EXPRESSWALA ACADEMY

Maths 1A: Heights and Distances

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. From a point 72 m away from the foot of a minar, the angle of elevation of its top is 60°. Find its height. [1 mark]

2. A chimney is 36 m high. From a point 36 m from its foot, find the angle of elevation of its top. [1 mark]

3. A building is 96 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 75°? [2 marks]

4. From the top of a bridge 54 m high, the angle of depression of a boat is 30°. How far is the boat from the foot of the bridge? [1 mark]

5. From the top of a building 12 m high, the angles of depression of two scooters on the same side of it, in line with its foot are 30° and 75°. Find the distance between them. [3 marks]

6. Two points 128 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1 mark]

7. Two points 24 m apart on level ground lie on opposite sides of a tree, in line with its foot. The angles of elevation of its top from them are 15° and 30°. Find its height. [2 marks]

8. Two points A and B, 90 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 30°. Find the height of the tower and the distances of A and B from its foot. [5 marks]

9. The angle of elevation of the top of a building from a point on level ground is 45°. After walking 30 m towards it, the angle becomes 60°. Find the height of the building. [2 marks]

10. A tree is 150 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 75°? [3 marks]

11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 30 m high, it is 30°. Find the height of the tower. [2 marks]

12. A is a point on level ground and B is 150 m vertically above A. The angles of elevation of the top of a tower from A and B are 45° and 15°. Find the height of the tower and its horizontal distance from A. [5 marks]

13. A flagstaff 20 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 60°. Find the height of the tower. [2 marks]

14. A tower is 18 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3 marks]

15. From a point 33 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2 marks]

16. An observer is 45 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 30°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5 marks]

17. From P the elevation of the top T of a tower is 60°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1 mark]

18. The angle of elevation of the top of a tower from a point is 30°. After walking 64 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5 marks]

19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 15^\circ$ and $\angle DCB = 30^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1 mark]

20. In triangle ABC, D is a point on AB with $AD : DB = 1 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 60^\circ$. Find $\cot\angle CDB$. [3 marks]

Key sheet

Q	Answer
1	$72\sqrt{3}\ \text{m}$
2	45°
3	$96(\sqrt{3}-1)\ \text{m}$
4	$54\sqrt{3}\ \text{m}$
5	$24(\sqrt{3}-1)\ \text{m}$
6	$\frac{64\sqrt{3}}{3}\ \text{m}$
7	$6(\sqrt{3}-1)\ \text{m}$
8	$h = 45$ m, $AF = 45(2-\sqrt{3})$ m, $BF = 45\sqrt{3}$ m
9	$15(\sqrt{3}+3)\ \text{m}$
10	$300\sqrt{3}\ \text{m}$
11	$\frac{15(\sqrt{3}+3)}{2}\ \text{m}$
12	$h = 75(\sqrt{3}+1)$ m, distance $= 75(\sqrt{3}+1)$ m
13	$10\ \text{m}$
14	$36(\sqrt{3}+1)\ \text{m}$
15	$33\sqrt{3}\ \text{m}$
16	$h = 45(\sqrt{3}+1)$ m, $x = 45(2\sqrt{3}+3)$ m
17	15°
18	$16\sqrt{6}\ \text{m}$
19	1
20	$\cot\theta = -\frac{\sqrt{3}}{6}$

Solutions

1. From a point 72 m away from the foot of a minar, the angle of elevation of its top is 60°. Find its height. [1]

$\tan 60^\circ = \frac{h}{72}$
$h = 72 \tan 60^\circ = 72 \times \sqrt{3} = 72\sqrt{3}$ m
Ans: $72\sqrt{3}$ m

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2. A chimney is 36 m high. From a point 36 m from its foot, find the angle of elevation of its top. [1]

$\tan \theta = \frac{36}{36} = 1$
$\theta = 45^\circ$
Ans: 45°

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3. A building is 96 m high. By how much is its shadow longer when the sun's elevation is 45° than when it is 75°? [2]

Shadow length $= h \cot(\text{elevation})$.
$96 \cot 45^\circ - 96 \cot 75^\circ = 96(1 - (2 - \sqrt{3}))$
$= 96(\sqrt{3} - 1)$ m
Ans: $96(\sqrt{3} - 1)$ m

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4. From the top of a bridge 54 m high, the angle of depression of a boat is 30°. How far is the boat from the foot of the bridge? [1]

