EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 12 m away from the foot of a temple gopuram, the angle of elevation of its top is 75°. Find its height. [1 mark]

2. A chimney is 120 m high. From a point $40\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A temple gopuram is 90 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 60°? [2 marks]

4. From the top of a bridge 54 m high, the angle of depression of a boat is 15°. What is the angle of elevation of the top of the bridge seen from the boat? [1 mark]

5. From the top of a lighthouse 33 m high, the angles of depression of two ships on the same side of it, in line with its foot are 45° and 75°. Find the distance between them. [3 marks]

6. Two points 300 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 60°. Find the height of the tower. [1 mark]

7. Two points 54 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 60° and 15°. Find its height. [2 marks]

8. Two points A and B, 21 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 45° and 30°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a building from a point on level ground is 15°. After walking 24 m towards it, the angle becomes 75°. Find the height of the building. [2 marks]

10. A temple gopuram is 21 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 30° to 45°? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 45 m high, it is 45°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 30 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 30°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 15 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 75°. Find the height of the tower. [2 marks]

14. A tower is 42 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 30°. Find the length of the flagstaff. [3 marks]

15. From a point 60 m above a lake, the angle of elevation of a cloud is 45° and the angle of depression of its reflection is 60°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 33 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 30°. After walking 21 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 75^\circ$ and $\angle DCB = 15^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$12(\sqrt{3}+2)$ m
2	60°
3	$30(3-\sqrt{3})$ m
4	15°
5	$33(\sqrt{3}-1)$ m
6	$150\sqrt{3}$ m
7	$27(2\sqrt{3}-3)$ m
8	$h=\frac{21(\sqrt{3}-1)}{2}$ m, $AF=\frac{21(\sqrt{3}-1)}{2}$ m, $BF=\frac{21(3-\sqrt{3})}{2}$ m
9	$4\sqrt{3}$ m
10	$21(\sqrt{3}-1)$ m
11	$\frac{45(\sqrt{3}+1)}{2}$ m
12	$h=45$ m, distance $=15\sqrt{3}$ m
13	$\frac{15(\sqrt{3}-1)}{4}$ m
14	$28\sqrt{3}$ m
15	$60(\sqrt{3}+2)$ m
16	$h=\frac{33(\sqrt{3}+1)}{2}$ m, $x=\frac{33(\sqrt{3}+1)}{2}$ m
17	120°
18	$\frac{21\sqrt{6}}{4}$ m
19	$-\sqrt{3}$
20	$\cot\theta=\frac{-9+2\sqrt{3}}{15}$

Solutions

1. From a point 12 m away from the foot of a temple gopuram, the angle of elevation of its top is 75°. Find its height. [1]

$\tan 75^\circ = \frac{h}{12}$

$h = 12\tan 75^\circ = 12 \times (2+\sqrt{3}) = 12(\sqrt{3}+2)$ m

Ans: $12(\sqrt{3}+2)$ m

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2. A chimney is 120 m high. From a point $40\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{120}{40\sqrt{3}} = \sqrt{3}$

$\theta = 60^\circ$

Ans: 60°

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3. A temple gopuram is 90 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 60°? [2]

Shadow length $= h\cot(\text{elevation})$.

$90\cot 45^\circ - 90\cot 60^\circ = 90(1-\frac{1}{\sqrt{3}})$

$= 30(3-\sqrt{3})$ m

Ans: $30(3-\sqrt{3})$ m

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4. From the top of a bridge 54 m high, the angle of depression of a boat is 15°. What is the angle of elevation of the top of the bridge seen from the boat? [1]

The horizontal lines at the two heights are parallel.

Angle of elevation from below $=$ angle of depression from above $= 15^\circ$.

Ans: 15°

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5. From the top of a lighthouse 33 m high, the angles of depression of two ships on the same side of it, in line with its foot are 45° and 75°. Find the distance between them. [3]

distances from the foot: $33\cot 45^\circ = 33,\ 33\cot 75^\circ = 33(2-\sqrt{3})$

Same side: subtract.

distance $= 33(\sqrt{3}-1)$ m

Ans: $33(\sqrt{3}-1)$ m

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6. Two points 300 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 60°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 150 m from it.

