EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 10

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 15 m away from the foot of a pole, the angle of elevation of its top is 15°. Find its height. [1 mark]

2. A temple gopuram is 54 m high. From a point $18\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1 mark]

3. A minar is 120 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 75°? [2 marks]

4. From the top of a hill 84 m high, the angle of depression of a hut is 60°. How far is the hut from the foot of the hill? [1 mark]

5. From the top of a building 90 m high, the angles of depression of two scooters on opposite sides of it, in line with its foot are 45° and 60°. Find the distance between them. [3 marks]

6. Two points 132 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 45°. Find the height of the tower. [1 mark]

7. Two points 30 m apart on level ground lie on opposite sides of a building, in line with its foot. The angles of elevation of its top from them are 15° and 45°. Find its height. [2 marks]

8. Two points A and B, 36 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 15° and 45°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a pole from a point on level ground is 15°. After walking 42 m towards it, the angle becomes 45°. Find the height of the pole. [2 marks]

10. The angle of elevation of the top of a chimney is 15°. After walking 45 m towards it, it is 60°. How far is the observer now from the foot of the chimney? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 12 m high, it is 30°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 64 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 45°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 12 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 45° and 75°. Find the height of the tower. [2 marks]

14. A tower is 75 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 60° and 75°. Find the length of the flagstaff. [3 marks]

15. From a point 30 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 45 m above the surface of a lake. The angle of elevation of a cloud is 45° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 60°. After walking 18 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $A = 30^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$15(2-\sqrt{3})$ m
2	60°
3	$120(\sqrt{3}-1)$ m
4	$28\sqrt{3}$ m
5	$30(\sqrt{3}+3)$ m
6	66 m
7	$5(3-\sqrt{3})$ m
8	$h = 6(3-\sqrt{3})$ m, $AF = 6(\sqrt{3}+3)$ m, $BF = 6(3-\sqrt{3})$ m
9	$21(\sqrt{3}-1)$ m
10	$\frac{45(\sqrt{3}-1)}{4}$ m
11	18 m
12	$h = 32(\sqrt{3}+3)$ m, distance $= 32(\sqrt{3}+1)$ m
13	$6(\sqrt{3}-1)$ m
14	$50\sqrt{3}$ m
15	$20\sqrt{3}$ m
16	$h = 45\sqrt{3}$ m, $x = 45(\sqrt{3}-1)$ m
17	15°
18	$\frac{27(\sqrt{6}+\sqrt{2})}{2}$ m
19	$\sqrt{3}-1$
20	$\cot\theta = -\frac{7\sqrt{3}}{15}$

Solutions

1. From a point 15 m away from the foot of a pole, the angle of elevation of its top is 15°. Find its height. [1]

$\tan 15^\circ = \frac{h}{15}$

$h = 15\tan 15^\circ = 15 \times (2-\sqrt{3}) = 15(2-\sqrt{3})$ m

Ans: $15(2-\sqrt{3})$ m

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2. A temple gopuram is 54 m high. From a point $18\sqrt{3}$ m from its foot, find the angle of elevation of its top. [1]

$\tan\theta = \frac{54}{18\sqrt{3}} = \sqrt{3}$

$\theta = 60^\circ$

Ans: 60°

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3. A minar is 120 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 75°? [2]

Shadow length $= h\cot(\text{elevation})$.

$120\cot 45^\circ - 120\cot 75^\circ = 120(1-(2-\sqrt{3}))$

$= 120(\sqrt{3}-1)$ m

Ans: $120(\sqrt{3}-1)$ m

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4. From the top of a hill 84 m high, the angle of depression of a hut is 60°. How far is the hut from the foot of the hill? [1]

The angle of elevation of the top from the hut is also 60°.

$x = 84\cot 60^\circ = 84 \times \frac{1}{\sqrt{3}} = 28\sqrt{3}$ m

Ans: $28\sqrt{3}$ m

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5. From the top of a building 90 m high, the angles of depression of two scooters on opposite sides of it, in line with its foot are 45° and 60°. Find the distance between them. [3]

distances from the foot: $90\cot 45^\circ = 90,\ 90\cot 60^\circ = 30\sqrt{3}$

Opposite sides: add.

distance $= 30(\sqrt{3}+3)$ m

Ans: $30(\sqrt{3}+3)$ m

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6. Two points 132 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 45°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 66 m from it.

