EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 90 m away from the foot of a building, the angle of elevation of its top is 75°. Find its height. [1 mark]

2. A temple gopuram is 27 m high. From a point $9\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A chimney is 96 m high. By how much is its shadow longer when the sun's elevation is 60° than when it is 75°? [2 marks]

4. From the top of a bridge 30 m high, the angle of depression of a boat is 30°. How far is the boat from the foot of the bridge? [1 mark]

5. From the top of a tower 120 m high, the angles of depression of two cars on opposite sides of it, in line with its foot are 15° and 75°. Find the distance between them. [3 marks]

6. Two points 60 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 150 m apart on level ground lie on opposite sides of a building, in line with its foot. The angles of elevation of its top from them are 15° and 75°. Find its height. [2 marks]

8. Two points A and B, 33 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 60°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a tower from a point on level ground is 30°. After walking 72 m towards it, the angle becomes 60°. Find the height of the tower. [2 marks]

10. The angle of elevation of the top of a building is 45°. After walking 33 m towards it, it is 60°. How far is the observer now from the foot of the building? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 45 m high, it is 30°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 25 m vertically above A. The angles of elevation of the top of a tower from A and B are 30° and 15°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 15 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 45°. Find the height of the tower. [2 marks]

14. A tower is 120 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3 marks]

15. From a point 24 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 150 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 45°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 45°. After walking 72 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $A = 45^\circ$ and $B = 15^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$90(\sqrt{3}+2)$ m
2	60°
3	$64(2\sqrt{3}-3)$ m
4	$30\sqrt{3}$ m
5	480 m
6	$10\sqrt{3}$ m
7	$\frac{75}{2}$ m
8	$h=\frac{33(\sqrt{3}+3)}{4}$ m, $AF=\frac{33(3-\sqrt{3})}{4}$ m, $BF=\frac{33(\sqrt{3}+1)}{4}$ m
9	$36\sqrt{3}$ m
10	$\frac{33(\sqrt{3}+1)}{2}$ m
11	$\frac{135}{2}$ m
12	$h=\frac{25(\sqrt{3}+2)}{2}$ m, distance $=\frac{25(2\sqrt{3}+3)}{2}$ m
13	$\frac{15(\sqrt{3}-1)}{2}$ m
14	$240(\sqrt{3}+1)$ m
15	$12(\sqrt{3}+1)$ m
16	$h=150\sqrt{3}$ m, $x=150(\sqrt{3}+1)$ m
17	15°
18	$36\sqrt{6}$ m
19	$\frac{-(\sqrt{3}+1)}{2}$
20	$\cot\theta=-\frac{\sqrt{3}}{5}$

Solutions

1. From a point 90 m away from the foot of a building, the angle of elevation of its top is 75°. Find its height. [1]

$\tan 75^\circ = \frac{h}{90}$

$h = 90 \tan 75^\circ = 90 \times (2+\sqrt{3}) = 90(\sqrt{3}+2)$ m

Ans: $90(\sqrt{3}+2)$ m

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2. A temple gopuram is 27 m high. From a point $9\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{27}{9\sqrt{3}} = \sqrt{3}$

$\theta = 60^\circ$

Ans: 60°

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3. A chimney is 96 m high. By how much is its shadow longer when the sun's elevation is 60° than when it is 75°? [2]

Shadow length $= h\cot(\text{elevation})$.

$96\cot 60^\circ - 96\cot 75^\circ = 96(\frac{1}{\sqrt{3}} - (2-\sqrt{3}))$

$= 64(2\sqrt{3}-3)$ m

Ans: $64(2\sqrt{3}-3)$ m

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4. From the top of a bridge 30 m high, the angle of depression of a boat is 30°. How far is the boat from the foot of the bridge? [1]

The angle of elevation of the top from the boat is also 30°.

$x = 30\cot 30^\circ = 30 \times \sqrt{3} = 30\sqrt{3}$ m

Ans: $30\sqrt{3}$ m

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5. From the top of a tower 120 m high, the angles of depression of two cars on opposite sides of it, in line with its foot are 15° and 75°. Find the distance between them. [3]

distances from the foot: $120\cot 15^\circ = 120(\sqrt{3}+2)$, $120\cot 75^\circ = 120(2-\sqrt{3})$

Opposite sides: add.

distance $= 480$ m

Ans: 480 m

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6. Two points 60 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 30 m from it.

