EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 72 m away from the foot of a tower, the angle of elevation of its top is 15°. Find its height. [1 mark]

2. A building is 48 m high. From a point 48 m from its foot, find the angle of elevation of its top. [1 mark]

3. A water tank is 60 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 45°? [2 marks]

4. From the top of a building 45 m high, the angle of depression of a scooter is 15°. What is the angle of elevation of the top of the building seen from the scooter? [1 mark]

5. From the top of a tower 75 m high, the angles of depression of two cars on opposite sides of it, in line with its foot are 15° and 75°. Find the distance between them. [3 marks]

6. Two points 240 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 36 m apart on level ground lie on opposite sides of a pole, in line with its foot. The angles of elevation of its top from them are 60° and 30°. Find its height. [2 marks]

8. Two points A and B, 66 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 60° and 45°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a tree from a point on level ground is 15°. After walking 54 m towards it, the angle becomes 75°. Find the height of the tree. [2 marks]

10. The angle of elevation of the top of a minar is 60°. After walking 30 m towards it, it is 75°. How far is the observer now from the foot of the minar? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 21 m high, it is 15°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 72 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 15°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 4 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 45°. Find the height of the tower. [2 marks]

14. A tower is 64 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 60°. Find the length of the flagstaff. [3 marks]

15. From a point 45 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 30°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 12 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 60°. After walking 75 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $A = 15^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 1$, $\angle ACD = 30^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$72(2-\sqrt{3})$ m
2	45°
3	$60(\sqrt{3}+1)$ m
4	15°
5	300 m
6	$40\sqrt{3}$ m
7	$9\sqrt{3}$ m
8	$h = 33(3-\sqrt{3})$ m, $AF = 33(\sqrt{3}-1)$ m, $BF = 33(3-\sqrt{3})$ m
9	$9\sqrt{3}$ m
10	$15\sqrt{3}$ m
11	$\frac{21(\sqrt{3}+3)}{4}$ m
12	$h = 12(2\sqrt{3}+3)$ m, distance $= 12\sqrt{3}$ m
13	$2(\sqrt{3}+1)$ m
14	$128(\sqrt{3}+1)$ m
15	$45(\sqrt{3}+1)$ m
16	$h = 24$ m, $x = 12\sqrt{3}$ m
17	120°
18	$\frac{225(\sqrt{6}+\sqrt{2})}{4}$ m
19	$\sqrt{3}$
20	$\cot\theta = \frac{-1+2\sqrt{3}}{3}$

Solutions

1. From a point 72 m away from the foot of a tower, the angle of elevation of its top is 15°. Find its height. [1]

$\tan 15^\circ = \frac{h}{72}$

$h = 72 \tan 15^\circ = 72 \times (2 - \sqrt{3}) = 72(2 - \sqrt{3})$ m

Ans: $72(2 - \sqrt{3})$ m

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2. A building is 48 m high. From a point 48 m from its foot, find the angle of elevation of its top. [1]

$\tan \theta = \frac{48}{48} = 1$

$\theta = 45^\circ$

Ans: 45°

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3. A water tank is 60 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 45°? [2]

Shadow length $= h \cot(\text{elevation})$.

$60 \cot 15^\circ - 60 \cot 45^\circ = 60((2 + \sqrt{3}) - 1)$

$= 60(\sqrt{3} + 1)$ m

Ans: $60(\sqrt{3} + 1)$ m

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4. From the top of a building 45 m high, the angle of depression of a scooter is 15°. What is the angle of elevation of the top of the building seen from the scooter? [1]

The horizontal lines at the two heights are parallel.

Angle of elevation from below = angle of depression from above $= 15^\circ$.

Ans: 15°

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5. From the top of a tower 75 m high, the angles of depression of two cars on opposite sides of it, in line with its foot are 15° and 75°. Find the distance between them. [3]

distances from the foot: $75 \cot 15^\circ = 75(\sqrt{3} + 2)$, $75 \cot 75^\circ = 75(2 - \sqrt{3})$

Opposite sides: add.

distance $= 300$ m

Ans: 300 m

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6. Two points 240 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 120 m from it.

$h = 120 \tan 30^\circ = 40\sqrt{3}$ m

Ans: $40\sqrt{3}$ m

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7. Two points 36 m apart on level ground lie on opposite sides of a pole, in line with its foot. The angles of elevation of its top from them are 60° and 30°. Find its height. [2]

$h = \frac{d}{\cot \alpha + \cot \beta} = \frac{36}{\frac{1}{\sqrt{3}} + \sqrt{3}}$

$= 9\sqrt{3}$ m

Ans: $9\sqrt{3}$ m

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8. Two points A and B, 66 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 60° and 45°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 60^\circ = \frac{1}{\sqrt{3}}\,h,\ BF = h\cot 45^\circ = 1\,h$