$h = 66\tan 45^\circ = 66$ m

Ans: 66 m

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7. Two points 30 m apart on level ground lie on opposite sides of a building, in line with its foot. The angles of elevation of its top from them are 15° and 45°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{30}{(2+\sqrt{3})+1}$

$= 5(3-\sqrt{3})$ m

Ans: $5(3-\sqrt{3})$ m

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8. Two points A and B, 36 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 15° and 45°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 15^\circ = (2+\sqrt{3})\,h,\ BF = h\cot 45^\circ = 1\,h$

$AF + BF = 36 \Rightarrow h((2+\sqrt{3})+1) = 36$

$h = \frac{36}{(2+\sqrt{3})+1} = 6(3-\sqrt{3})$ m

$AF = 6(\sqrt{3}+3)$ m, $BF = 6(3-\sqrt{3})$ m

Ans: $h = 6(3-\sqrt{3})$ m, $AF = 6(\sqrt{3}+3)$ m, $BF = 6(3-\sqrt{3})$ m

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9. The angle of elevation of the top of a pole from a point on level ground is 15°. After walking 42 m towards it, the angle becomes 45°. Find the height of the pole. [2]

$h\cot 15^\circ - h\cot 45^\circ = 42$

$h = \frac{42}{(2+\sqrt{3})-1} = 21(\sqrt{3}-1)$ m

Ans: $21(\sqrt{3}-1)$ m

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10. The angle of elevation of the top of a chimney is 15°. After walking 45 m towards it, it is 60°. How far is the observer now from the foot of the chimney? [3]

$h = \frac{45}{(2+\sqrt{3})-\frac{1}{\sqrt{3}}} = \frac{45(3-\sqrt{3})}{4}$

$\text{distance} = h\cot 60^\circ = \frac{45(3-\sqrt{3})}{4} \times \frac{1}{\sqrt{3}} = \frac{45(\sqrt{3}-1)}{4}$ m

Ans: $\frac{45(\sqrt{3}-1)}{4}$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 60°, and from the top of the building, 12 m high, it is 30°. Find the height of the tower. [2]

$h = x\tan 60^\circ,\ h - 12 = x\tan 30^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{12\times\sqrt{3}}{\sqrt{3}-\frac{1}{\sqrt{3}}}$

$= 18$ m

Ans: 18 m

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12. A is a point on level ground and B is 64 m vertically above A. The angles of elevation of the top of a tower from A and B are 60° and 45°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 60^\circ = \sqrt{3}\,x$

$h - 64 = x\tan 45^\circ = 1\,x$

Subtracting: $64 = x(\sqrt{3}-1)$

$x = 32(\sqrt{3}+1)$ m

$h = 32(\sqrt{3}+3)$ m

Ans: $h = 32(\sqrt{3}+3)$ **m, distance** $= 32(\sqrt{3}+1)$ **m**

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13. A flagstaff 12 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 45° and 75°. Find the height of the tower. [2]

$h = x\tan 45^\circ,\ h + 12 = x\tan 75^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{12\times(2-\sqrt{3})}{1-(2-\sqrt{3})}$

$= 6(\sqrt{3}-1)$ m

Ans: $6(\sqrt{3}-1)$ m

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14. A tower is 75 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 60° and 75°. Find the length of the flagstaff. [3]

$x = 75\cot 60^\circ = 25\sqrt{3}$

$75 + l = x\tan 75^\circ = 25\sqrt{3}\times(2+\sqrt{3}) = 25(2\sqrt{3}+3)$

$l = 50\sqrt{3}$ m

Ans: $50\sqrt{3}$ m

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15. From a point 30 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 75°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{30\sin 90^\circ}{\sin 60^\circ}$

$= 20\sqrt{3}$ m

Ans: $20\sqrt{3}$ m

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16. An observer is 45 m above the surface of a lake. The angle of elevation of a cloud is 45° and the angle of depression of its reflection in the lake is 75°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 45 = x\tan 45^\circ,\ h + 45 = x\tan 75^\circ$

Subtracting: $90 = x((2+\sqrt{3})-1)$

$x = 45(\sqrt{3}-1)$ m

$h = 45 + x\tan 45^\circ = 45\sqrt{3}$ m

Ans: $h = 45\sqrt{3}$ **m,** $x = 45(\sqrt{3}-1)$ **m**

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17. From P the elevation of the top T of a tower is 45°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 60°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1]

The angle at T is $\gamma - \alpha$.

$= 15^\circ$

Ans: 15°

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18. The angle of elevation of the top of a tower from a point is 60°. After walking 18 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:

$\angle TPQ = 60^\circ - 15^\circ = 45^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$

$\angle PTQ = 75^\circ - 60^\circ = 15^\circ$

Sine rule: $PT = \frac{18 \sin 60^\circ}{\sin 15^\circ}$

$h = PT \sin 60^\circ = \frac{18 \sin 60^\circ \sin 60^\circ}{\sin 15^\circ}$

$= \frac{27(\sqrt{6}+\sqrt{2})}{2}$ m

Ans: $\frac{27(\sqrt{6}+\sqrt{2})}{2}$ m

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19. In triangle ABC, D is the midpoint of AB, $A = 30^\circ$ and $B = 75^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = n\cot A - m\cot B,\ m = n = 1$

$2\cot\theta = \sqrt{3} - (2 - \sqrt{3}) \Rightarrow \cot\theta = \sqrt{3} - 1$

Ans: $\sqrt{3} - 1$

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20. In triangle ABC, D is a point on AB with $AD : DB = 2 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 30^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$5\cot\theta = 2(\frac{1}{\sqrt{3}}) - 3(\sqrt{3})$

$\cot\theta = -\frac{7\sqrt{3}}{15}$

Ans: $\cot\theta = -\frac{7\sqrt{3}}{15}$
