EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 66 m away from the foot of a minar, the angle of elevation of its top is 30°. Find its height. [1 mark]

2. A tree is 66 m high. From a point 66 m from its foot, find the angle of elevation of its top. [1 mark]

3. A pole is 72 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 30°? [2 marks]

4. From the top of a hill 27 m high, the angle of depression of a hut is 60°. What is the angle of elevation of the top of the hill seen from the hut? [1 mark]

5. From the top of a tower 30 m high, the angles of depression of two cars on the same side of it, in line with its foot are 15° and 45°. Find the distance between them. [3 marks]

6. Two points 192 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 66 m apart on level ground lie on opposite sides of a chimney, in line with its foot. The angles of elevation of its top from them are 15° and 45°. Find its height. [2 marks]

8. Two points A and B, 96 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 45°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a tower from a point on level ground is 15°. After walking 30 m towards it, the angle becomes 60°. Find the height of the tower. [2 marks]

10. A water tank is 75 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 75°? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 150 m high, it is 60°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 24 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 15°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 18 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 45°. Find the height of the tower. [2 marks]

14. A tower is 66 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 45°. Find the length of the flagstaff. [3 marks]

15. From a point 66 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 30°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 120 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 30°. After walking 25 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 45^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$22\sqrt{3}\ \text{m}$
2	45°
3	$144\ \text{m}$
4	60°
5	$30(\sqrt{3}+1)\ \text{m}$
6	$32\sqrt{3}\ \text{m}$
7	$11(3-\sqrt{3})\ \text{m}$
8	$h = 16(\sqrt{3}+3)$ m, $AF = 16(3-\sqrt{3})$ m, $BF = 16(\sqrt{3}+3)$ m
9	$\frac{15(3-\sqrt{3})}{2}\ \text{m}$
10	$150\sqrt{3}\ \text{m}$
11	$75(\sqrt{3}+2)\ \text{m}$
12	$h = 6(\sqrt{3}+3)$ m, distance $= 6(\sqrt{3}+1)$ m
13	$9(\sqrt{3}-1)\ \text{m}$
14	$66(\sqrt{3}-1)\ \text{m}$
15	$66(\sqrt{3}+1)\ \text{m}$
16	$h = 60(\sqrt{3}+1)$ m, $x = 60(3-\sqrt{3})$ m
17	150°
18	$\frac{25\sqrt{6}}{4}\ \text{m}$
19	$\frac{2\sqrt{3}-3}{3}$
20	$\cot\theta = \frac{3-2\sqrt{3}}{5}$

Solutions

1. From a point 66 m away from the foot of a minar, the angle of elevation of its top is 30°. Find its height. [1]

$\tan 30^\circ = \frac{h}{66}$

$h = 66 \tan 30^\circ = 66 \times \frac{1}{\sqrt{3}} = 22\sqrt{3}$ m

Ans: $22\sqrt{3}$ m

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2. A tree is 66 m high. From a point 66 m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{66}{66} = 1$

$\theta = 45^\circ$

Ans: 45°

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3. A pole is 72 m high. By how much is its shadow longer when the sun's elevation is 15° than when it is 30°? [2]

Shadow length $= h\cot(\text{elevation})$.

$72\cot 15^\circ - 72\cot 30^\circ = 72((2+\sqrt{3}) - \sqrt{3})$

$= 144$ m

Ans: 144 m

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4. From the top of a hill 27 m high, the angle of depression of a hut is 60°. What is the angle of elevation of the top of the hill seen from the hut? [1]

The horizontal lines at the two heights are parallel.

Angle of elevation from below $=$ angle of depression from above $= 60^\circ$.

Ans: 60°

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5. From the top of a tower 30 m high, the angles of depression of two cars on the same side of it, in line with its foot are 15° and 45°. Find the distance between them. [3]

distances from the foot: $30\cot 15^\circ = 30(\sqrt{3}+2)$, $30\cot 45^\circ = 30$

Same side: subtract.

distance $= 30(\sqrt{3}+1)$ m

Ans: $30(\sqrt{3}+1)$ m

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6. Two points 192 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 96 m from it.

