EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 66 m away from the foot of a pole, the angle of elevation of its top is 60°. Find its height. [1 mark]

2. A tower is 54 m high. From a point $54\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A building is 30 m high. By how much is its shadow longer when the sun's elevation is 30° than when it is 75°? [2 marks]

4. From the top of a tower 90 m high, the angle of depression of a car is 60°. How far is the car from the foot of the tower? [1 mark]

5. From the top of a cliff 120 m high, the angles of depression of two boats on opposite sides of it, in line with its foot are 30° and 60°. Find the distance between them. [3 marks]

6. Two points 96 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 12 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 30° and 60°. Find its height. [2 marks]

8. Two points A and B, 64 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 30° and 45°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a tree from a point on level ground is 45°. After walking 72 m towards it, the angle becomes 60°. Find the height of the tree. [2 marks]

10. The angle of elevation of the top of a temple gopuram is 30°. After walking 24 m towards it, it is 75°. How far is the observer now from the foot of the temple gopuram? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 45°, and from the top of the building, 48 m high, it is 30°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 45 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 60°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 10 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 45°. Find the height of the tower. [2 marks]

14. A tower is 36 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 60°. Find the length of the flagstaff. [3 marks]

15. From a point 33 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 36 m above the surface of a lake. The angle of elevation of a cloud is 45° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 60°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 60°. After walking 36 m towards the tower up a path inclined at 30° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $A = 30^\circ$ and $B = 60^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 60^\circ$ and $\angle DCB = 60^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$66\sqrt{3}\ \text{m}$
2	30°
3	$60(\sqrt{3}-1)\ \text{m}$
4	$30\sqrt{3}\ \text{m}$
5	$160\sqrt{3}\ \text{m}$
6	$16\sqrt{3}\ \text{m}$
7	$3\sqrt{3}\ \text{m}$
8	$h = 32(\sqrt{3}-1)$ m, $AF = 32(3-\sqrt{3})$ m, $BF = 32(\sqrt{3}-1)$ m
9	$36(\sqrt{3}+3)\ \text{m}$
10	$6(\sqrt{3}-1)\ \text{m}$
11	$24(\sqrt{3}+3)\ \text{m}$
12	$h = \frac{45(\sqrt{3}+2)}{2}$ m, distance $= \frac{45}{2}$ m
13	$5(\sqrt{3}+1)\ \text{m}$
14	$72\ \text{m}$
15	$\frac{33(\sqrt{3}+1)}{2}\ \text{m}$
16	$h = 36\sqrt{3}$ m, $x = 36(\sqrt{3}-1)$ m
17	45°
18	$18(\sqrt{3}+3)\ \text{m}$
19	$\frac{\sqrt{3}}{3}$
20	$\cot\theta = \frac{\sqrt{3}}{15}$

Solutions

1. From a point 66 m away from the foot of a pole, the angle of elevation of its top is 60°. Find its height. [1]

$\tan 60^\circ = \frac{h}{66}$

$h = 66 \tan 60^\circ = 66 \times \sqrt{3} = 66\sqrt{3}$ m

Ans: $66\sqrt{3}$ m

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2. A tower is 54 m high. From a point $54\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{54}{54\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

Ans: 30°

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3. A building is 30 m high. By how much is its shadow longer when the sun's elevation is 30° than when it is 75°? [2]

Shadow length $= h\cot(\text{elevation})$.

$30\cot 30^\circ - 30\cot 75^\circ = 30(\sqrt{3} - (2-\sqrt{3}))$

$= 60(\sqrt{3}-1)$ m

Ans: $60(\sqrt{3}-1)$ m

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4. From the top of a tower 90 m high, the angle of depression of a car is 60°. How far is the car from the foot of the tower? [1]

The angle of elevation of the top from the car is also 60°.

$x = 90\cot 60^\circ = 90 \times \frac{1}{\sqrt{3}} = 30\sqrt{3}$ m

Ans: $30\sqrt{3}$ m

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5. From the top of a cliff 120 m high, the angles of depression of two boats on opposite sides of it, in line with its foot are 30° and 60°. Find the distance between them. [3]

distances from the foot: $120\cot 30^\circ = 120\sqrt{3},\ 120\cot 60^\circ = 40\sqrt{3}$

Opposite sides: add.

distance $= 160\sqrt{3}$ m

Ans: $160\sqrt{3}$ m

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6. Two points 96 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 48 m from it.

$h = 48\tan 30^\circ = 16\sqrt{3}$ m

Ans: $16\sqrt{3}$ m

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7. Two points 12 m apart on level ground lie on opposite sides of a tower, in line with its foot. The angles of elevation of its top from them are 30° and 60°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{12}{\sqrt{3}+\frac{1}{\sqrt{3}}}$

$= 3\sqrt{3}$ m

Ans: $3\sqrt{3}$ m

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8. Two points A and B, 64 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 30° and 45°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 30^\circ = \sqrt{3}\,h,\ BF = h\cot 45^\circ = 1\,h$