$h = 150\tan 60^\circ = 150\sqrt{3}$ m

Ans: $150\sqrt{3}$ m

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7. Two points 54 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 60° and 15°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{54}{\frac{1}{\sqrt{3}} + (2+\sqrt{3})}$

$= 27(2\sqrt{3} - 3)$ m

Ans: $27(2\sqrt{3} - 3)$ m

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8. Two points A and B, 21 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 45° and 30°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 45^\circ = 1\,h,\ BF = h\cot 30^\circ = \sqrt{3}\,h$

$AF + BF = 21 \Rightarrow h(1+\sqrt{3}) = 21$

$h = \frac{21}{1+\sqrt{3}} = \frac{21(\sqrt{3}-1)}{2}$ m

$AF = \frac{21(\sqrt{3}-1)}{2}$ m, $BF = \frac{21(3-\sqrt{3})}{2}$ m

Ans: $h = \frac{21(\sqrt{3}-1)}{2}$ m, $AF = \frac{21(\sqrt{3}-1)}{2}$ m, $BF = \frac{21(3-\sqrt{3})}{2}$ m

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9. The angle of elevation of the top of a building from a point on level ground is 15°. After walking 24 m towards it, the angle becomes 75°. Find the height of the building. [2]

$h\cot 15^\circ - h\cot 75^\circ = 24$

$h = \frac{24}{(2+\sqrt{3})-(2-\sqrt{3})} = 4\sqrt{3}$ m

Ans: $4\sqrt{3}$ m

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10. A temple gopuram is 21 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 30° to 45°? [3]

$\text{distance} = h\cot 30^\circ - h\cot 45^\circ = 21(\sqrt{3}-1)$

$= 21(\sqrt{3}-1)$ m

Ans: $21(\sqrt{3}-1)$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 45 m high, it is 45°. Find the height of the tower. [2]

$h = x\tan 75^\circ,\ h - 45 = x\tan 45^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{45\times 1}{1-(2-\sqrt{3})}$

$= \frac{45(\sqrt{3}+1)}{2}$ m

Ans: $\frac{45(\sqrt{3}+1)}{2}$ m

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12. A is a point on level ground and B is 30 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 30°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 60^\circ = \sqrt{3}\,x$

$h - 30 = x\tan 30^\circ = \frac{1}{\sqrt{3}}\,x$

Subtracting: $30 = x(\sqrt{3} - \frac{1}{\sqrt{3}})$

$x = 15\sqrt{3}$ m

$h = 45$ m

Ans: $h = 45$ **m, distance** $= 15\sqrt{3}$ **m**

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13. A flagstaff 15 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 75°. Find the height of the tower. [2]

$h = x\tan 30^\circ,\ h + 15 = x\tan 75^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{15\times(2-\sqrt{3})}{\sqrt{3}-(2-\sqrt{3})}$

$= \frac{15(\sqrt{3}-1)}{4}$ m

Ans: $\frac{15(\sqrt{3}-1)}{4}$ m

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14. A tower is 42 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 30°. Find the length of the flagstaff. [3]

$x = 42\cot 15^\circ = 42(\sqrt{3}+2)$

$42 + l = x\tan 30^\circ = 42(\sqrt{3}+2)\times\frac{1}{\sqrt{3}} = 14(2\sqrt{3}+3)$

$l = 28\sqrt{3}$ m

Ans: $28\sqrt{3}$ m

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15. From a point 60 m above a lake, the angle of elevation of a cloud is 45° and the angle of depression of its reflection is 60°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{60\sin 105^\circ}{\sin 15^\circ}$

$= 60(\sqrt{3}+2)$ m

Ans: $60(\sqrt{3}+2)$ m

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16. An observer is 33 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 33 = x\tan 15^\circ,\ h + 33 = x\tan 60^\circ$

Subtracting: $66 = x(\sqrt{3} - (2-\sqrt{3}))$

$x = \frac{33(\sqrt{3}+1)}{2}$ m

$h = 33 + x\tan 15^\circ = \frac{33(\sqrt{3}+1)}{2}$ m

Ans: $h = \frac{33(\sqrt{3}+1)}{2}$ **m,** $x = \frac{33(\sqrt{3}+1)}{2}$ **m**

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17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1]

The angle at Q is $180^\circ - (\gamma - \beta)$.
$= 120^\circ$

Ans: 120°

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18. The angle of elevation of the top of a tower from a point is 30°. After walking 21 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:
$\angle TPQ = 30^\circ - 15^\circ = 15^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$
$\angle PTQ = 75^\circ - 30^\circ = 45^\circ$
Sine rule: $PT = \frac{21 \sin 60^\circ}{\sin 45^\circ}$
$h = PT \sin 30^\circ = \frac{21 \sin 60^\circ \sin 30^\circ}{\sin 45^\circ}$
$= \frac{21\sqrt{6}}{4}$ m

Ans: $\frac{21\sqrt{6}}{4}$ m

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19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 75^\circ$ and $\angle DCB = 15^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta,\ m = n = 1$
$2\cot\theta = (2-\sqrt{3}) - (2+\sqrt{3}) \Rightarrow \cot\theta = -\sqrt{3}$

Ans: $-\sqrt{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
$5\cot\theta = 2(\frac{1}{\sqrt{3}}) - 3(1)$
$\cot\theta = \frac{-9+2\sqrt{3}}{15}$

Ans: $\cot\theta = \frac{-9+2\sqrt{3}}{15}$
