EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 48 m away from the foot of a minar, the angle of elevation of its top is 75°. Find its height. [1 mark]

2. A tower is 27 m high. From a point 27 m from its foot, find the angle of elevation of its top. [1 mark]

3. A pole is 45 m high. By how much is its shadow longer when the sun's elevation is 30° than when it is 45°? [2 marks]

4. From the top of a lighthouse 18 m high, the angle of depression of a ship is 60°. How far is the ship from the foot of the lighthouse? [1 mark]

5. From the top of a lighthouse 27 m high, the angles of depression of two ships on the same side of it, in line with its foot are 45° and 60°. Find the distance between them. [3 marks]

6. Two points 30 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 60°. Find the height of the tower. [1 mark]

7. Two points 42 m apart on level ground lie on opposite sides of a temple gopuram, in line with its foot. The angles of elevation of its top from them are 60° and 15°. Find its height. [2 marks]

8. Two points A and B, 150 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 60° and 15°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a chimney from a point on level ground is 15°. After walking 150 m towards it, the angle becomes 30°. Find the height of the chimney. [2 marks]

10. A building is 12 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 45° to 75°? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 45°, and from the top of the building, 15 m high, it is 15°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 12 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 60°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 8 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 75°. Find the height of the tower. [2 marks]

14. A tower is 42 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 75°. Find the length of the flagstaff. [3 marks]

15. From a point 48 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 36 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 30°. After walking 27 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 60°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 45^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 1$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$48(\sqrt{3}+2)$ m
2	45°
3	$45(\sqrt{3}-1)$ m
4	$6\sqrt{3}$ m
5	$9(3-\sqrt{3})$ m
6	$15\sqrt{3}$ m
7	$21(2\sqrt{3}-3)$ m
8	$h = 75(2\sqrt{3}-3)$ m, $AF = 75(2-\sqrt{3})$ m, $BF = 75\sqrt{3}$ m
9	75 m
10	$12(\sqrt{3}-1)$ m
11	$\frac{15(\sqrt{3}+1)}{2}$ m
12	$h = 6(\sqrt{3}+2)$ m, distance $= 6$ m
13	$\frac{4(2\sqrt{3}-3)}{3}$ m
14	$84(2\sqrt{3}+3)$ m
15	$32\sqrt{3}$ m
16	$h = 18(\sqrt{3}+1)$ m, $x = 18(\sqrt{3}+1)$ m
17	45°
18	$\frac{27\sqrt{2}}{2}$ m
19	$\frac{\sqrt{3}-3}{6}$
20	$\cot\theta = -\frac{\sqrt{3}}{9}$

Solutions

1. From a point 48 m away from the foot of a minar, the angle of elevation of its top is 75°. Find its height. [1]

$\tan 75^\circ = \frac{h}{48}$

$h = 48 \tan 75^\circ = 48 \times (2+\sqrt{3}) = 48(\sqrt{3}+2)$ m

Ans: $48(\sqrt{3}+2)$ m

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2. A tower is 27 m high. From a point 27 m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{27}{27} = 1$

$\theta = 45^\circ$

Ans: 45°

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3. A pole is 45 m high. By how much is its shadow longer when the sun's elevation is 30° than when it is 45°? [2]

Shadow length $= h\cot(\text{elevation})$.

$45\cot 30^\circ - 45\cot 45^\circ = 45(\sqrt{3}-1)$

$= 45(\sqrt{3}-1)$ m

Ans: $45(\sqrt{3}-1)$ m

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4. From the top of a lighthouse 18 m high, the angle of depression of a ship is 60°. How far is the ship from the foot of the lighthouse? [1]

The angle of elevation of the top from the ship is also 60°.

$x = 18\cot 60^\circ = 18 \times \frac{1}{\sqrt{3}} = 6\sqrt{3}$ m

Ans: $6\sqrt{3}$ m

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5. From the top of a lighthouse 27 m high, the angles of depression of two ships on the same side of it, in line with its foot are 45° and 60°. Find the distance between them. [3]

distances from the foot: $27\cot 45^\circ = 27,\ 27\cot 60^\circ = 9\sqrt{3}$

Same side: subtract.

distance $= 9(3-\sqrt{3})$ m

Ans: $9(3-\sqrt{3})$ m

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6. Two points 30 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 60°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 15 m from it.

