EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 25 m away from the foot of a temple gopuram, the angle of elevation of its top is 75°. Find its height. [1 mark]

2. A pole is 64 m high. From a point $\frac{64\sqrt{3}}{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A building is 54 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 45°? [2 marks]

4. From the top of a building 25 m high, the angle of depression of a scooter is 75°. How far is the scooter from the foot of the building? [1 mark]

5. From the top of a lighthouse 84 m high, the angles of depression of two ships on opposite sides of it, in line with its foot are 15° and 45°. Find the distance between them. [3 marks]

6. Two points 48 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 45 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 15° and 60°. Find its height. [2 marks]

8. Two points A and B, 60 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 60°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a chimney from a point on level ground is 30°. After walking 24 m towards it, the angle becomes 60°. Find the height of the chimney. [2 marks]

10. The angle of elevation of the top of a pole is 15°. After walking 18 m towards it, it is 75°. How far is the observer now from the foot of the pole? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 75 m high, it is 45°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 25 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 30°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 6 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 45° and 75°. Find the height of the tower. [2 marks]

14. A tower is 12 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3 marks]

15. From a point 60 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 48 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 45°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 45°. After walking 25 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $A = 60^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 45^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$25(\sqrt{3}+2)$ m
2	60°
3	$54(\sqrt{3}+1)$ m
4	$25(2-\sqrt{3})$ m
5	$84(\sqrt{3}+3)$ m
6	$8\sqrt{3}$ m
7	$\frac{45(2\sqrt{3}-3)}{2}$ m
8	$h = 15(\sqrt{3}+3)$ m, $AF = 15(3-\sqrt{3})$ m, $BF = 15(\sqrt{3}+1)$ m
9	$12\sqrt{3}$ m
10	$3(2\sqrt{3}-3)$ m
11	$\frac{75(\sqrt{3}+1)}{2}$ m
12	$h = \frac{75}{2}$ m, distance $= \frac{25\sqrt{3}}{2}$ m
13	$3(\sqrt{3}-1)$ m
14	$24(\sqrt{3}+1)$ m
15	$60\sqrt{3}$ m
16	$h = 48(\sqrt{3}+2)$ m, $x = 48(\sqrt{3}+3)$ m
17	30°
18	$\frac{25\sqrt{6}}{2}$ m
19	$\frac{2\sqrt{3}-3}{3}$
20	$\cot\theta = -\frac{1}{5}$

Solutions

1. From a point 25 m away from the foot of a temple gopuram, the angle of elevation of its top is 75°. Find its height. [1]

$\tan 75^\circ = \frac{h}{25}$

$h = 25 \tan 75^\circ = 25 \times (2+\sqrt{3}) = 25(\sqrt{3}+2)$ m

Ans: $25(\sqrt{3}+2)$ m

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2. A pole is 64 m high. From a point $\frac{64\sqrt{3}}{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{64}{\frac{64\sqrt{3}}{3}} = \sqrt{3}$

$\theta = 60^\circ$

Ans: 60°

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3. A building is 54 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 45°? [2]

Shadow length $= h\cot(\text{elevation})$.

$54\cot 15^\circ - 54\cot 45^\circ = 54((2+\sqrt{3}) - 1)$

$= 54(\sqrt{3}+1)$ m

Ans: $54(\sqrt{3}+1)$ m

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4. From the top of a building 25 m high, the angle of depression of a scooter is 75°. How far is the scooter from the foot of the building? [1]

The angle of elevation of the top from the scooter is also 75°.

$x = 25\cot 75^\circ = 25 \times (2-\sqrt{3}) = 25(2-\sqrt{3})$ m

Ans: $25(2-\sqrt{3})$ m

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5. From the top of a lighthouse 84 m high, the angles of depression of two ships on opposite sides of it, in line with its foot are 15° and 45°. Find the distance between them. [3]

distances from the foot: $84\cot 15^\circ = 84(\sqrt{3}+2)$, $84\cot 45^\circ = 84$

Opposite sides: add.

distance $= 84(\sqrt{3}+3)$ m

Ans: $84(\sqrt{3}+3)$ m

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6. Two points 48 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 24 m from it.

