3}$

Ans: (B) $\frac{7}{3}$

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8. If $\tanh x = -\frac{4}{5}$, find $\operatorname{sech} x$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{16}{25} = \frac{9}{25}$

$\operatorname{sech} x > 0$, so $\operatorname{sech} x = \frac{3}{5}$

Ans: (A) $\frac{3}{5}$

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9. If $\cosh x = \frac{5}{4}$ and x is positive, find $\sinh x$.

$\sinh^2 x = \cosh^2 x - 1 = \frac{25}{16} - 1 = \frac{9}{16}$

x is positive, so $\sinh x$ is positive: $\sinh x = \frac{3}{4}$

Ans: (B) $\frac{3}{4}$

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10. If $\cosh x = \frac{37}{12}$ and $\sinh y = \frac{3}{4}$, with $x, y > 0$, find $\cosh(x+y)$.

$\sinh x = \sqrt{\frac{1369}{144} - 1} = \frac{35}{12}$, $\cosh y = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$= \frac{37}{12} \cdot \frac{5}{4} + \frac{35}{12} \cdot \frac{3}{4} = \frac{145}{24}$

Ans: (A) $\frac{145}{24}$

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11. If $\cosh x = \frac{29}{21}$ and $\sinh y = \frac{4}{3}$, with $x, y > 0$, find $\cosh(x-y)$.

$\sinh x = \sqrt{\frac{841}{441} - 1} = \frac{20}{21}$, $\cosh y = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{29}{21} \cdot \frac{5}{3} - \frac{20}{21} \cdot \frac{4}{3} = \frac{65}{63}$

Ans: (D) $\frac{65}{63}$

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12. If $\cosh x = \frac{29}{20}$ and $\sinh y = \frac{12}{5}$, with $x, y > 0$, find $\cosh(x-y)$.

$\sinh x = \sqrt{\frac{841}{400} - 1} = \frac{21}{20}$, $\cosh y = \sqrt{1 + \frac{144}{25}} = \frac{13}{5}$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

$= \frac{29}{20} \cdot \frac{13}{5} - \frac{21}{20} \cdot \frac{12}{5} = \frac{5}{4}$

Ans: (A) $\frac{5}{4}$

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13. If $\sinh x = \frac{35}{12}$, find $\cosh 2x$.

$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2 \cdot \frac{1225}{144} = \frac{1297}{72}$

Ans: (A) $\frac{1297}{72}$

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14. If $\sinh x = 2$, find $\sinh 3x$.

$\sinh 3x = 3\sinh x + 4\sinh^3 x$

$= 3(2) + 4(2)^3 = 38$

Ans: (C) 38

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15. If $\tanh x = \frac{2}{3}$, find $\sinh 2x$.

$\sinh 2x = \frac{2\tanh x}{1 - \tanh^2 x} = \frac{\frac{4}{3}}{1 - \frac{4}{9}} = \frac{12}{5}$

Ans: (C) $\frac{12}{5}$

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16. Find $(\cosh x + \sinh x)^4$ when $x = \ln 10$.

$(\cosh x + \sinh x)^4 = e^{4x}$

$= e^{4 \ln 10} = 10^4 = 10000$

Ans: (D) 10000

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17. Find the coefficient of x^4 in the series for $\cosh x$.

$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

The coefficient is $\frac{1}{4!} = \frac{1}{24}$.

Ans: (B) $\frac{1}{24}$

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18. If $\sinh x = \frac{8}{15}$ and $\sinh y = \frac{15}{8}$, find $\sinh(x+y)\sinh(x-y)$.

$\sinh(x+y)\sinh(x-y) = \sinh^2 x - \sinh^2 y$

$= \frac{64}{225} - \frac{225}{64} = -\frac{46529}{14400}$

Ans: (A) $-\frac{46529}{14400}$

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19. Express $\sinh 5x - \sinh 3x$ as a product.

$\sinh 5x - \sinh 3x = 2\cosh \frac{5+3}{2}x \ \sinh \frac{5-3}{2}x$

$= 2\cosh 4x \sinh x$

Ans: (D) $2\cosh 4x \sinh x$

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20. If $\sinh x = \frac{4}{3}$ and $\cosh y = \frac{25}{24}$, find $\cosh(x+y)\cosh(x-y)$.

$\cosh(x+y)\cosh(x-y) = \sinh^2 x + \cosh^2 y$

$= \frac{16}{9} + \frac{625}{576} = \frac{1649}{576}$

Ans: (D) $\frac{1649}{576}$

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21. Find $\coth^{-1}(2)$.

$\coth^{-1} x = \frac{1}{2}\log_e \frac{x+1}{x-1}$

$= \frac{1}{2}\log_e \frac{3}{1} = \frac{\log_e 3}{2}$

Ans: (C) $\frac{\log_e 3}{2}$

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22. Find $\coth^{-1}(3)$.

$\coth^{-1} x = \frac{1}{2}\log_e \frac{x+1}{x-1}$

$= \frac{1}{2}\log_e \frac{4}{2} = \frac{\log_e 2}{2}$

Ans: (D) $\frac{\log_e 2}{2}$

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23. Find $\sinh^{-1}\left(-\frac{5}{12}\right)$.

$\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

$\sqrt{\frac{25}{144}+1} = \frac{13}{12}$

$\sinh^{-1}\left(-\frac{5}{12}\right) = \log_e\left(-\frac{5}{12} + \frac{13}{12}\right) = -\log_e \frac{3}{2}$

Ans: (B) $-\log_e \frac{3}{2}$

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24. Find the domain of $f(x) = \operatorname{sech}^{-1}\left(\frac{x}{2}\right)$.

$\operatorname{sech}^{-1} u$ needs $0 < u \leq 1$.

$0 < \frac{x}{2} \leq 1 \Rightarrow 0 < x \leq 2$

Ans: (C) $(0, 2]$

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25. The range of $\tanh^{-1} x$ is

From the table of inverse hyperbolic functions: $\mathbb{R}$.

Ans: (C) $\mathbb{R}$
