EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 36 m away from the foot of a building, the angle of elevation of its top is 60°. Find its height. [1 mark]

2. A building is 12 m high. From a point $4\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A building is 33 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 75°? [2 marks]

4. From the top of a cliff 30 m high, the angle of depression of a boat is 75°. How far is the boat from the foot of the cliff? [1 mark]

5. From the top of a cliff 120 m high, the angles of depression of two boats on the same side of it, in line with its foot are 60° and 75°. Find the distance between them. [3 marks]

6. Two points 144 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 66 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 75° and 30°. Find its height. [2 marks]

8. Two points A and B, 36 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 45° and 15°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a tower from a point on level ground is 30°. After walking 90 m towards it, the angle becomes 45°. Find the height of the tower. [2 marks]

10. A pole is 72 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 30°? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 33 m high, it is 45°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 90 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 45°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 12 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 60° and 75°. Find the height of the tower. [2 marks]

14. A tower is 21 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 45°. Find the length of the flagstaff. [3 marks]

15. From a point 90 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 120 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 30°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 45°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 30°. After walking 54 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 45°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 1 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$36\sqrt{3}\ \text{m}$
2	60°
3	$66\sqrt{3}\ \text{m}$
4	$30(2-\sqrt{3})\ \text{m}$
5	$80(2\sqrt{3}-3)\ \text{m}$
6	$24\sqrt{3}\ \text{m}$
7	$33\ \text{m}$
8	$h = 6(3-\sqrt{3})$ m, $AF = 6(3-\sqrt{3})$ m, $BF = 6(\sqrt{3}+3)$ m
9	$45(\sqrt{3}+1)\ \text{m}$
10	$144\ \text{m}$
11	$\frac{33(\sqrt{3}+3)}{2}\ \text{m}$
12	$h = 45(\sqrt{3}+3)$ m, distance $= 45(\sqrt{3}+1)$ m
13	$6\sqrt{3}\ \text{m}$
14	$21(\sqrt{3}-1)\ \text{m}$
15	$90(\sqrt{3}+2)\ \text{m}$
16	$h = 120(\sqrt{3}+1)$ m, $x = 120(2\sqrt{3}+3)$ m
17	15°
18	$\frac{27(\sqrt{6}+\sqrt{2})}{2}\ \text{m}$
19	$-\frac{\sqrt{3}}{3}$
20	$\cot\theta = -\frac{2\sqrt{3}}{3}$

Solutions

1. From a point 36 m away from the foot of a building, the angle of elevation of its top is 60°. Find its height. [1]

$\tan 60^\circ = \frac{h}{36}$

$h = 36 \tan 60^\circ = 36 \times \sqrt{3} = 36\sqrt{3}$ m

Ans: $36\sqrt{3}$ m

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2. A building is 12 m high. From a point $4\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan \theta = \frac{12}{4\sqrt{3}} = \sqrt{3}$

$\theta = 60^\circ$

Ans: 60°

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3. A building is 33 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 75°? [2]

Shadow length $= h \cot(\text{elevation})$.

$33 \cot 15^\circ - 33 \cot 75^\circ = 33((2+\sqrt{3}) - (2-\sqrt{3}))$

$= 66\sqrt{3}$ m

Ans: $66\sqrt{3}$ m

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4. From the top of a cliff 30 m high, the angle of depression of a boat is 75°. How far is the boat from the foot of the cliff? [1]

The angle of elevation of the top from the boat is also 75°.

$x = 30 \cot 75^\circ = 30 \times (2-\sqrt{3}) = 30(2-\sqrt{3})$ m

Ans: $30(2-\sqrt{3})$ m

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5. From the top of a cliff 120 m high, the angles of depression of two boats on the same side of it, in line with its foot are 60° and 75°. Find the distance between them. [3]

distances from the foot: $120 \cot 60^\circ = 40\sqrt{3},\ 120 \cot 75^\circ = 120(2-\sqrt{3})$

Same side: subtract.

$\text{distance} = 80(2\sqrt{3} - 3)$ m

Ans: $80(2\sqrt{3} - 3)$ m

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6. Two points 144 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 72 m from it.

