EXPRESSWALA ACADEMY

Maths 1A: Functions

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{1, 5, 6\}$ and $B = \{p, q, r, t, v\}$, how many functions are there from A to B? [1 mark]

2. Find the range of $f : A \to \mathbb{Z}$, $f(x) = |2x + 1|$, where $A = \{-3, 0, 1, 3, 4\}$. [1 mark]

3. Find the point where the line $y = -x - 8$ meets the x-axis. [1 mark]

4. Find the range of $f(x) = -x^2 - 8x - 9$, $x \in \mathbb{R}$. [2 marks]

5. Solve $x^2 + 2x + 3 > 0$. [2 marks]

6. Solve $|3x - 3| \leq 4$. [2 marks]

7. Find $[-3.5] + [3.5]$. [1 mark]

8. Solve $[x]^2 + [x] - 12 = 0$, where $[x]$ is the greatest integer function. [3 marks]

9. Find the value of $5^{3\log_5 14}$. [1 mark]

10. If $f(x) = 4x + 3$, find $(f \circ f)(x)$. [2 marks]

11. If $f(x) = \dfrac{-4x - 1}{x + 4}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-2)$. [5 marks]

12. Write $f(x) = -4x^3 - 4x^2 + x - 2$ as the sum of an even function and an odd function. [3 marks]

13. If $f(x) = x + 2$ and $g(x) = x^2 - 5$, find $(fg)(3)$ and $\left(\dfrac{f}{g}\right)(5)$. [2 marks]

14. A polynomial f satisfies $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65$. Find $f(-1)$. [3 marks]

15. How many onto functions are there from $A = \{5, 6, 7, 9, 10, 11, 12\}$ to $B = \{d, e\}$? [2 marks]

16. Let $A = \mathbb{R} - \left\{-\frac{1}{3}\right\}$ and $B = \mathbb{R} - \{1\}$. Show that $f : A \to B$, $f(x) = \frac{3x+7}{3x+1}$ is a bijection and find f^{-1}. [5 marks]

17. Find the domain of $f(x) = \sqrt{x^2 + 5x - 6} + \dfrac{1}{x + 9}$. [5 marks]

18. Find the range of $f(x) = \dfrac{x^2 - x + 1}{x^2 - 2x + 2}$, $x \in \mathbb{R}$. [5 marks]

19. The point $(4, -3)$ lies on $y = f(x)$. Which point must lie on $y = f(x + 2)$? [1 mark]

20. Where does the graph of $y = |x^2 - 2x - 8|$ meet the x-axis, and what is the highest point of the arch between these points? [3 marks]

Key sheet

Q	Answer
1	125
2	$\text{Range} = \{1, 3, 5, 7, 9\}$
3	$(-8, 0)$
4	$(-\infty, 7]$
5	$x \in \mathbb{R}$
6	$x \in [-\frac{1}{3}, \frac{7}{3}]$
7	-1
8	$x \in [-4, -3) \cup [3, 4)$
9	2744
10	$(f \circ f)(x) = 16x + 15$
11	$(f \circ f)(x) = x$; $(f \circ f \circ f)(-2) = \frac{7}{2}$
12	Even part $-4x^2 - 2$, odd part $-4x^3 + x$
13	$(fg)(3) = 20$, $\left(\frac{f}{g}\right)(5) = \frac{7}{20}$
14	$f(-1) = 0$
15	126
16	$f^{-1}(x) = \frac{7-x}{3x-3}$
17	$(-\infty, -9) \cup (-9, -6] \cup [1, \infty)$
18	$[\frac{1}{2}, \frac{3}{2}]$
19	$(2, -3)$
20	At $x = -2$ and $x = 4$; the top of the arch is $(1, 9)$

Solutions

1. If $A = \{1, 5, 6\}$ and $B = \{p, q, r, t, v\}$, how many functions are there from A to B? [1]

$n(A) = 3,\ n(B) = 5$

$\text{Number of functions} = \{n(B)\}^{n(A)} = 5^3 = 125$

Ans: 125

---

2. Find the range of $f : A \to \mathbb{Z}$, $f(x) = |2x + 1|$, where $A = \{-3, 0, 1, 3, 4\}$. [1]

$f(-3) = 5,\ f(0) = 1,\ f(1) = 3,\ f(3) = 7,\ f(4) = 9$

Range $= \{1, 3, 5, 7, 9\}$

Ans: $\text{Range} = \{1, 3, 5, 7, 9\}$

---

3. Find the point where the line $y = -x - 8$ meets the x-axis. [1]

$y = 0 \Rightarrow -x - 8 = 0$

$\Rightarrow x = -8$

The point is $(-8, 0)$.