The angle of elevation of the top from the boat is also 30°.
$x = 54 \cot 30^\circ = 54 \times \sqrt{3} = 54\sqrt{3}$ m
Ans: $54\sqrt{3}$ m

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5. From the top of a building 12 m high, the angles of depression of two scooters on the same side of it, in line with its foot are 30° and 75°. Find the distance between them. [3]

distances from the foot: $12 \cot 30^\circ = 12\sqrt{3},\ 12 \cot 75^\circ = 12(2 - \sqrt{3})$
Same side: subtract.
distance $= 24(\sqrt{3} - 1)$ m
Ans: $24(\sqrt{3} - 1)$ m

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6. Two points 128 m apart lie on opposite sides of a tower, in line with its foot. The top is seen from both at an elevation of 30°. Find the height of the tower. [1]

The angles are equal, so the foot is midway: each point is 64 m from it.
$h = 64 \tan 30^\circ = \frac{64\sqrt{3}}{3}$ m
Ans: $\frac{64\sqrt{3}}{3}$ m

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7. Two points 24 m apart on level ground lie on opposite sides of a tree, in line with its foot. The angles of elevation of its top from them are 15° and 30°. Find its height. [2]

$h = \frac{d}{\cot\alpha + \cot\beta} = \frac{24}{(2+\sqrt{3})+\sqrt{3}}$
$= 6(\sqrt{3} - 1)$ m
Ans: $6(\sqrt{3} - 1)$ m

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8. Two points A and B, 90 m apart, lie on opposite sides of a tower and in line with its foot. The angles of elevation of the top of the tower from A and B are 75° and 30°. Find the height of the tower and the distances of A and B from its foot. [5]

Let the height be h and the foot F.

$AF = h\cot 75^\circ = (2-\sqrt{3})\,h,\ BF = h\cot 30^\circ = \sqrt{3}\,h$

$AF + BF = 90 \Rightarrow h((2-\sqrt{3})+\sqrt{3}) = 90$

$h = \frac{90}{(2-\sqrt{3})+\sqrt{3}} = 45$ m

$AF = 45(2-\sqrt{3})$ m, $BF = 45\sqrt{3}$ m

Ans: $h = 45$ **m,** $AF = 45(2-\sqrt{3})$ **m,** $BF = 45\sqrt{3}$ **m**

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9. The angle of elevation of the top of a building from a point on level ground is 45°. After walking 30 m towards it, the angle becomes 60°. Find the height of the building. [2]

$h\cot 45^\circ - h\cot 60^\circ = 30$

$h = \frac{30}{1-\frac{1}{\sqrt{3}}} = 15(\sqrt{3}+3)$ m

Ans: $15(\sqrt{3}+3)$ m

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10. A tree is 150 m high. How far must an observer walk towards it for the angle of elevation of its top to change from 15° to 75°? [3]

$\text{distance} = h\cot 15^\circ - h\cot 75^\circ = 150((2+\sqrt{3}) - (2-\sqrt{3}))$

$= 300\sqrt{3}$ m

Ans: $300\sqrt{3}$ m

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11. From the foot of a building the angle of elevation of the top of a tower is 75°, and from the top of the building, 30 m high, it is 30°. Find the height of the tower. [2]

$h = x\tan 75^\circ,\ h - 30 = x\tan 30^\circ$

$h = \frac{d\cot\alpha}{\cot\alpha - \cot\beta} = \frac{30\times\sqrt{3}}{\sqrt{3}-(2-\sqrt{3})}$

$= \frac{15(\sqrt{3}+3)}{2}$ m

Ans: $\frac{15(\sqrt{3}+3)}{2}$ m

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12. A is a point on level ground and B is 150 m vertically above A. The angles of elevation of the top of a tower from A and B are 45° and 15°. Find the height of the tower and its horizontal distance from A. [5]

Let h be the height and x the horizontal distance.