$h = 72 \tan 30^\circ = 24\sqrt{3}$ m

Ans: $24\sqrt{3}$ m

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7. Two points 66 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 75° and 30°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{66}{(2-\sqrt{3})+\sqrt{3}}$

$= 33$ m

Ans: 33 m

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8. Two points A and B, 36 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 45° and 15°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 45^\circ = 1\,h,\ BF = h\cot 15^\circ = (2+\sqrt{3})\,h$

$AF + BF = 36 \Rightarrow h(1+(2+\sqrt{3})) = 36$

$h = \frac{36}{1+(2+\sqrt{3})} = 6(3-\sqrt{3})$ m

$AF = 6(3-\sqrt{3})$ m, $BF = 6(\sqrt{3}+3)$ m

Ans: $h = 6(3-\sqrt{3})$ **m,** $AF = 6(3-\sqrt{3})$ **m,** $BF = 6(\sqrt{3}+3)$ **m**

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9. The angle of elevation of the top of a tower from a point on level ground is 30°. After walking 90 m towards it, the angle becomes 45°. Find the height of the tower. [2]

$h\cot 30^\circ - h\cot 45^\circ = 90$

$h = \frac{90}{\sqrt{3}-1} = 45(\sqrt{3}+1)$ m

Ans: $45(\sqrt{3}+1)$ m

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10. A pole is 72 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 30°? [3]

$\text{distance} = h\cot 15^\circ - h\cot 30^\circ = 72((2+\sqrt{3}) - \sqrt{3})$

$= 144$ m

Ans: 144 m

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11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 33 m high, it is 45°. Find the height of the tower. [2]

$h = x\tan 60^\circ,\ h - 33 = x\tan 45^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{33\times 1}{1-\frac{1}{\sqrt{3}}}$

$= \frac{33(\sqrt{3}+3)}{2}$ m

Ans: $\frac{33(\sqrt{3}+3)}{2}$ m

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12. A is a point on level ground and B is 90 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 45°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 60^\circ = \sqrt{3}\,x$

$h - 90 = x\tan 45^\circ = 1\,x$

Subtracting: $90 = x(\sqrt{3}-1)$

$x = 45(\sqrt{3}+1)$ m

$h = 45(\sqrt{3}+3)$ m

Ans: $h = 45(\sqrt{3}+3)$ **m, distance** $= 45(\sqrt{3}+1)$ **m**

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13. A flagstaff 12 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 60° and 75°. Find the height of the tower. [2]

$h = x\tan 60^\circ,\ h + 12 = x\tan 75^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{12\times(2-\sqrt{3})}{\frac{1}{\sqrt{3}}-(2-\sqrt{3})}$

$= 6\sqrt{3}\ \text{m}$

Ans: $6\sqrt{3}\ \text{m}$

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14. A tower is 21 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 45°. Find the length of the flagstaff. [3]

$x = 21\cot 30^\circ = 21\sqrt{3}$

$21 + l = x\tan 45^\circ = 21\sqrt{3}\times 1 = 21\sqrt{3}$

$l = 21(\sqrt{3}-1)\ \text{m}$

Ans: $21(\sqrt{3}-1)\ \text{m}$

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15. From a point 90 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{90\sin 75^\circ}{\sin 15^\circ}$

$= 90(\sqrt{3}+2)\ \text{m}$

Ans: $90(\sqrt{3}+2)\ \text{m}$

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16. An observer is 120 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 30°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 120 = x\tan 15^\circ,\ h + 120 = x\tan 30^\circ$

Subtracting: $240 = x(\frac{1}{\sqrt{3}} - (2-\sqrt{3}))$

$x = 120(2\sqrt{3}+3)\ \text{m}$

$h = 120 + x\tan 15^\circ = 120(\sqrt{3}+1)\ \text{m}$

Ans: $h = 120(\sqrt{3}+1)$ m, $x = 120(2\sqrt{3}+3)$ m

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17. From P the elevation of the top T of a tower is 30°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 45°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1]

The angle at P is $\alpha - \beta$.

$= 15^\circ$

Ans: 15°

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18. The angle of elevation of the top of a tower from a point is 30°. After walking 54 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 45°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:
$\angle TPQ = 30^\circ - 15^\circ = 15^\circ,\ \angle PQT = 180^\circ - (45^\circ - 15^\circ) = 150^\circ$
$\angle PTQ = 45^\circ - 30^\circ = 15^\circ$
Sine rule: $PT = \frac{54 \sin 30^\circ}{\sin 15^\circ}$
$h = PT \sin 30^\circ = \frac{54 \sin 30^\circ \sin 30^\circ}{\sin 15^\circ}$
$= \frac{27(\sqrt{6}+\sqrt{2})}{2}$ m
Ans: $\frac{27(\sqrt{6}+\sqrt{2})}{2}$ m

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19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta,\ m = n = 1$
$2\cot\theta = \frac{1}{\sqrt{3}} - \sqrt{3} \Rightarrow \cot\theta = -\frac{\sqrt{3}}{3}$
Ans: $-\frac{\sqrt{3}}{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 1 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
$4\cot\theta = 1(\frac{1}{\sqrt{3}}) - 3(\sqrt{3})$
$\cot\theta = -\frac{2\sqrt{3}}{3}$
Ans: $\cot\theta = -\frac{2\sqrt{3}}{3}$