Ans: $(-8, 0)$

---

4. Find the range of $f(x) = -x^2 - 8x - 9$, $x \in \mathbb{R}$. [2]

$a = -1 < 0$, so the parabola opens downward.

$f(x) = -(x + 4)^2 + 7$

The maximum value is 7 at $x = -4$.

Range $= (-\infty, 7]$

Ans: $(-\infty, 7]$

---

5. Solve $x^2 + 2x + 3 > 0$. [2]

$\Delta = -8 < 0$

So the expression has the sign of $a = 1$ for every real x.

Solution set: $\mathbb{R}$

Ans: $x \in \mathbb{R}$

---

6. Solve $|3x - 3| \leq 4$. [2]

$-4 \leq 3x - 3 \leq 4$

Solution set: $[-\frac{1}{3}, \frac{7}{3}]$

Ans: $x \in [-\frac{1}{3}, \frac{7}{3}]$

---

7. Find $[-3.5] + [3.5]$. [1]

$[-3.5] = -4,\ [3.5] = 3$

The sum is -1 (for a non-integer x, $[x] + [-x] = -1$).

Ans: -1

---

8. Solve $[x]^2 + [x] - 12 = 0$, where $[x]$ is the greatest integer function. [3]

Put $t = [x]$: $t^2 + t - 12 = 0$

$(t+4)(t-3) = 0 \Rightarrow t = -4$ or 3

$[x] = -4 \Rightarrow x \in [-4, -3)$, $[x] = 3 \Rightarrow x \in [3, 4)$

So $x \in [-4, -3) \cup [3, 4)$.

Ans: $x \in [-4, -3) \cup [3, 4)$

---

9. Find the value of $5^{3\log_5 14}$. [1]

$5^{3\log_5 14} = 5^{\log_5 14^3}$

$= 14^3 = 2744$ (using $a^{\log_a N} = N$)

Ans: 2744

---

10. If $f(x) = 4x + 3$, find $(f \circ f)(x)$. [2]

$(f \circ f)(x) = f(4x+3) = 4(4x+3) + 3$

$= 16x + 15$

Ans: $(f \circ f)(x) = 16x + 15$

---

11. If $f(x) = \dfrac{-4x-1}{x+4}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-2)$. [5]

$(f \circ f)(x) = f\left(\frac{-4x-1}{x+4}\right) = \dfrac{-4\left(\frac{-4x-1}{x+4}\right) - 1}{1\left(\frac{-4x-1}{x+4}\right) + 4}$

Multiply the numerator and the denominator by $x+4$:

$= \dfrac{(-x-4) + (16x+4)}{(-4x-1) + (4x+16)} = \dfrac{15x}{15}$

$= x$

So $f \circ f = I$, and $(f \circ f \circ f)(x) = f(x)$.

$(f \circ f \circ f)(-2) = f(-2) = \frac{7}{2} = \frac{7}{2}$

Ans: $(f \circ f)(x) = x$; $(f \circ f \circ f)(-2) = \frac{7}{2}$

---

12. Write $f(x) = -4x^3 - 4x^2 + x - 2$ as the sum of an even function and an odd function. [3]

$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$

$f(-x) = 4x^3 - 4x^2 - x - 2$

$\frac{1}{2}[f(x) + f(-x)] = -4x^2 - 2$ (even)

$\frac{1}{2}[f(x) - f(-x)] = -4x^3 + x$ (odd)

$f(x) = \left(-4x^2 - 2\right) + \left(-4x^3 + x\right)$

Ans: Even part $-4x^2 - 2$**, odd part** $-4x^3 + x$

---

13. If $f(x) = x + 2$ and $g(x) = x^2 - 5$, find $(fg)(3)$ and $\left(\dfrac{f}{g}\right)(5)$. [2]

$(fg)(3) = f(3)\, g(3) = (5)(4) = 20$

$\left(\frac{f}{g}\right)(5) = \frac{f(5)}{g(5)} = \frac{7}{20} = \frac{7}{20}$

Ans: $(fg)(3) = 20$, $\left(\frac{f}{g}\right)(5) = \frac{7}{20}$

---

14. A polynomial f satisfies $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65$. Find $f(-1)$. [3]

So $f(x) = 1 \pm x^n$.