$h = 30\tan 30^\circ = 10\sqrt{3}$ m

Ans: $10\sqrt{3}$ m

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7. Two points 150 m apart on level ground lie on opposite sides of a building, in line with its foot. The angles of elevation of its top from them are 15° and 75°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{150}{(2+\sqrt{3})+(2-\sqrt{3})}$

$= \frac{75}{2}$ m

Ans: $\frac{75}{2}$ m

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8. Two points A and B, 33 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 60°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 75^\circ = (2-\sqrt{3})\,h,\ BF = h\cot 60^\circ = \frac{1}{\sqrt{3}}\,h$

$AF + BF = 33 \Rightarrow h\left((2-\sqrt{3}) + \frac{1}{\sqrt{3}}\right) = 33$

$h = \frac{33}{(2-\sqrt{3})+\frac{1}{\sqrt{3}}} = \frac{33(\sqrt{3}+3)}{4}$ m

$AF = \frac{33(3-\sqrt{3})}{4}$ m, $BF = \frac{33(\sqrt{3}+1)}{4}$ m

Ans: $h = \frac{33(\sqrt{3}+3)}{4}$ m, $AF = \frac{33(3-\sqrt{3})}{4}$ m, $BF = \frac{33(\sqrt{3}+1)}{4}$ m

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9. The angle of elevation of the top of a tower from a point on level ground is 30°. After walking 72 m towards it, the angle becomes 60°. Find the height of the tower. [2]

$h\cot 30^\circ - h\cot 60^\circ = 72$

$h = \frac{72}{\sqrt{3}-\frac{1}{\sqrt{3}}} = 36\sqrt{3}$ m

Ans: $36\sqrt{3}$ m

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10. The angle of elevation of the top of a building is 45°. After walking 33 m towards it, it is 60°. How far is the observer now from the foot of the building? [3]

$h = \frac{33}{1-\frac{1}{\sqrt{3}}} = \frac{33(\sqrt{3}+3)}{2}$

$\text{distance} = h\cot 60^\circ = \frac{33(\sqrt{3}+3)}{2} \times \frac{1}{\sqrt{3}} = \frac{33(\sqrt{3}+1)}{2}$ m

Ans: $\frac{33(\sqrt{3}+1)}{2}$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 45 m high, it is 30°. Find the height of the tower. [2]

$h = x\tan 60^\circ,\ h - 45 = x\tan 30^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{45\times\sqrt{3}}{\sqrt{3}-\frac{1}{\sqrt{3}}}$

$= \frac{135}{2}$ m

Ans: $\frac{135}{2}$ m

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12. A is a point on level ground and B is 25 m vertically above A. The angles of elevation of the top of a tower from A and B are 30° and 15°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 30^\circ = \frac{1}{\sqrt{3}}\,x$

$h - 25 = x\tan 15^\circ = (2-\sqrt{3})\,x$

Subtracting: $25 = x(\frac{1}{\sqrt{3}} - (2-\sqrt{3}))$

$x = \frac{25(2\sqrt{3}+3)}{2}$ m

$h = \frac{25(\sqrt{3}+2)}{2}$ m

Ans: $h = \frac{25(\sqrt{3}+2)}{2}$ m, distance $= \frac{25(2\sqrt{3}+3)}{2}$ m

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13. A flagstaff 15 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 45°. Find the height of the tower. [2]

$h = x\tan 15^\circ,\ h + 15 = x\tan 45^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{15\times 1}{(2+\sqrt{3})-1}$

$= \frac{15(\sqrt{3}-1)}{2}$ m

Ans: $\frac{15(\sqrt{3}-1)}{2}$ m

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14. A tower is 120 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3]

$x = 120\cot 30^\circ = 120\sqrt{3}$

$120 + l = x\tan 75^\circ = 120\sqrt{3}\times(2+\sqrt{3}) = 120(2\sqrt{3}+3)$

$l = 240(\sqrt{3}+1)$ m

Ans: $240(\sqrt{3}+1)$ m

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15. From a point 24 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{24\sin 105^\circ}{\sin 45^\circ}$

$= 12(\sqrt{3}+1)$ m

Ans: $12(\sqrt{3}+1)$ m

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16. An observer is 150 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 45°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 150 = x\tan 15^\circ,\ h + 150 = x\tan 45^\circ$

Subtracting: $300 = x(1-(2-\sqrt{3}))$

$x = 150(\sqrt{3}+1)$ m

$h = 150 + x\tan 15^\circ = 150\sqrt{3}$ m

Ans: $h = 150\sqrt{3}$ m, $x = 150(\sqrt{3}+1)$ m

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17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1]

The angle at P is $\alpha - \beta$.

$= 15^\circ$

Ans: 15°

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18. The angle of elevation of the top of a tower from a point is 45°. After walking 72 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:

$\angle TPQ = 45^\circ - 15^\circ = 30^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$

$\angle PTQ = 75^\circ - 45^\circ = 30^\circ$

Sine rule: $PT = \frac{72 \sin 60^\circ}{\sin 30^\circ}$

$h = PT \sin 45^\circ = \frac{72 \sin 60^\circ \sin 45^\circ}{\sin 30^\circ}$

$= 36\sqrt{6}$ m

Ans: $36\sqrt{6}$ m

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19. In triangle ABC, D is the midpoint of AB, $A = 45^\circ$ and $B = 15^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = n\cot A - m\cot B,\ m = n = 1$

$2\cot\theta = 1 - (2+\sqrt{3}) \Rightarrow \cot\theta = \frac{-(\sqrt{3}+1)}{2}$

Ans: $\frac{-(\sqrt{3}+1)}{2}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$5\cot\theta = 3(\frac{1}{\sqrt{3}}) - 2(\sqrt{3})$

$\cot\theta = -\frac{\sqrt{3}}{5}$

Ans: $\cot\theta = -\frac{\sqrt{3}}{5}$