$AF + BF = 64 \Rightarrow h(\sqrt{3}+1) = 64$

$h = \frac{64}{\sqrt{3}+1} = 32(\sqrt{3}-1)$ m

$AF = 32(3-\sqrt{3})$ m, $BF = 32(\sqrt{3}-1)$ m

Ans: $h = 32(\sqrt{3}-1)$ m, $AF = 32(3-\sqrt{3})$ m, $BF = 32(\sqrt{3}-1)$ m

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9. The angle of elevation of the top of a tree from a point on level ground is 45°. After walking 72 m towards it, the angle becomes 60°. Find the height of the tree. [2]

$h\cot 45^\circ - h\cot 60^\circ = 72$

$h = \frac{72}{1-\frac{1}{\sqrt{3}}} = 36(\sqrt{3}+3)$ m

Ans: $36(\sqrt{3}+3)$ m

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10. The angle of elevation of the top of a temple gopuram is 30°. After walking 24 m towards it, it is 75°. How far is the observer now from the foot of the temple gopuram? [3]

$h = \frac{24}{\sqrt{3}-(2-\sqrt{3})} = 6(\sqrt{3}+1)$

$\text{distance} = h\cot 75^\circ = 6(\sqrt{3}+1)\times(2-\sqrt{3}) = 6(\sqrt{3}-1)$ m

Ans: $6(\sqrt{3}-1)$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 45°, and from the top of the building, 48 m high, it is 30°. Find the height of the tower. [2]

$h = x\tan 45^\circ,\ h - 48 = x\tan 30^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{48\times\sqrt{3}}{\sqrt{3}-1}$

$= 24(\sqrt{3}+3)$ m

Ans: $24(\sqrt{3}+3)$ m

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12. A is a point on level ground and B is 45 m vertically above A. The angles of elevation of the top of a tower from A and B are 75° and 60°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 75^\circ = (2+\sqrt{3})\,x$

$h - 45 = x\tan 60^\circ = \sqrt{3}\,x$

Subtracting: $45 = x((2+\sqrt{3}) - \sqrt{3})$

$x = \frac{45}{2}$ m

$h = \frac{45(\sqrt{3}+2)}{2}$ m

Ans: $h = \frac{45(\sqrt{3}+2)}{2}$ m, distance $= \frac{45}{2}$ m

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13. A flagstaff 10 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 45°. Find the height of the tower. [2]

$h = x\tan 30^\circ,\ h + 10 = x\tan 45^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{10\times 1}{\sqrt{3}-1}$

$= 5(\sqrt{3}+1)$ m

Ans: $5(\sqrt{3}+1)$ m

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14. A tower is 36 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 60°. Find the length of the flagstaff. [3]

$x = 36\cot 30^\circ = 36\sqrt{3}$

$36 + l = x\tan 60^\circ = 36\sqrt{3}\times\sqrt{3} = 108$

$l = 72$ m

Ans: 72 m

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15. From a point 33 m above a lake, the angle of elevation of a cloud is 30° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{33\sin 105^\circ}{\sin 45^\circ}$

$= \frac{33(\sqrt{3}+1)}{2}$ m

Ans: $\frac{33(\sqrt{3}+1)}{2}$ m

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16. An observer is 36 m above the surface of a lake. The angle of elevation of a cloud is 45° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 36 = x\tan 45^\circ,\ h + 36 = x\tan 75^\circ$

Subtracting: $72 = x((2+\sqrt{3}) - 1)$

$x = 36(\sqrt{3}-1)$ m

$h = 36 + x\tan 45^\circ = 36\sqrt{3}$ m

Ans: $h = 36\sqrt{3}$ m, $x = 36(\sqrt{3}-1)$ m

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17. From P the elevation of the top T of a tower is 60°. Walking up a path inclined at 15° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle TPQ$, at the starting point. Find this angle. [1]

The angle at P is $\alpha - \beta$.

$= 45^\circ$

Ans: 45°

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18. The angle of elevation of the top of a tower from a point is 60°. After walking 36 m towards the tower up a path inclined at 30° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:

$\angle TPQ = 60^\circ - 30^\circ = 30^\circ,\ \angle PQT = 180^\circ - (75^\circ - 30^\circ) = 135^\circ$

$\angle PTQ = 75^\circ - 60^\circ = 15^\circ$

Sine rule: $PT = \frac{36 \sin 45^\circ}{\sin 15^\circ}$

$h = PT \sin 60^\circ = \frac{36 \sin 45^\circ \sin 60^\circ}{\sin 15^\circ}$

$= 18(\sqrt{3} + 3)$ m

Ans: $18(\sqrt{3} + 3)$ m

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19. In triangle ABC, D is the midpoint of AB, $A = 30^\circ$ and $B = 60^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = n\cot A - m\cot B,\ m = n = 1$

$2\cot\theta = \sqrt{3} - \frac{1}{\sqrt{3}} \Rightarrow \cot\theta = \frac{\sqrt{3}}{3}$

Ans: $\frac{\sqrt{3}}{3}$

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20. In triangle ABC, D is a point on AB with $AD : DB = 3 : 2$, $\angle ACD = 60^\circ$ and $\angle DCB = 60^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$5\cot\theta = 3(\frac{1}{\sqrt{3}}) - 2(\frac{1}{\sqrt{3}})$

$\cot\theta = \frac{\sqrt{3}}{15}$

Ans: $\cot\theta = \frac{\sqrt{3}}{15}$