$1 + 4^n = 65 \Rightarrow 4^n = 64 \Rightarrow n = 3$

$f(x) = 1 + x^3$

$f(-1) = 1 + (-1)^3 = 0$

Ans: $f(-1) = 0$

---

15. How many onto functions are there from $A = \{5, 6, 7, 9, 10, 11, 12\}$ to $B = \{d, e\}$? [2]

$n(A) = 7,\ n(B) = 2$

$\text{Number of onto functions} = 2^n - 2 = 2^7 - 2 = 126$

Ans: 126

---

16. Let $A = \mathbb{R} - \left\{-\frac{1}{3}\right\}$ and $B = \mathbb{R} - \{1\}$. Show that $f : A \to B$, $f(x) = \frac{3x+7}{3x+1}$ is a bijection and find f^{-1}. [5]

One-one: $f(x_1) = f(x_2) \Rightarrow (3x_1 + 7)(3x_2 + 1) = (3x_2 + 7)(3x_1 + 1)$

$\Rightarrow -18(x_1 - x_2) = 0 \Rightarrow x_1 = x_2$, so f is one-one.

Onto: for $y \in B$, $y = \frac{3x+7}{3x+1} \Rightarrow y(3x + 1) = 3x + 7$

$\Rightarrow x = \frac{7-y}{3y-3}$, defined since $y \neq 1$, and $x \neq -\frac{1}{3}$ (else $3 = 21$).

So $x \in A$ and $f(x) = y$: f is onto. Hence f is a bijection.

$f^{-1}(x) = \frac{7-x}{3x-3}$

Ans: $f^{-1}(x) = \frac{7-x}{3x-3}$

---

17. Find the domain of $f(x) = \sqrt{x^2 + 5x - 6} + \dfrac{1}{x + 9}$. [5]

We need $x^2 + 5x - 6 \geq 0$ and $x + 9 \neq 0$.

$x^2 + 5x - 6 = (x - 1)(x + 6) \geq 0$: the roots are $-6 < 1$ and $a > 0$.

$\Rightarrow x \in (-\infty, -6] \cup [1, \infty)$

$x + 9 \neq 0 \Rightarrow x \neq -9$

Common part: domain $= (-\infty, -9) \cup (-9, -6] \cup [1, \infty)$

Ans: $(-\infty, -9) \cup (-9, -6] \cup [1, \infty)$

---

18. Find the range of $f(x) = \dfrac{x^2 - x + 1}{x^2 - 2x + 2}$, $x \in \mathbb{R}$. [5]

The denominator $x^2 - 2x + 2$ has $\Delta = -4 < 0$, so it is never zero: the domain is $\mathbb{R}$.

$y = \frac{x^2-x+1}{x^2-2x+2} \Rightarrow y(x^2 - 2x + 2) = x^2 - x + 1$

$\Rightarrow (y - 1)x^2 + (1 - 2y)x + (2y - 1) = 0$

For $y \neq 1$, x is real, so $\Delta = (1 - 2y)^2 - 4(y - 1)(2y - 1) \geq 0$

$\Rightarrow -4y^2 + 8y - 3 \geq 0$, $\Rightarrow \frac{1}{2} \leq y \leq \frac{3}{2}$

The value $y = 1$ is taken at $x = 1$, and 1 lies in this interval.

Range $= [\frac{1}{2}, \frac{3}{2}]$

Ans: $[\frac{1}{2}, \frac{3}{2}]$

---

19. The point $(4, -3)$ lies on $y = f(x)$. Which point must lie on $y = f(x + 2)$? [1]

The graph of $y = f(x + 2)$ is obtained by shift left by 2: $(x, y) \to (x - 2, y)$.

So $(4, -3) \to (2, -3)$.

Ans: $(2, -3)$

---

20. Where does the graph of $y = |x^2 - 2x - 8|$ meet the x-axis, and what is the highest point of the arch between these points? [3]

$y = x^2 - 2x - 8$ cuts the x-axis where $(x - 4)(x + 2) = 0$: $x = -2, 4$.

Between them it dips to -9 at $x = 1$.

Taking $|f(x)|$ flips this part up: the arch has top $(1, 9)$.

Ans: At $x = -2$ and $x = 4$; the top of the arch is $(1, 9)$