$h = x\tan 45^\circ = 1\,x$

$h - 150 = x\tan 15^\circ = (2-\sqrt{3})\,x$

Subtracting: $150 = x(1-(2-\sqrt{3}))$

$x = 75(\sqrt{3}+1)$ m

$h = 75(\sqrt{3}+1)$ m

Ans: $h = 75(\sqrt{3}+1)$ **m, distance** $= 75(\sqrt{3}+1)$ **m**

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13. A flagstaff 20 m long stands on a tower. From a point on the ground, the angles of elevation of the foot and the top of the flagstaff are 30° and 60°. Find the height of the tower. [2]

$h = x\tan 30^\circ,\ h + 20 = x\tan 60^\circ$

$h = \frac{d\cot\beta}{\cot\alpha - \cot\beta} = \frac{20\times\frac{1}{\sqrt{3}}}{\sqrt{3}-\frac{1}{\sqrt{3}}}$

$= 10\ \text{m}$

Ans: $10\ \text{m}$

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14. A tower is 18 m high and a flagstaff stands on it. From a point on the ground the elevations of the top of the tower and the top of the flagstaff are 30° and 75°. Find the length of the flagstaff. [3]

$x = 18\cot 30^\circ = 18\sqrt{3}$

$18 + l = x\tan 75^\circ = 18\sqrt{3}\times(2+\sqrt{3}) = 18(2\sqrt{3}+3)$

$l = 36(\sqrt{3}+1)\ \text{m}$

Ans: $36(\sqrt{3}+1)\ \text{m}$

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15. From a point 33 m above a lake, the angle of elevation of a cloud is 15° and the angle of depression of its reflection is 45°. Find the height of the cloud above the lake. [2]

$h = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)} = \frac{33\sin 60^\circ}{\sin 30^\circ}$

$= 33\sqrt{3}\ \text{m}$

Ans: $33\sqrt{3}\ \text{m}$

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16. An observer is 45 m above the surface of a lake. The angle of elevation of a cloud is 15° and the angle of depression of its reflection in the lake is 30°. Find the height of the cloud above the lake and its horizontal distance from the observer. [5]

Let the cloud be h m above the lake; its image is h m below the surface. Let x be the horizontal distance.

$h - 45 = x\tan 15^\circ,\ h + 45 = x\tan 30^\circ$

Subtracting: $90 = x\left(\frac{1}{\sqrt{3}} - (2-\sqrt{3})\right)$

$x = 45(2\sqrt{3}+3)\ \text{m}$

$h = 45 + x\tan 15^\circ = 45(\sqrt{3}+1)\ \text{m}$

Ans: $h = 45(\sqrt{3}+1)$ **m,** $x = 45(2\sqrt{3}+3)$ **m**

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17. From P the elevation of the top T of a tower is 60°. Walking up a path inclined at 30° towards the tower to Q, the elevation becomes 75°. In triangle PQT, $\angle PTQ$, at the top of the tower. Find this angle. [1]

The angle at T is $\gamma - \alpha$.

$= 15^\circ$

Ans: 15°

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18. The angle of elevation of the top of a tower from a point is 30°. After walking 64 m towards the tower up a path inclined at 15° to the horizontal, the angle of elevation becomes 75°. Find the height of the tower. [5]

Let P be the start, Q the point reached and T the top. In triangle PQT:
$\angle TPQ = 30^\circ - 15^\circ = 15^\circ,\ \angle PQT = 180^\circ - (75^\circ - 15^\circ) = 120^\circ$
$\angle PTQ = 75^\circ - 30^\circ = 45^\circ$
Sine rule: $PT = \frac{64 \sin 60^\circ}{\sin 45^\circ}$
$h = PT \sin 30^\circ = \frac{64 \sin 60^\circ \sin 30^\circ}{\sin 45^\circ}$
$= 16\sqrt{6}$ m
Ans: $16\sqrt{6}$ m

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19. In triangle ABC, D is the midpoint of AB, $\angle ACD = 15^\circ$ and $\angle DCB = 30^\circ$. If $\theta = \angle CDB$, find $\cot\theta$. [1]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta,\ m = n = 1$
$2\cot\theta = (2+\sqrt{3}) - \sqrt{3} \Rightarrow \cot\theta = 1$
Ans: 1

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20. In triangle ABC, D is a point on AB with $AD : DB = 1 : 3$, $\angle ACD = 60^\circ$ and $\angle DCB = 60^\circ$. Find $\cot\angle CDB$. [3]

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$
$4\cot\theta = 1(\frac{1}{\sqrt{3}}) - 3(\frac{1}{\sqrt{3}})$
$\cot\theta = -\frac{\sqrt{3}}{6}$
Ans: $\cot\theta = -\frac{\sqrt{3}}{6}$