$h = 24\tan 30^\circ = 8\sqrt{3}$ m

Ans: $8\sqrt{3}$ m

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7. Two points 45 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 15° and 60°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{45}{(2+\sqrt{3})+\frac{1}{\sqrt{3}}}$

$= \frac{45(2\sqrt{3}-3)}{2}$ m

Ans: $\frac{45(2\sqrt{3}-3)}{2}$ m

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8. Two points A and B, 60 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 60°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 75^\circ = (2-\sqrt{3})\,h,\ BF = h\cot 60^\circ = \frac{1}{\sqrt{3}}\,h$

$AF + BF = 60 \Rightarrow h((2-\sqrt{3}) + \frac{1}{\sqrt{3}}) = 60$

$h = \frac{60}{(2-\sqrt{3})+\frac{1}{\sqrt{3}}} = 15(\sqrt{3}+3)$ m

$AF = 15(3-\sqrt{3})$ m, $BF = 15(\sqrt{3}+1)$ m

Ans: $h = 15(\sqrt{3}+3)$ m, $AF = 15(3-\sqrt{3})$ m, $BF = 15(\sqrt{3}+1)$ m

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9. The angle of elevation of the top of a chimney from a point on level ground is 30°. After walking 24 m towards it, the angle becomes 60°. Find the height of the chimney. [2]

$h\cot 30^\circ - h\cot 60^\circ = 24$

$h = \frac{24}{\sqrt{3}-\frac{1}{\sqrt{3}}} = 12\sqrt{3}$ m

Ans: $12\sqrt{3}$ m

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10. The angle of elevation of the top of a pole is 15°. After walking 18 m towards it, it is 75°. How far is the observer now from the foot of the pole? [3]

$h = \frac{18}{(2+\sqrt{3})-(2-\sqrt{3})} = 3\sqrt{3}$

$\text{distance} = h\cot 75^\circ = 3\sqrt{3}\times(2-\sqrt{3}) = 3(2\sqrt{3}-3)$ m

Ans: $3(2\sqrt{3}-3)$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 75 m high, it is 45°. Find the height of the tower. [2]

$h = x\tan 75^\circ,\ h - 75 = x\tan 45^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{75\times 1}{1-(2-\sqrt{3})}$

$= \frac{75(\sqrt{3}+1)}{2}$ m

Ans: $\frac{75(\sqrt{3}+1)}{2}$ m

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12. A is a point on level ground and B is 25 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 30°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 60^\circ = \sqrt{3}\,x$

$h - 25 = x\tan 30^\circ = \frac{1}{\sqrt{3}}\,x$

Subtracting: $25 = x(\sqrt{3} - \frac{1}{\sqrt{3}})$

$x = \frac{25\sqrt{3}}{2}$ m

$h = \frac{75}{2}$ m

Ans: $h = \frac{75}{2}$ **m, distance** $= \frac{25\sqrt{3}}{2}$ **m**

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13. A flagstaff 6 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 45° and 75°. Find the height of the tower. [2]

$h = x\tan 45^\circ,\ h + 6 = x\tan 75^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{6\times(2-\sqrt{3})}{1-(2-\sqrt{3})}$

$= 3(\sqrt{3} - 1)$ m

Ans: $3(\sqrt{3} - 1)$ m

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14. A tower is 12 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3]

$x = 12\cot 30^\circ = 12\sqrt{3}$

$12 + l = x\tan 75^\circ = 12\sqrt{3}\times(2+\sqrt{3}) = 12(2\sqrt{3}+3)$

$l = 24(\sqrt{3}+1)$ m

Ans: $24(\sqrt{3}+1)$ m

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15. From a point 60 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{60\sin 60^\circ}{\sin 30^\circ}$

$= 60\sqrt{3}$ m

Ans: $60\sqrt{3}$ m

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16. An observer is 48 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 45°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 48 = x\tan 30^\circ,\ h + 48 = x\tan 45^\circ$

Subtracting: $96 = x(1 - \frac{1}{\sqrt{3}})$

$x = 48(\sqrt{3}+3)$ m

$h = 48 + x\tan 30^\circ = 48(\sqrt{3}+2)$ m

Ans: $h = 48(\sqrt{3}+2)$ **m,** $x = 48(\sqrt{3}+3)$ **m**

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17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1]

The angle at T is $\gamma - \alpha$.

$= 30^\circ$

Ans: 30°

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18. The angle of elevation of the top of a tower from a point is 45°. After walking 25 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:

$\angle TPQ = 45^\circ - 15^\circ = 30^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$

$\angle PTQ = 75^\circ - 45^\circ = 30^\circ$

Sine rule: $PT = \frac{25 \sin 60^\circ}{\sin 30^\circ}$

$h = PT \sin 45^\circ = \frac{25 \sin 60^\circ \sin 45^\circ}{\sin 30^\circ}$

$= \frac{25\sqrt{6}}{2}$ m

Ans: $\frac{25\sqrt{6}}{2}$ m

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19. In triangle ABC, D is the midpoint of AB, $A = 60^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = n\cot A - m\cot B,\ m = n = 1$

$2\cot\theta = \frac{1}{\sqrt{3}} - (2 - \sqrt{3}) \Rightarrow \cot\theta = \frac{2\sqrt{3}-3}{3}$

Ans: $\frac{2\sqrt{3}-3}{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 45^\circ$ and $\angle DCB = 45^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$5\cot\theta = 2(1) - 3(1)$

$\cot\theta = -\frac{1}{5}$

Ans: $\cot\theta = -\frac{1}{5}$