$AF + BF = 66 \Rightarrow h(\frac{1}{\sqrt{3}} + 1) = 66$

$h = \frac{66}{\frac{1}{\sqrt{3}}+1} = 33(3-\sqrt{3})$ m

$AF = 33(\sqrt{3}-1)$ m, $BF = 33(3-\sqrt{3})$ m

Ans: $h = 33(3-\sqrt{3})$ m, $AF = 33(\sqrt{3}-1)$ m, $BF = 33(3-\sqrt{3})$ m

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9. The angle of elevation of the top of a tree from a point on level ground is 15°. After walking 54 m towards it, the angle becomes 75°. Find the height of the tree. [2]

$h\cot 15^\circ - h\cot 75^\circ = 54$

$h = \frac{54}{(2+\sqrt{3})-(2-\sqrt{3})} = 9\sqrt{3}$ m

Ans: $9\sqrt{3}$ m

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10. The angle of elevation of the top of a minar is 60°. After walking 30 m towards it, it is 75°. How far is the observer now from the foot of the minar? [3]

$h = \frac{30}{\frac{1}{\sqrt{3}}-(2-\sqrt{3})} = 15(2\sqrt{3}+3)$

$\text{distance} = h\cot 75^\circ = 15(2\sqrt{3}+3)\times(2-\sqrt{3}) = 15\sqrt{3}$ m

Ans: $15\sqrt{3}$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 21 m high, it is 15°. Find the height of the tower. [2]

$h = x\tan 60^\circ,\ h - 21 = x\tan 15^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{21\times(2+\sqrt{3})}{(2+\sqrt{3})-\frac{1}{\sqrt{3}}}$

$= \frac{21(\sqrt{3}+3)}{4}$ m

Ans: $\frac{21(\sqrt{3}+3)}{4}$ m

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12. A is a point on level ground and B is 72 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 15°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 75^\circ = (2+\sqrt{3})\,x$

$h - 72 = x\tan 15^\circ = (2-\sqrt{3})\,x$

Subtracting: $72 = x((2+\sqrt{3})-(2-\sqrt{3}))$

$x = 12\sqrt{3}$ m

$h = 12(2\sqrt{3}+3)$ m

Ans: $h = 12(2\sqrt{3}+3)$ m, **distance** $= 12\sqrt{3}$ m

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13. A flagstaff 4 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 45°. Find the height of the tower. [2]

$h = x\tan 30^\circ,\ h+4 = x\tan 45^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{4\times 1}{\sqrt{3}-1}$

$= 2(\sqrt{3}+1)$ m

Ans: $2(\sqrt{3}+1)$ m

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14. A tower is 64 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 15° and 60°. Find the length of the flagstaff. [3]

$x = 64\cot 15^\circ = 64(\sqrt{3}+2)$

$64 + l = x\tan 60^\circ = 64(\sqrt{3}+2)\times\sqrt{3} = 64(2\sqrt{3}+3)$

$l = 128(\sqrt{3}+1)$ m

Ans: $128(\sqrt{3}+1)$ m

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15. From a point 45 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 30°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{45\sin 45^\circ}{\sin 15^\circ}$

$= 45(\sqrt{3}+1)$ m

Ans: $45(\sqrt{3}+1)$ m

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16. An observer is 12 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 12 = x\tan 30^\circ,\ h + 12 = x\tan 60^\circ$

Subtracting: $24 = x(\sqrt{3} - \frac{1}{\sqrt{3}})$

$x = 12\sqrt{3}$ m

$h = 12 + x\tan 30^\circ = 24$ m

Ans: $h = 24$ m, $x = 12\sqrt{3}$ m

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17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1]

The angle at Q is $180^\circ - (\gamma - \beta)$.

$= 120^\circ$

Ans: 120°

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18. The angle of elevation of the top of a tower from a point is 60°. After walking 75 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:
$\angle TPQ = 60^\circ - 15^\circ = 45^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$
$\angle PTQ = 75^\circ - 60^\circ = 15^\circ$
Sine rule: $PT = \frac{75 \sin 60^\circ}{\sin 15^\circ}$
$h = PT \sin 60^\circ = \frac{75 \sin 60^\circ \sin 60^\circ}{\sin 15^\circ}$
$= \frac{225(\sqrt{6}+\sqrt{2})}{4}$ m
Ans: $\frac{225(\sqrt{6}+\sqrt{2})}{4}$ m

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19. In triangle ABC, D is the midpoint of AB, $A = 15^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = n\cot A - m\cot B,\ m = n = 1$
$2\cot\theta = (2+\sqrt{3}) - (2-\sqrt{3}) \Rightarrow \cot\theta = \sqrt{3}$
Ans: $\sqrt{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 2:1$, $\angle ACD = 30^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
$3\cot\theta = 2(\sqrt{3}) - 1(1)$
$\cot\theta = \frac{-1+2\sqrt{3}}{3}$
Ans: $\cot\theta = \frac{-1+2\sqrt{3}}{3}$