$h = 15\tan 60^\circ = 15\sqrt{3}$ m

Ans: $15\sqrt{3}$ m

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7. Two points 42 m apart on level ground lie on opposite sides of a temple gopuram, in line with its foot. The angles of elevation of its top from them are 60° and 15°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{42}{\frac{1}{\sqrt{3}} + (2+\sqrt{3})}$

$= 21(2\sqrt{3} - 3)$ m

Ans: $21(2\sqrt{3} - 3)$ m

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8. Two points A and B, 150 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 60° and 15°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 60^\circ = \frac{1}{\sqrt{3}}h,\ BF = h\cot 15^\circ = (2+\sqrt{3})\,h$

$AF + BF = 150 \Rightarrow h(\frac{1}{\sqrt{3}} + (2+\sqrt{3})) = 150$

$h = \frac{150}{\frac{1}{\sqrt{3}} + (2+\sqrt{3})} = 75(2\sqrt{3} - 3)$ m

$AF = 75(2-\sqrt{3})$ m, $BF = 75\sqrt{3}$ m

Ans: $h = 75(2\sqrt{3} - 3)$ m, $AF = 75(2-\sqrt{3})$ m, $BF = 75\sqrt{3}$ m

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9. The angle of elevation of the top of a chimney from a point on level ground is 15°. After walking 150 m towards it, the angle becomes 30°. Find the height of the chimney. [2]

$h\cot 15^\circ - h\cot 30^\circ = 150$

$h = \frac{150}{(2+\sqrt{3}) - \sqrt{3}} = 75$ m

Ans: 75 m

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10. A building is 12 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 45° to 75°? [3]

$\text{distance} = h\cot 45^\circ - h\cot 75^\circ = 12(1 - (2-\sqrt{3}))$

$= 12(\sqrt{3} - 1)$ m

Ans: $12(\sqrt{3} - 1)$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 45°, and from the top of the building, 15 m high, it is 15°. Find the height of the tower. [2]

$h = x\tan 45^\circ,\ h - 15 = x\tan 15^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{15\times(2+\sqrt{3})}{(2+\sqrt{3}) - 1}$

$= \frac{15(\sqrt{3}+1)}{2}$ m

Ans: $\frac{15(\sqrt{3}+1)}{2}$ m

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12. A is a point on level ground and B is 12 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 60°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 75^\circ = (2+\sqrt{3})\,x$

$h - 12 = x\tan 60^\circ = \sqrt{3}\,x$

Subtracting: $12 = x((2+\sqrt{3}) - \sqrt{3})$

$x = 6$ m

$h = 6(\sqrt{3}+2)$ m

Ans: $h = 6(\sqrt{3}+2)$ **m, distance** $= 6$ **m**

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13. A flagstaff 8 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 75°. Find the height of the tower. [2]

$h = x\tan 15^\circ,\ h + 8 = x\tan 75^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{8\times(2-\sqrt{3})}{(2+\sqrt{3})-(2-\sqrt{3})}$

$= \frac{4(2\sqrt{3}-3)}{3}$ m

Ans: $\frac{4(2\sqrt{3}-3)}{3}$ m

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14. A tower is 42 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 75°. Find the length of the flagstaff. [3]

$x = 42\cot 15^\circ = 42(\sqrt{3}+2)$

$42 + l = x\tan 75^\circ = 42(\sqrt{3}+2)\times(2+\sqrt{3}) = 42(4\sqrt{3}+7)$

$l = 84(2\sqrt{3}+3)$ m

Ans: $84(2\sqrt{3}+3)$ m

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15. From a point 48 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{48\sin 90^\circ}{\sin 60^\circ}$

$= 32\sqrt{3}$ m

Ans: $32\sqrt{3}$ m

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16. An observer is 36 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 36 = x\tan 15^\circ,\ h + 36 = x\tan 60^\circ$

Subtracting: $72 = x(\sqrt{3} - (2-\sqrt{3}))$

$x = 18(\sqrt{3}+1)$ m

$h = 36 + x\tan 15^\circ = 18(\sqrt{3}+1)$ m

Ans: $h = 18(\sqrt{3}+1)$ **m,** $x = 18(\sqrt{3}+1)$ **m**

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17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1]

The angle at T is $\gamma - \alpha$.

$= 45^\circ$

Ans: 45°

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18. The angle of elevation of the top of a tower from a point is 30°. After walking 27 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 60°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:

$\angle TPQ = 30^\circ - 15^\circ = 15^\circ,\ \angle PQT = 180^\circ - (60^\circ - 15^\circ) = 135^\circ$

$\angle PTQ = 60^\circ - 30^\circ = 30^\circ$

Sine rule: $PT = \frac{27 \sin 45^\circ}{\sin 30^\circ}$

$h = PT \sin 30^\circ = \frac{27 \sin 45^\circ \sin 30^\circ}{\sin 30^\circ}$

$= \frac{27\sqrt{2}}{2}$ m

Ans: $\frac{27\sqrt{2}}{2}$ m

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19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 45^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta,\ m = n = 1$

$2\cot\theta = \frac{1}{\sqrt{3}} - 1 \Rightarrow \cot\theta = \frac{\sqrt{3}-3}{6}$

Ans: $\frac{\sqrt{3}-3}{6}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 1$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$3\cot\theta = 2(\frac{1}{\sqrt{3}}) - 1(\sqrt{3})$

$\cot\theta = -\frac{\sqrt{3}}{9}$

Ans: $\cot\theta = -\frac{\sqrt{3}}{9}$