$h = 96\tan 30^\circ = 32\sqrt{3}$ m

Ans: $32\sqrt{3}$ m

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7. Two points 66 m apart on level ground lie on opposite sides of a chimney, in line with its foot. The angles of elevation of its top from them are 15° and 45°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{66}{(2+\sqrt{3})+1}$

$= 11(3-\sqrt{3})$ m

Ans: $11(3-\sqrt{3})$ m

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8. Two points A and B, 96 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 45°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 75^\circ = (2-\sqrt{3})\,h,\ BF = h\cot 45^\circ = 1\,h$

$AF + BF = 96 \Rightarrow h((2-\sqrt{3})+1) = 96$

$h = \frac{96}{(2-\sqrt{3})+1} = 16(\sqrt{3}+3)$ m

$AF = 16(3-\sqrt{3})$ m, $BF = 16(\sqrt{3}+3)$ m

Ans: $h = 16(\sqrt{3}+3)$ **m,** $AF = 16(3-\sqrt{3})$ **m,** $BF = 16(\sqrt{3}+3)$ **m**

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9. The angle of elevation of the top of a tower from a point on level ground is 15°. After walking 30 m towards it, the angle becomes 60°. Find the height of the tower. [2]

$h\cot 15^\circ - h\cot 60^\circ = 30$

$h = \frac{30}{(2+\sqrt{3})-\frac{1}{\sqrt{3}}} = \frac{15(3-\sqrt{3})}{2}$ m

Ans: $\frac{15(3-\sqrt{3})}{2}$ m

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10. A water tank is 75 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 75°? [3]

$\text{distance} = h\cot 15^\circ - h\cot 75^\circ = 75((2+\sqrt{3}) - (2-\sqrt{3}))$

$= 150\sqrt{3}$ m

Ans: $150\sqrt{3}$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 150 m high, it is 60°. Find the height of the tower. [2]

$h = x\tan 75^\circ,\ h - 150 = x\tan 60^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{150\times\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}-(2-\sqrt{3})}$

$= 75(\sqrt{3}+2)$ m

Ans: $75(\sqrt{3}+2)$ m

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12. A is a point on level ground and B is 24 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 15°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 60^\circ = \sqrt{3}\,x$

$h - 24 = x\tan 15^\circ = (2-\sqrt{3})\,x$

Subtracting: $24 = x(\sqrt{3} - (2-\sqrt{3}))$

$x = 6(\sqrt{3}+1)$ m

$h = 6(\sqrt{3}+3)$ m

Ans: $h = 6(\sqrt{3}+3)$ **m, distance** $= 6(\sqrt{3}+1)$ **m**

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13. A flagstaff 18 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 15° and 45°. Find the height of the tower. [2]

$h = x\tan 15^\circ,\ h + 18 = x\tan 45^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{18\times 1}{(2+\sqrt{3})-1}$

$= 9(\sqrt{3}-1)$ m

Ans: $9(\sqrt{3}-1)$ m

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14. A tower is 66 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 45°. Find the length of the flagstaff. [3]

$x = 66\cot 30^\circ = 66\sqrt{3}$

$66 + l = x\tan 45^\circ = 66\sqrt{3}\times 1 = 66\sqrt{3}$

$l = 66(\sqrt{3}-1)$ m

Ans: $66(\sqrt{3}-1)$ m

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15. From a point 66 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 30°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{66\sin 45^\circ}{\sin 15^\circ}$

$= 66(\sqrt{3}+1)$ m

Ans: $66(\sqrt{3}+1)$ m

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16. An observer is 120 m above the surface of a lake. The angle of elevation of a cloud is 30° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 120 = x\tan 30^\circ,\ h + 120 = x\tan 75^\circ$

Subtracting: $240 = x((2+\sqrt{3}) - \frac{1}{\sqrt{3}})$

$x = 60(3-\sqrt{3})$ m

$h = 120 + x\tan 30^\circ = 60(\sqrt{3}+1)$ m

Ans: $h = 60(\sqrt{3}+1)$ m, $x = 60(3-\sqrt{3})$ m

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17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle PQT$, at the point reached. Find this angle. [1]

The angle at Q is $180^\circ - (\gamma - \beta)$.

$= 150^\circ$

Ans: 150°

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18. The angle of elevation of the top of a tower from a point is 30°. After walking 25 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:
$\angle TPQ = 30^\circ - 15^\circ = 15^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$
$\angle PTQ = 75^\circ - 30^\circ = 45^\circ$
Sine rule: $PT = \frac{25\sin 60^\circ}{\sin 45^\circ}$
$h = PT\sin 30^\circ = \frac{25\sin 60^\circ \sin 30^\circ}{\sin 45^\circ}$
$= \frac{25\sqrt{6}}{4}$ m
Ans: $\frac{25\sqrt{6}}{4}$ m

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19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 60^\circ$ and $\angle DCB = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta,\ m = n = 1$
$2\cot\theta = \frac{1}{\sqrt{3}} - (2-\sqrt{3}) \Rightarrow \cot\theta = \frac{2\sqrt{3}-3}{3}$
Ans: $\frac{2\sqrt{3}-3}{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 45^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
$5\cot\theta = 3(1) - 2(\sqrt{3})$
$\cot\theta = \frac{3-2\sqrt{3}}{5}$
Ans: $\cot\theta = \frac{3-2\sqrt{3}}{5}$
